α_s from the hadronic width of the Z

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DESY

Introduction

- QCD corrections to $\sigma(\mathrm{e^+e^-} \rightarrow$ (hadrons) are known since long
- At lower energies usually $R = \frac{\sigma(\mathrm{e^+e^-}\rightarrow\mathrm{hadrons})}{\sigma(\mathrm{e^+e^-}\rightarrow\mu^+\mu^-)}$ has been used for α_s determination
- Similar corrections arise on the Z-resonance
- These corrections modify the partial width of the Z decaying to hadrons, Γ_{had}, and via this the total Z-width Γ_{Z}
- Relevant observables:

  $R^0_\ell = \frac{\Gamma_{\mathrm{had}}}{\Gamma_\ell}$**:** Ratio of had. and lept. partial width $\simeq$ ratio of cross sections

  $\sigma^{\mathrm{had}}_0 = \frac{12\pi}{m_Z}\frac{\Gamma_e\Gamma_{\mathrm{had}}}{\Gamma^2_Z}$**:** Hadronic peak cross section, almost insensitive due to cancellation of α_s effects in Γ_{had} and Γ_{Z}

  Γ_{Z}**:** Total Z-width, measured with complementary systematics

  $\sigma^0_\ell = \frac{12\pi}{m_Z}\frac{\Gamma^2_\ell}{\Gamma^2_Z}$**:** Leptonic peak cross section. Very sensitive but included automatically in a fit to R^0_ℓ, σ^{had}_0, ...

Standard Model formulae

- The partial widths have a non-trivial dependence on the other SM parameters
- On Born level one has

$$\Gamma_f = N_c^f \frac{G_f m_Z^3}{6\sqrt{2}\pi}(g_{A,f}^2 + g_{V,f}^2)$$
$$g_{A,f} = T_3^f$$
$$g_{A,f} = g_{A,f}(1 - 4|q_f|\sin^2\theta_W)$$

- On loop level this gets to

$$g_{A,f} \rightarrow \sqrt{1 + \Delta\rho_f g_{A,f}}$$
$$\sin^2\theta_W \rightarrow \sqrt{1 + \Delta\kappa_f \sin^2\theta_W} = \sin^2\theta_{\mathrm{eff}}^f$$

➠ Unknown SM (and BSM) parameters enter prediction of Γ_{had}

The structure of the radiative corrections

- In general the $\Delta\rho_f$ and $\Delta\kappa_f$ are flavour independent apart from small constant terms and some possible contributions to the b-quark observables
- $\sin^2\theta^l_{eff}$ can be measured from various asymmetries at LEP and SLD
- In the SM the only unknown parameter is m_{H} and $\sin^2\theta^l_{eff}$ and m_{W} can be used to constrain it.
- In a more general case $\Delta\rho$ can be obtained from Γ_ℓ and $\sin^2\theta^l_{eff}$ from the asymmetries
- If one also allows for free corrections to the bbZ vertex they can be constrained by R_{b}
- If all vertex corrections are left free nothing can be said

The QCD corrections to Γ_{had}

- The massless corrections are known to 4th order in QCD (P. A. Baikov, K. G. Chetyrkin, J. H. Kühn, arXiv:0801.1821)

$$\Gamma_{\mathrm{had}} = \Gamma_{\mathrm{had}}^{\mathrm{no\,QCD}} \left[1 + \frac{\alpha_s}{\pi} + 1.4 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.7 \left(\frac{\alpha_s}{\pi}\right)^3 - 80.0 \left(\frac{\alpha_s}{\pi}\right)^4 \right]$$

- The massive corrections are known to numerically better precision (K. G. Chetyrkin, J. H. Kühn and A. Kwiatkowski, hep-ph/9503396)
- Also numerically important $\alpha\alpha_s$ terms in the electroweak corrections are taken into account

The Data

- Main input: hadronic and leptonic cross sections at and around the Z-pole from LEP1 (1991-1995)
- Analysis of beam energy in the Z-scans

