

QCD Sum Rules and the Determination of Fundamental Parameters*

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We present a new QCD sum rule with high sensitivity to the continuum regions of charm and bottom quark pair production. Combining this sum rule with existing ones yields very stable results for the $\overline{\text{MS}}$ quark masses, $\hat{m}_c(\hat{m}_c)$ and $\hat{m}_b(\hat{m}_b)$. Comparison of our approach with experimental data allows for a robust theoretical error estimate. We have also provided a new evaluation of the lifetime of the τ lepton, τ_τ , serving as a strong constraint on α_s .

I. INTRODUCTION

The determination of the fundamental Standard Model (SM) parameters is important in its own right. It provides a test of the SM when results from various sources are compared, which can foster our understanding of SM dynamics (such as strong QCD effects). This may also lead to hints of new physics beyond the SM, when precise values of the SM parameters are compared against the predictions of more fundamental theories. For example, gauge couplings do not unify within the SM. This gives extra evidence against simple grand unification theories (GUTs) such as $SU(5)$ without supersymmetry, in addition to the non-observation of proton decay. On the other hand, gauge couplings seem to unify at a scale $\sim 2 \times 10^{16}$ GeV in the minimal supersymmetric standard model, which can be interpreted

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as a hint for supersymmetry as well as GUTs [1]. Most GUTs [2] also predict the mass ratio m_b/m_τ .

It is generally difficult to obtain reliable information on quark masses. The Particle Data Group [3] lists only ranges for their values, indicating a lack of confidence in methods used to evaluate them. Indeed, α_s is quite large at the mass scales of the bottom and charm quarks, questioning the convergence of perturbative QCD (PQCD). Furthermore, non-perturbative effects governed by the scale $\Lambda_{\text{QCD}} \sim 0.5$ GeV could be large, thus potentially compromising the validity of perturbative calculations. Two types of conditions are known to improve the situation: high energy or inclusiveness. As an example for the former, α_s and m_b can be determined at LEP energies using PQCD. This yields $\alpha_s(M_Z) = 0.1200 \pm 0.0028$ [4] with very little theoretical uncertainty. But $b(c)$ quark effects are small, so that $\hat{m}_b(M_Z) = 2.67 \pm 0.50$ GeV [5] is not well constrained.

In a recent work [6], we computed α_s from τ_τ , by definition an inclusive quantity and known to be quite insensitive to effects from non-perturbative QCD (NPQCD) [7]. Likewise, we used a set of inclusive QCD sum rules to derive values for $\hat{m}_c(\hat{m}_c)$ and $\hat{m}_b(\hat{m}_b)$. One of these sum rules is new, and its use together with existing ones [8, 9] proves to be a powerful tool to constrain the continuum region of quark pair production. This will be particularly helpful for the case of the b quark for which precise measurements of $R(s)$ (the inclusive hadronic cross section normalized to the leptonic point cross section) or of $R_b(s)$ (exclusive cross section for $b\bar{b}$ pairs) are unavailable.

In section 2, we determine the heavy quark masses based upon the new sum rule in addition to known ones. We point out a puzzling discrepancy between theory and the recent BES data. The BES data seem to be lower than theoretical predictions by 30% consistently across the moments. In section 3, we compute the τ lifetime. In section 4, we summarize our results.

II. SUM RULES AND HEAVY QUARK MASSES

On the basis of an unsubtracted dispersion relation (UDR) it was shown in Ref. [10] that knowledge of m_c , m_b , and α_s is sufficient to compute the charm and bottom quark contributions to the QED coupling $\alpha(\sqrt{t} = M_Z)$. Conversely, comparison of this UDR with the more traditional approaches using a subtracted dispersion relation (SDR) offers

information on m_c and m_b . The resulting equation relates an inclusive integrated cross section to a difference of vacuum polarization tensors, *viz.*

$$12\pi^2 \left[\hat{\Pi}_q(0) - \hat{\Pi}_q(-t) \right] = t \int_{4m_q^2}^{\infty} \frac{ds}{s} \frac{R_q(s)}{s+t}. \quad (1)$$

