α_s with GAPP

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Workshop on Precision Measurements of α_s

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Outline

- GAPP
- α_s from Z decays
- α_s from τ decays
- α_s from the global EW fit

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- *$\overline{MS}$ scheme* used throughout

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- uncertainties: statistical, systematic, parametric and theoretical *errors & correlations* estimated and included

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- axial-vector singlet: $\mathcal{O}(\alpha_s^2)$ ***Kniehl, Kühn 1990*** $\Delta\alpha_s = +0.0027$
  $\mathcal{O}(\alpha_s^3)$ ***Larin, v. Ritbergen, Vermaseren 1995*** $\Delta\alpha_s = +0.00043$
  $\mathcal{O}(\alpha_s^4) \sim \mathcal{O}(\alpha_s^3)^2/\mathcal{O}(\alpha_s^2) \Rightarrow \Delta\alpha_s = \pm 0.00007$ (*dominant*)

α_s from Z decays: results

	$\Delta\alpha_s(M_Z)$	$\alpha_s(M_Z)$
Γ_Z, σ_{had}, R_l	0.1188	±0.0076
LEP 1	0.1213	±0.0028
LEP 1 + SLC	0.1198	±0.0028
Z pole + M_W	0.1196	±0.0027
EW fit (excl. τ_τ)	0.1203	±0.0027

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- allowing special new physics corrections to Zbb-vertex:
  $\alpha_s = 0.1167 \pm 0.0038$ ($\chi^2_{min} = 16.3/21$)

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- δ_q: quark condensates ***Maltman, Yavin 2008*** and finite m_c, m_b & m_s effects ***Chetyrkin 1993; Larin, van Ritbergen, Vermaseren 1995***

τ lifetime average

$\tau[\mathcal{B}_e] = \hbar\, \mathcal{B}_e^{\text{expt}} / \Gamma_e^{\text{theo}}$

$\mathcal{B}_e^{\text{expt}}$: $0.1785 \pm 0.0005 \Rightarrow \tau[\mathcal{B}_e^{\text{expt}}] = 291.47 \pm 0.82$ fs

$\mathcal{B}_\mu^{\text{expt}}$: $0.1736 \pm 0.0005 \Rightarrow \tau[\mathcal{B}_\mu^{\text{expt}}] = 291.00 \pm 0.84$ fs

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$\mathcal{B}_{e,\mu}^{\text{expt}}$ $(\rho_{e\mu} = -0.13) \Rightarrow \tau[\mathcal{B}_{e,\mu}^{\text{expt}}] = 291.24 \pm 0.55$ fs

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$\tau_{\text{direct}}^{\text{expt}} = 290.6 \pm 1.0$ fs

$\tau^{\text{expt}} \equiv \tau[\mathcal{B}_{e,\mu}^{\text{expt}}, \tau_{\text{direct}}^{\text{expt}}] = 291.09 \pm 0.48$ fs $\Rightarrow$

$\mathcal{R} \equiv \Gamma_{ud} / \Gamma_e = 3.475 \pm 0.010 \Rightarrow \delta_{\text{QCD}} = 0.1963 \pm 0.0034$

α_s [τ_τ]: experimental errors

source	uncertainty	$\Delta\tau$ [fs]	$\Delta\alpha_s(M_Z)$
$\Delta\tau^{expt}$	$\pm$ 0.48 fs	$\pm$ 0.48	$\mp$ 0.00039
$\Delta\mathcal{B}_s{}^{expt}$	$\pm$ 0.0007	$\mp$ 0.21	$\mp$ 0.00017
ΔV_{ud}	$\pm$ 0.00022	$\mp$ 0.08	$\mp$ 0.00007
Δm_τ	$\pm$ 0.17 MeV	$\mp$ 0.03	$\mp$ 0.00002
total		0.53	0.00043

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total			$+0.00178$ -0.00150

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Padé

- many *Padé approximants, summations, or predictions* (for the next term) can be defined. For illustration only:
- Padé [1/1]* $\equiv 1 + (a + 0.133\ a^2)/(1 - 5.069\ a) - 6.6\ a^4$

  higher order coefficients positive
- Padé [2/2] $\equiv (1 - 13.6\ a + 40\ a^2)/(1 - 14.6\ a + 49.5\ a^2)$

  $\Rightarrow \alpha_s(M_Z) = 0.1173$

  coefficients turn negative at order a^7

α_s $[\tau_\tau]$: summation schemes

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$\alpha_s(M_Z)$
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$\Delta_\beta\alpha_s(M_Z)$
$\Delta_{total}\alpha_s(M_Z)$

α_s [τ_τ]: summation schemes

	FOPT
$\alpha_s(M_Z)$	0.1174
$\Delta\delta_{PQCD}$	∓ 0.0119
$\Delta_{PQCD}\alpha_s(M_Z)$	$^{+0.00167}_{-0.00137}$
$\Delta_\beta\alpha_s(M_Z)$	± 0.00036
$\Delta_{total}\alpha_s(M_Z)$	$^{+0.00177}_{-0.00153}$

α_s [τ_τ]: summation schemes

	FOPT	CIPT
$\alpha_s(M_Z)$	0.1174	0.1193
$\Delta\delta_{PQCD}$	∓0.0119	∓0.0076
$\Delta_{PQCD}\alpha_s(M_Z)$	+0.00167 −0.00137	+0.00124 −0.00110
$\Delta_\beta\alpha_s(M_Z)$	±0.00036	±0.00050
$\Delta_{total}\alpha_s(M_Z)$	+0.00177 −0.00153	+0.00156 −0.00147

α_s [τ_τ]: summation schemes

	FOPT	CIPT	Padé [1/1]*
$\alpha_s(M_Z)$	0.1174	0.1193	0.1161
$\Delta\delta_{PQCD}$	∓ 0.0119	∓ 0.0076	∓ 0.0005
$\Delta_{PQCD}\alpha_s(M_Z)$	$^{+0.00167}_{-0.00137}$	$^{+0.00124}_{-0.00110}$	± 0.00006
$\Delta_\beta\alpha_s(M_Z)$	± 0.00036	± 0.00050	± 0.00036
$\Delta_{total}\alpha_s(M_Z)$	$^{+0.00177}_{-0.00153}$	$^{+0.00156}_{-0.00147}$	$^{+0.00067}_{-0.00070}$

α_s from the global EW fit

Z decays: $\alpha_s = 0.1198 \pm 0.0028$

τ decays: $\alpha_s = 0.1174^{+0.0018}_{-0.0015}$

Z & τ decays: $\alpha_s = 0.1181^{+0.0015}_{-0.0013}$

global fit: $\alpha_s = 0.1183^{+0.0016}_{-0.0014}$

Summary

- global EW fit: $\alpha_s = 0.1183^{+0.0016}_{-0.0015}$ (*no PDG scaling*)
- M_H: modest sensitivity of Z-pole value unless M_H close to unitarity limit of ~ 800 GeV ($\Delta\alpha_s = +0.0021$) but then we *know* that new physics is upsetting the SM fit
- new physics: negligible (significant) sensitivity of Z-pole value to (non-)universal corrections
- α_s [τ_τ]: theory uncertainty itself α_s dependent $\Rightarrow$ asymmetric error (re-calculated in each call in the fits)