Express results in minimally correlated parameters:

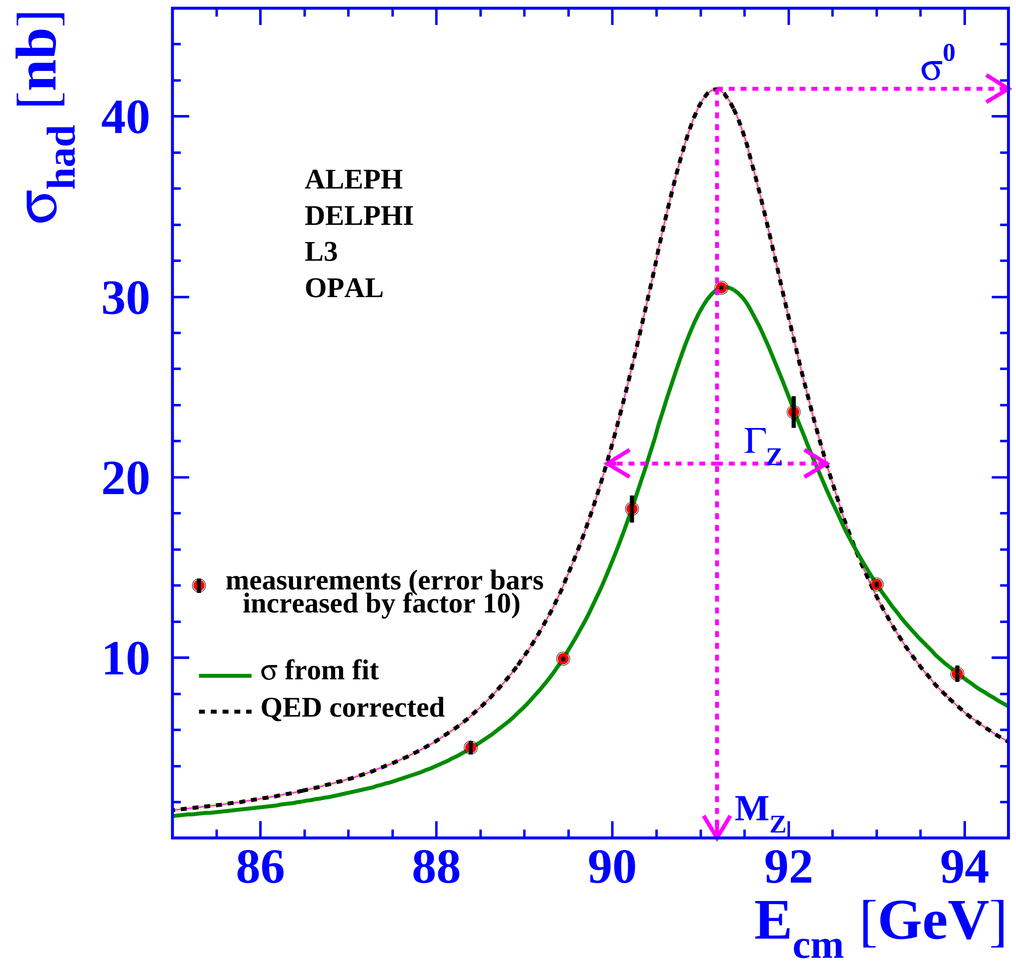
$$m_Z,\ \Gamma_Z$$

$$\sigma_0^{\rm had} = \frac{12\pi}{m_Z}\frac{\Gamma_e\Gamma_{\rm had}}{\Gamma_Z^2}$$