Eq. (1) defines a continuous set of sum rules parametrized by t , where the limit $t \rightarrow 0$ coincides with the first moment of $\Pi_q(t)$. Similarly, for each higher moment, \mathcal{M}_n , one has [8, 9, 11, 12, 13].

$$\frac{12\pi^2}{n!} \frac{d^n}{dt^n} \Pi_q(t) \Big|_{t=0} = \int_{4m_q^2}^{\infty} \frac{ds}{s^{n+1}} R_q(s). \quad (2)$$

We now take the opposite limit in Eq. (1), $t \rightarrow \infty$, and regularize the divergent expression,

$$\begin{aligned} \frac{R_q(s)}{3Q_q^2} \longrightarrow \frac{R_q(s)}{3Q_q^2} - \lambda_1^q(s) &\equiv \frac{R_q(s)}{3Q_q^2} - 1 - \frac{\alpha_s(\sqrt{s})}{\pi} \\ &- \left[\frac{\alpha_s(\sqrt{s})}{\pi} \right]^2 \left[\frac{365}{24} - 11\zeta(3) + n_q \left(\frac{2}{3}\zeta(3) - \frac{11}{12} \right) \right]. \end{aligned} \quad (3)$$

Q_q and n_q are the quark charge and the number of active flavors. Using expressions derived in Refs. [14, 15] and taking the limit $t \rightarrow \infty$, the sum rule (1) becomes:

$$\begin{aligned} \sum_{\text{resonances}} \frac{3\pi\Gamma_R^e}{Q_q^2 M_R \hat{\alpha}^2(M_R)} + \int_{4M^2}^{\infty} \frac{ds}{s} \frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{ds}{s} \lambda_1^q(s) &= -\frac{5}{3} + \frac{\hat{\alpha}_s}{\pi} \left[4\zeta(3) - \frac{7}{2} \right] \\ + \frac{\hat{\alpha}_s^2}{\pi^2} \left[\frac{11}{4}\zeta(2) + \frac{2429}{48}\zeta(3) - \frac{25}{3}\zeta(5) - \frac{2543}{48} + n_q \left(\frac{677}{216} - \frac{\zeta(2)}{6} - \frac{19}{9}\zeta(3) \right) \right]. \end{aligned} \quad (4)$$

Here, M_R and Γ_R^e are the mass and the electronic partial width of resonance R , and R_q^{cont} denote the continuum regions integrated from $M = M_{B^\pm}$ for b and $M = M_{D^0}$ for c . The regularization (3) together with the scale choices $\hat{m}_q = \hat{m}_q(\hat{m}_q)$ and $\hat{\alpha}_s = \hat{\alpha}_s(\hat{m}_q)$ eliminates (resums) all logarithmic terms in Eq. (4). Unlike in any of the sum rules (2), R_q^{cont} appears unsuppressed in Eq. (4) so that \hat{m}_q varies exponentially with the experimental information on the resonances. We will use Eq. (4) to constrain the continuum region and work with the following *ansatz*:

$$\begin{aligned} \frac{R_q^{\text{cont}}(s)}{3Q_q^2} &= \lambda_1^q(s) \sqrt{1 - \frac{4\hat{m}_q^2(2M)}{s'}} \left[1 + \lambda_3^q \frac{2\hat{m}_q^2(2M)}{s'} \right] \\ &\approx \lambda_1^q(4M^2) \sqrt{1 - \frac{4\hat{m}_q^2}{s'}} \left[1 + \lambda_3^q \frac{2\hat{m}_q^2}{s'} \right] - \frac{\hat{\alpha}_s}{\pi} \frac{\lambda_2^q(s)}{1 + \lambda_2^q(s)}, \end{aligned} \quad (5)$$

where now $\hat{\alpha}_s = \hat{\alpha}_s(2M)$, $s' \equiv s + 4(\hat{m}_q^2(2M) - M^2)$, and

$$\lambda_2^q(s) = \frac{\hat{\alpha}_s}{\pi} \beta_0 \ln \frac{s}{4M^2} = \frac{\hat{\alpha}_s(2M)}{\pi} \left(\frac{11}{4} - \frac{n_q}{6} \right) \ln \frac{s}{4M^2}.$$