$$R_l = \frac{\Gamma_{\rm had}}{\Gamma_l}$$

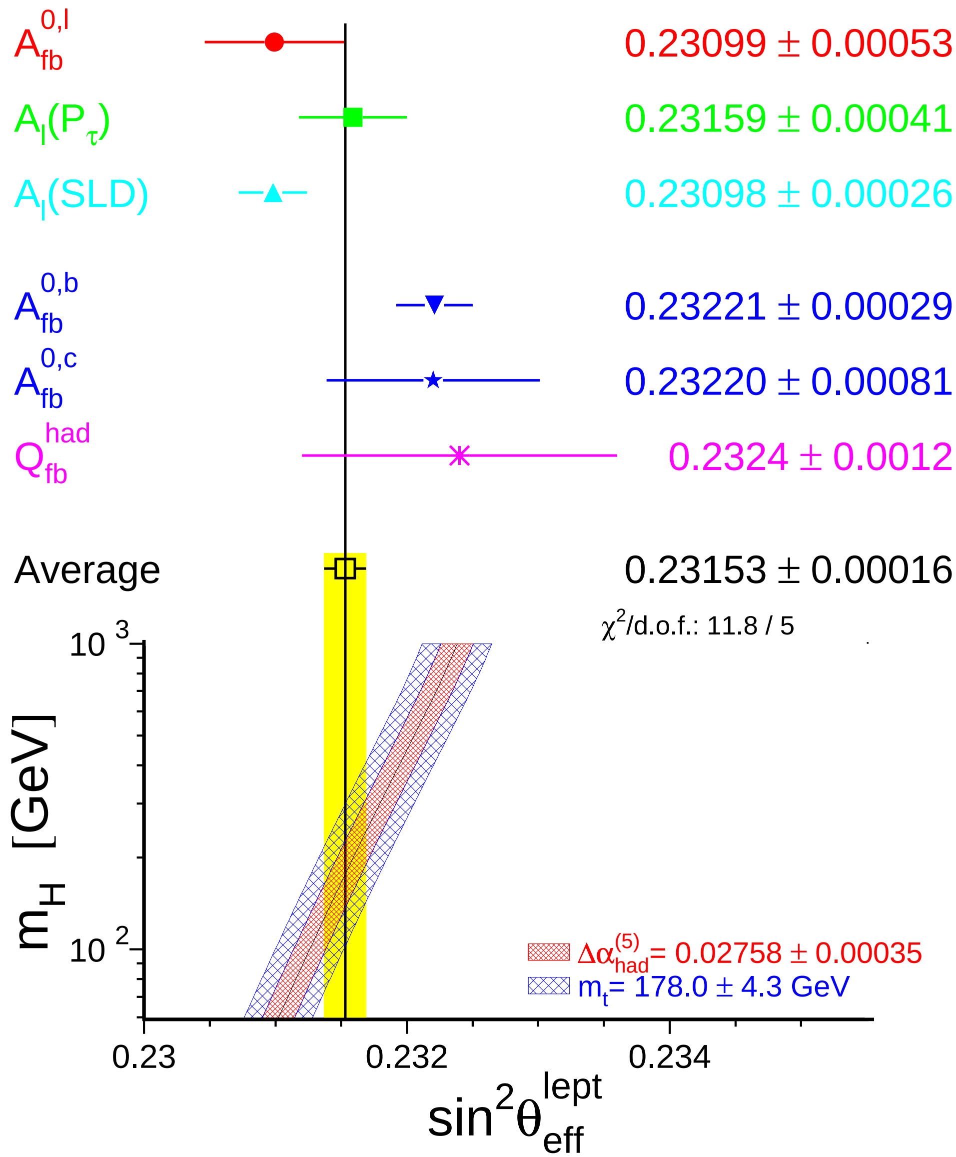
Results:

$$m_Z = 91.1875 \pm 0.0021\,{\rm GeV}$$
$$\Gamma_Z = 2.4952 \pm 0.0023\,{\rm GeV}$$
$$\sigma_0^{\rm had} = 41.540 \pm 0.037\,{\rm nb}$$
$$R_l = 20.767 \pm 0.025$$