We will use the form in the second line (applying it to all moments) of Eq. (5) with the corresponding change in the regularization in Eq. (4). This keeps only the leading logarithms resummed but allows for an analytical integration. Eq. (5) coincides asymptotically with the predictions of PQCD for massless quarks and interpolates smoothly between the vanishing phase space at the pseudo-scalar threshold and the strong onset of fermion pair production. Unlike when PQCD is applied to $R(s)$ directly and relatively close to the resonance region, we minimize the exposure to local quark-hadron duality violations by using QCD *inclusively* and by merely requiring stable results across the moments. No claim is being made about the *local shape* of R_q — we only need theoretical information about *global averages*. It should be pointed out that the new sum rule (4) and the choice of *ansatz* (5) are logically independent ingredients. An explicit *ansatz* facilitates the discussion, but our results are essentially independent of the shape of the continuum.

We use the narrow resonance data [3], J/Ψ , $\Psi(2S)$ for the c quark and $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ for the b quark, as the only experimental input. The wider resonances in the continuum region are assumed to be accounted for by our *ansatz* (5) because (i) they decay almost exclusively into flavored hadrons; (ii) they interfere with the non-resonating part of the continuum rendering a common treatment virtually impossible; (iii) the δ -function approximation (which is perfect for the narrow resonances) becomes successively worse; (iv) the philosophy of our *ansatz* supposes that it averages over local cross-section fluctuations; and (v) we wish to compare Eq. (5) directly to experimental data on the charm continuum region such as from Beijing [17]. The narrow resonance contribution to the various moments is shown in the second column of Table I. The 3rd column gives the continuum contribution, and the 4th column shows the totals to be compared with the theoretical moments in the last column, *viz.*

$$\mathcal{M}_n^{\text{theory}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n. \quad (6)$$

The \bar{C}_n are known up to $\mathcal{O}(\alpha_s^2)$ and taken from Refs. [14, 18] where they were computed for arbitrary renormalization scale μ . It seems appropriate to choose $\mu = \hat{m}_q(\hat{m}_q)$, eliminating all logarithmic terms as there is only one scale in the problem. Indeed, the authors of Ref. [13], who have chosen $\mu = 3$ (10) GeV for the charm (bottom) quark and then evolved to $\mu = \hat{m}_q$, report a variation over the first 5 (7) moments of 122 (312) MeV. (For larger moments the α_s expansion [19] of the gluon condensate contribution [9] breaks down.) Using

n	resonances	continuum	total	theory
0	1.16 (6)	-3.03 ± 0.37	-1.86 ± 0.37	input (4)
1	1.12 (6)	1.04 ± 0.14	2.16 ± 0.16	2.19 (6)
2	1.10 (7)	0.37 ± 0.07	1.47 ± 0.10	1.49 (9)
3	1.10 (7)	0.17 ± 0.04	1.27 ± 0.08	1.26 (14)
4	1.11 (7)	0.09 ± 0.02	1.20 ± 0.08	1.16 (20)
5	1.13 (7)	0.05 ± 0.01	1.18 ± 0.08	1.10 (31)
0	1.17 (5)	-52.44 ± 1.24	-51.27 ± 1.24	input (2)
1	1.24 (5)	3.12 ± 0.53	4.36 ± 0.54	4.51 (2)
2	1.31 (5)	1.33 ± 0.30	2.64 ± 0.31	2.79 (3)
3	1.40 (5)	0.75 ± 0.19	2.15 ± 0.20	2.27 (5)
4	1.50 (5)	0.48 ± 0.13	1.98 ± 0.14	2.06 (7)
5	1.61 (5)	0.33 ± 0.10	1.94 ± 0.11	1.99 (10)
6	1.74 (6)	0.23 ± 0.07	1.98 ± 0.09	1.98 (14)
7	1.89 (6)	0.17 ± 0.05	2.06 ± 0.08	2.03 (19)

TABLE I: Results for the lowest moments, \mathcal{M}_n , defined in Eq. (1) for $n = 0$ ($t \rightarrow \infty$) and Eq. (2) for $n \geq 1$. The upper (lower) half of the Table corresponds to the charm (bottom) quark. Each moment has been multiplied by 10^nGeV^{2n} ($10^{2n+1} \text{GeV}^{2n}$). The continuum error is from $\Delta\lambda_3^{b,c} = \pm 1.47$. The last column shows the theoretical prediction for $\hat{m}_c(\hat{m}_c) = 1.289 \text{ GeV}$, $\hat{m}_b(\hat{m}_b) = 4.207 \text{ GeV}$, and $\alpha_s(M_Z) = 0.1211$, where the uncertainty is our estimate for the truncation error (see text).

the same moments [13] but choosing $\mu = \hat{m}_q$ instead, we observe a variation of less than 27 (16) MeV. This impressive improvement clearly overcompensates for the larger $\hat{\alpha}_s$. We will choose $\mu = \hat{m}_q$ in the following. As for the theoretical uncertainty associated with the truncation of the perturbative series, we use the method suggested in Ref. [20]. In our case this yields the error estimate,

$$\pm N_C Q_q^2 C_F C_A^2 \frac{\hat{\alpha}_s^3(\hat{m}_q)}{\pi^3} \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n}, \quad (7)$$

($N_C = C_A = 4C_F = 3$) corresponding to $\pm 16\hat{\alpha}_s^3/\pi^3$ in the \bar{C}_n . Comparing the corresponding estimate against the exactly known coefficients of the first eight moments up to order α_s^2 [14, 18] shows that with $\mu = \hat{m}_q$, 23 of 24 coefficients are within the estimate, while only

n	BES	$\lambda_c^3 = 0.50$	$\lambda_c^3 = 1.97$	BES	$\Psi(3S)$
0	5.51 (35)	5.50	7.19	0.215 (39)	0.348 (54)
1	3.02 (19)	3.01	3.98	0.151 (27)	0.245 (38)
2	1.68 (11)	1.68	2.25	0.106 (19)	0.172 (27)
3	0.95 (6)	0.96	1.29	0.074 (13)	0.121 (19)
4	0.55 (4)	0.55	0.76	0.052 (9)	0.085 (13)
5	0.32 (2)	0.33	0.45	0.037 (6)	0.060 (9)

TABLE II: The left part shows contributions to the charm moments ($\times 10^{n+1} \text{GeV}^{2n}$) from $2M_{D^0} \leq \sqrt{s} \leq 4.8 \text{ GeV}$, and the right part from $2M_{D^0} \leq \sqrt{s} \leq 3.83 \text{ GeV}$. Following Ref. [13], we computed the columns labeled BES by subtracting from the threshold data on $R(s)$ the average, \bar{R} , below threshold. (We applied corrections for the leading s -dependence.) The errors combine the statistical and uncorrelated systematic ones of \bar{R} with those in the continuum region and with the common systematics ($\leq 3.5\%$) of the difference.

one coefficient would have been underestimated by a factor ≈ 1.437 . This seems to be a reasonable state of affairs for a 1σ error estimate and corresponds to $\pm 20 \text{ MeV}$ for $\hat{m}_c(\hat{m}_c)$ from \mathcal{M}_1 , while variation of the renormalization scale [13] assesses this error to only 1 MeV , which is optimistic. We show the estimate (7) in the last column of Table I.