Additional inputs to fix $\Delta\rho$ and $\Delta\kappa$

- $\sin^2\theta^l_{eff}$ measurements from LEP/SLD:
  $\sin^2\theta^l_{eff} = 0.23153 \pm 0.00016$
- W-mass from LEP/Tevatron
  $m_{\rm W} = 80.399 \pm 0.023\,{\rm GeV}$
- Top-mass from Tevatron
  $m_{\rm t} = 173.3 \pm 1.1\,{\rm GeV}$
- Hadronic vacuum polarisation in the running of α
  $\Delta\alpha^{(5)}_{\rm had}(m_{\rm Z}) = 0.02749 \pm 0.000098$
- (Higgs mass limit from LEP
  $m_{\rm H} > 114.4\,{\rm GeV}$)



The Fit-Programs

Gfitter:

- Object-oriented fit-program built on root
- All known electroweak and QCD corrections implemented
- SM fits with and without external Higgs constraint
- Also STU fits implemented

ZFITTER:

- ZFITTER is a program that predicts electroweak pseudo-observables (and cross sections with cuts)
- Fits using ZFITTER are implemented by several people
- Contains (almost) all known electroweak corrections
- Contains all QCD corrections apart from α_s^4 term, however implemented in private version

Both programs give consistent results

SM fit results

Without Higgs constraint:

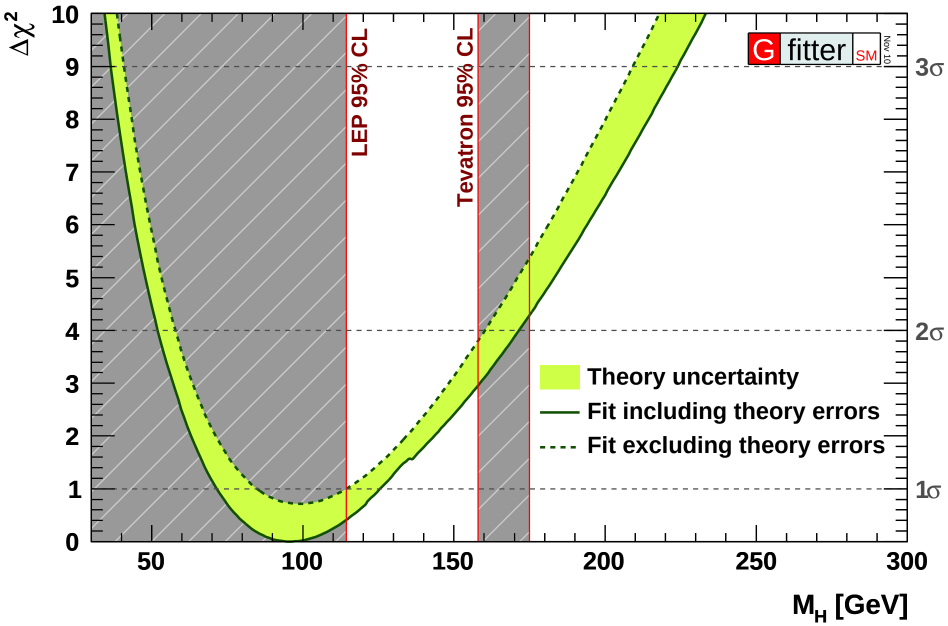
$$m_{\rm H} = 96^{+31}_{-24}\,\text{GeV}$$
$$\alpha_s(m_{\rm Z}) = 0.1192 \pm 0.0028$$

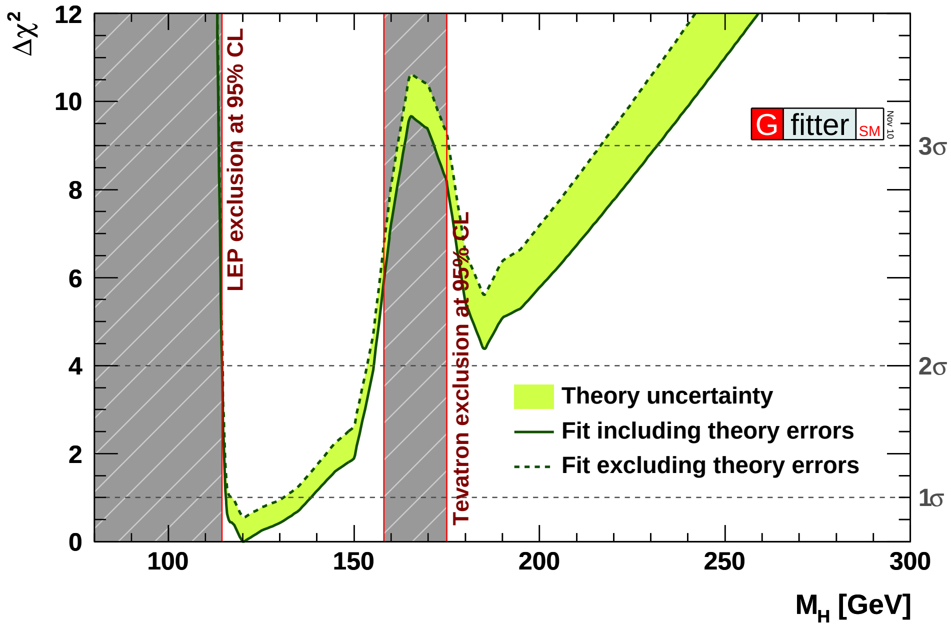
$$\chi^2/ndf = 16.6/13$$
$$\Rightarrow \text{Prob} = 22\%$$

With Higgs constraint:

$$m_{\rm H} = 121^{+18}_{-5}\,\text{GeV}$$
$$\alpha_s(m_{\rm Z}) = 0.1193 \pm 0.0028$$

$$\chi^2/ndf = 17.5/14$$
$$\Rightarrow \text{Prob} = 23\%$$





Uncertainties on α_s

QCD uncertainties:

- Higher orders: assume 5th order term of $k_5 = \frac{k_4}{k_3} k_4$ on Adler function coefficients $\Rightarrow c_5 \sim 50 \Rightarrow \Delta\alpha_s(m_Z) = 0.00002$
- If the same estimate is done on the perturbative series directly uncertainty is a factor 10 larger but still negligible.
- (Dropping the α_s^4 term completely would give $\Delta\alpha_s(m_Z) = 0.00036$)
- Mass corrections $\Rightarrow \Delta\alpha_s(m_Z) = 0.00006$
- QCD uncertainties seem completely negligible

Fit program:

- Repeat fit with ZFITTER (incl. α_s^4-term) $\Rightarrow \Delta\alpha_s(m_Z) = 0.0003$

Electroweak uncertainties:

- Theoretical uncertainties on $\sin^2\theta^l_{eff}$ and m_{W} are already included in fit (and are negligible)
- However there might be uncertainties from unknown physics
- Way out: model independent approach (STU- or ε-parameters)
- Fit with ST instead of m_{H}: $\Delta\alpha_s(m_{\mathrm{Z}}) = 0.0009 \pm 0.0007$
(error is independent statistical error due to less used information)
- Fit with STU: $\Delta\alpha_s(m_{\mathrm{Z}}) = 0.0001 \pm 0.0015$
- Fit with STU and free R_b: $\Delta\alpha_s(m_{\mathrm{Z}}) = 0.0011 \pm 0.0028$
- Of course further free vertex corrections cannot be assumed
- However the ST-fit is safe within most models, including SUSY
- (Fit without $A^{0,\,\mathrm{b}}_{\mathrm{FB}}$: $\Delta\alpha_s(m_{\mathrm{Z}}) = 0.0006$)

Summary

- From a fit to the electroweak precision observables one obtains $\alpha_s(m_Z) = 0.1192 \pm 0.0028$

- Additional QCD corrections are below 0.0001

- The fit is also valid in a large class of BSM models with an extra uncertainty of only 0.0007