The last two columns of that Table would agree within errors even if we had chosen significantly smaller variations in λ_3^b and especially λ_3^c ($\Delta\lambda_3^{b,c} = \pm 1.47$ accounts for the error introduced by our *ansatz* and is above and beyond the variations induced by the fit parameters). The reason for our more conservative error is shown in Table II. It shows that Eq. (5) with $\lambda_3^c = 0.50$ reproduces the n dependence of the moments computed from recent data by the BES Collaboration [17] remarkably well. However, our method favors $\lambda_3^c \approx \lambda_3^b \approx 1.97$, and thus 30 to 40% larger contributions. Note that the quark parton model predicts $\lambda_3^q = 1$, while from third order massive QCD corrections [16] one expects $\lambda_3^q > 1$ (in agreement with our results). Table II also compares the BES data to the $\Psi(3S)$ contribution [3] in the narrow width approximation. Even assuming that the $\Psi(3S)$ resonance ($M_{\Psi(3S)} = 3.7699 \text{ GeV}$) saturates the charm cross-section in that region, we observe a direct *experimental* 2σ discrepancy between Ref. [17] and $\Gamma_{\Psi(3S)}^e = 0.26 \pm 0.04 \text{ keV}$ [3]. Thus there is a discrepancy between perturbative QCD (our set of sum rules) and the BES data, while

QCD appears to be consistent with the $\Psi(3S)$ data within 1σ . The BES data seem to be lower than theoretical predictions by 30% consistently across the moments. This constitutes a great puzzle which needs to be resolved in the future. We may be able to quote smaller errors after this situation has been resolved. Nevertheless, the quark masses can still be determined precisely enough through the sum rule approach.

There is a possible contribution from the gluon condensate [9]. It is known up to $\mathcal{O}(\alpha_s)$ [19], but its actual value is not well known. Its inclusion lowers the extracted quark masses, increases λ_3^c , and sharpens the discrepancy with the BES data. We can bound its value to $\lesssim 0.07 \text{ GeV}^4$ by demanding n independent results within the uncertainties. We use this bound (with a central value of zero) to account collectively for non-perturbative uncertainties. They induce errors of about 29 MeV into $\hat{m}_c(\hat{m}_c)$ ($n = 2$) and 2.4 MeV into $\hat{m}_b(\hat{m}_b)$ ($n = 6$).

The parametric uncertainties from α_s and the quark masses themselves are correlated in a complicated way (i) across the moments, (ii) across the two quark flavors, (iii) between the theoretical moments and the continuum contribution, and (iv) with each other. In practice, all this is accounted for by performing fits to the moments. Heavy quark radiation by light quarks [21] is not resonating and problems associated with singlet contributions [21, 22] appear only at $\mathcal{O}(\alpha_s^3)$, so these issues should not introduce further uncertainties into our analysis. We will present our final results after discussing the τ lifetime.

III. τ LIFETIME

It was pointed out long ago that the total hadronic decay width of the τ lepton can be reliably computed in the framework of perturbative QCD [23]. Employing the ratio

$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{Hadrons})}{\Gamma(\tau \rightarrow \nu_\tau e \nu_e)} \quad (8)$$

which is predicted to be $N_c = 3$ in the lowest order and in the absence of Cabbibo mixing, the perturbative corrections can be obtained from QCD calculations of the two point current-current correlator $\Pi_{\mu\nu}(q)$ (*i.e.*, the hadronic contributions to the W boson vacuum polarization tensor). Electroweak radiative corrections were calculated in Ref. [24]. Non-perturbative effects have also been estimated and found to be small [7], thus inducing little uncertainty. Therefore R_τ with its small experimental uncertainty provides a solid venue to

determine the strong coupling constant precisely.

R_τ^{QCD} can be expressed as a contour integral [7] along $|s| = m_\tau^2$ in the complex s -plane,

$$R_\tau^{\text{QCD}} = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \left[1 - 2\frac{s}{m_\tau^2} + 2\left(\frac{s}{m_\tau^2}\right)^3 - \left(\frac{s}{m_\tau^2}\right)^4 \right] s \frac{d}{ds} \Pi(s), \quad (9)$$

where the Adler function has been calculated to the third order in α_S [16],

$$s \frac{d}{ds} \Pi(s) = 1 + \frac{\alpha_S(-s)}{\pi} + K_2 \left[\frac{\alpha_S(-s)}{\pi} \right]^2 + K_3 \left[\frac{\alpha_S(-s)}{\pi} \right]^3 + \dots, \quad (10)$$

where $K_2 = 1.6398$ and $K_3 = 6.371$, assuming three massless flavors of quarks.

One way to evaluate R_τ^{QCD} is to expand $\alpha_S(-s)$ perturbatively in terms of $\alpha_S(m_\tau)$ with the help of the renormalization group equation (RGE) of α_S . However, this results in a series of $\alpha_S(m_\tau)$ with poor convergence. It proves to be expedient to keep the following contour integrals in Eq. (9) [25]

$$A_n = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \left[1 - 2\frac{s}{m_\tau^2} + 2\left(\frac{s}{m_\tau^2}\right)^3 - \left(\frac{s}{m_\tau^2}\right)^4 \right] \left[\frac{\alpha_S(-s)}{\pi} \right]^n, \quad (11)$$

which are complicated functions of α_S but well-behaved if numerically integrated with the help of the RGE on the complex plane. In general,

$$|A_n| \sim \left[\frac{\alpha_S(m_\tau)}{\pi} \right]^n \quad (12)$$

which we calculate numerically up to 4-loop order in the β function [26].

We have included one-loop electroweak radiative corrections [24]:

$$S_{EW} = \left(1 + \frac{\alpha}{\pi} \ln \frac{M_Z^2}{m_\tau^2} \right) \left(1 + \frac{5}{12} \frac{\alpha(m_\tau)}{\pi} \right), \quad (13)$$

where the large log is resummed by the corresponding RGE [27]. We have also included QED (phase space) corrections [28], quark condensate contributions, as well as c quark effects in an expansion in $m_\tau^2/4m_c^2$ [29]. In total, the partial width into hadrons with vanishing net strangeness is

$$\begin{aligned} \Gamma_\tau^{ud} = & \frac{G_F^2 m_\tau^5 |V_{ud}|^2}{64\pi^3} S_{EW} \left(1 + \frac{3m_\tau^2}{5M_W^2} \right) \left[\frac{R_\tau^{\text{QCD}}}{3} + \frac{\hat{\alpha}}{\pi} \left(\frac{85}{24} - \frac{\pi^2}{2} \right) \right. \\ & \left. - 0.09 \frac{m_u^2 + m_d^2}{m_s^2 - m_d^2} - \frac{f_{\pi^\pm}^2}{m_\tau^4} [m_{\pi^\pm}^2 (8\pi^2 + 23\alpha_s^2) - 4m_{K^\pm}^2 \alpha_s^2] \right]. \quad (14) \end{aligned}$$

For our analysis, the experimental input is the τ lifetime,

$$\tau_\tau = \frac{\hbar}{\Gamma_\tau} = \hbar \frac{1 - \mathcal{B}_S}{\Gamma_\tau^e + \Gamma_\tau^\mu + \Gamma_\tau^{ud}} = 290.96 \pm 0.59 \text{ fs}, \quad (15)$$

evaluating the partial widths into leptons, $\Gamma_\tau^e + \Gamma_\tau^\mu$, as well as Γ_τ^{ud} theoretically. The relative fraction of decays with $\Delta S = -1$, $\mathcal{B}_S = 0.0286 \pm 0.0009$ [3], is based on experimental data, since the value for the strange quark mass, $\hat{m}_s(m_\tau)$, is not well known, and the PQCD expansion, $C_{QCD}^{D=2}$, proportional to m_s^2 converges poorly and cannot be trusted. $C_{QCD}^{D=2}$ also multiplies the corresponding $m_{u,d}^2$ terms in Γ_τ^{ud} , posing the same but numerically less important problem there. We solved it, by relating $C_{QCD}^{D=2}$ to the ratio $\Gamma_\tau^{us}|V_{ud}|^2/(\Gamma_\tau^{ud}|V_{us}|^2) = 0.896 \pm 0.034$ [3] (in which to linear order all universal terms cancel), and find $C_{QCD}^{D=2}(m_s^2 - m_d^2) = m_\tau^2(0.091 \pm 0.046)$.

We computed the world average (15) by combining the direct value, $\tau_\tau = 290.6 \pm 1.1$ fs [3], with $\tau_\tau(\mathcal{B}_e, \mathcal{B}_\mu) = 291.1 \pm 0.7$ fs derived from the leptonic branching ratios $\mathcal{B}_e = 0.1784(6)$ and $\mathcal{B}_\mu = 0.1737(6)$ [3] taking into account their 1% correlation. The dominant theoretical error induced by the unknown coefficient $d_3 = 0 \pm 77$ [20] is itself strongly α_s -dependent, is recalculated in each call within a fit, and induces an asymmetric α_s error.

Other experimental uncertainties arise from [3] $m_\tau = 1.77699(28)$ GeV, $|V_{ud}| = 0.97485(46)$, and \mathcal{B}_S . Uncertainties from higher dimensional terms in the operator product expansion, OPE, are taken from Ref. [30] and add up to $\Delta\tau_\tau(\text{OPE}) = \pm 0.64$ fs. We assume that an uncertainty of the same size is induced by possible OPE breaking effects[32]. The unknown five-loop β -function coefficient, $\beta_4 = 0 \pm 579$ [20], contributes mainly to the evolution of $\alpha_s(m_\tau)$ to $\alpha_s(M_Z)$ and less to the A_i . The sub-leading errors listed in this paragraph amount to ± 1.2 fs. We find, $\alpha_s(m_\tau) = 0.356_{-0.021}^{+0.027}$ and $\alpha_s(M_Z) = 0.1221_{-0.0023}^{+0.0026}$, in excellent agreement with $\alpha_s(M_Z) = 0.1200 \pm 0.0028$ from Z -decays [4] and most other recent evaluations of τ_τ [30, 31].

IV. SUMMARY

We have presented a new QCD sum rule with high sensitivity to the continuum regions of charm and bottom quark pair production. Combining this sum rule with existing ones yields very stable results for the $\overline{\text{MS}}$ quark masses, $\hat{m}_c(\hat{m}_c)$ and $\hat{m}_b(\hat{m}_b)$. Comparison of our approach with experimental data allows for a robust theoretical error estimate. We have also

provided a new evaluation of the lifetime of the τ lepton, τ_τ , serving as a strong constraint on α_s . Including τ_τ , and the $n = 2$ and $n = 6$ moments for the c and b quark, respectively, as constraints in a fit to all data [4] yields,

$$\begin{aligned}\alpha_s(M_Z) &= 0.1211^{+0.0018}_{-0.0017}, \\ \hat{m}_c(\hat{m}_c) &= 1.289^{+0.040}_{-0.045} \text{ GeV}, \\ \hat{m}_b(\hat{m}_b) &= 4.207^{+0.030}_{-0.031} \text{ GeV}.\end{aligned}\tag{16}$$

These results reduce the error [10] in $\alpha(M_Z)$ by 25%.

Finally, we stress again that there is a discrepancy between the BES data on one hand, and QCD and the $\Psi(3S)$ electronic width on the other. The BES data seem to be lower than theoretical predictions by 30% consistently across the moments. This constitutes a great puzzle. We may be able to quote smaller errors after this situation has been resolved.

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