Five jets at LEP @ NLO and $\alpha_s(M_Z)$

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Jet observables at LEP

$e^+e^- \to$ hadrons is one of the most studied process in high-energy physics. Analyses at PEP, KEK, PETRA, SLD, and LEP were instrumental for

- ☑ understanding jet structures, developing jet algorithms
- ☑ improving perturbative calculations
- ☑ developing, testing, and tuning of parton showers
- ☑ investigating non-perturbative effects
- ☑ fits of the strong-coupling constant α_s

At LEP exclusive processes with up to five jets were measured. Five-jet cross-sections start at $O(\alpha_s^3) \Rightarrow$ very sensitive to α_s. But, no previous fit of α_s from five jets, because of large uncertainties of LO predictions

Calculation of five jets @ NLO

We performed the full NLO calculation of five jets by computing

☑ virtual corrections using D-dimensional unitarity as implemented in Rocket

Ellis et al., '07; Giele et al. '08

☑ using MadFKS for real radiation, subtraction, and phase-space integration

Alwall et al. '07; Frederix et al., '09

This calculation started as an academic exercise:

- For Rocket: how does FKS subtraction compare to Catani-Seymour subtraction used previously?
- For MadFKS: test the performance of the new automated subtraction code with a highly non-trivial calculation.

NB: all elements, but for the finite part of the virtual, are obtained by MadFKS through calls to Madgraph routines

Five-jet observables

Definitions:

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{s}(1-\cos\theta_{ij})$$

$$\frac{1}{\sigma_{\rm tot}}\int_{y_{\rm cut}}^{1} {\rm d}y_{45}\,\frac{{\rm d}\sigma}{{\rm d}y_{45}} = \frac{\sigma_{\rm incl}^{5-\rm jet}(y_{\rm cut})}{\sigma_{\rm tot}}$$

$$R_5(y_{\rm cut}) = \frac{\sigma_{\rm excl}^{5-\rm jet}(y_{\rm cut})}{\sigma_{\rm tot}},$$

Durham jet-algorithm

Power series in the coupling constant:

$$\sigma_{\rm tot}^{-1}\frac{{\rm d}\sigma}{{\rm d}y_{45}} = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 A_{45}(y_{45}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^4 \left(B_{45}(y_{45}) + 3b_0 A_{45}(y_{45})\ln\frac{\mu}{\sqrt{s}}\right)$$

$$R_5(y_{\rm cut}) = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 A_5(y_{\rm cut}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^4 \left(B_5(y_{\rm cut}) + 3b_0 A_5(y_{\rm cut})\ln\frac{\mu}{\sqrt{s}}\right)$$

NB: $A_{45,5}$ $B_{45,5}$ do not depend on the CM energy

Hadronization corrections

Because of factorization of long- and short-distance physics, hadronization corrections to infrared safe observables can be estimated by running an event generator at parton and hadron level:

$$H^i[\mathcal{O}] = \frac{\mathcal{O}^i_{\rm hadr}}{\mathcal{O}^i_{\rm part}}$$

Once an improved PT prediction $\mathcal{O}_{\rm pt}$ is available, one defines

$$\mathcal{O}_{\rm impr} = H^i[\mathcal{O}]\ \mathcal{O}_{\rm pt}$$

and compares this with data

This procedure is widespread, but it is clear for a number of reasons that it cannot be fully valid. This issue becomes particularly important for high jet-multiplicity and when the size of hadronization corrections is large

Comparison with PS

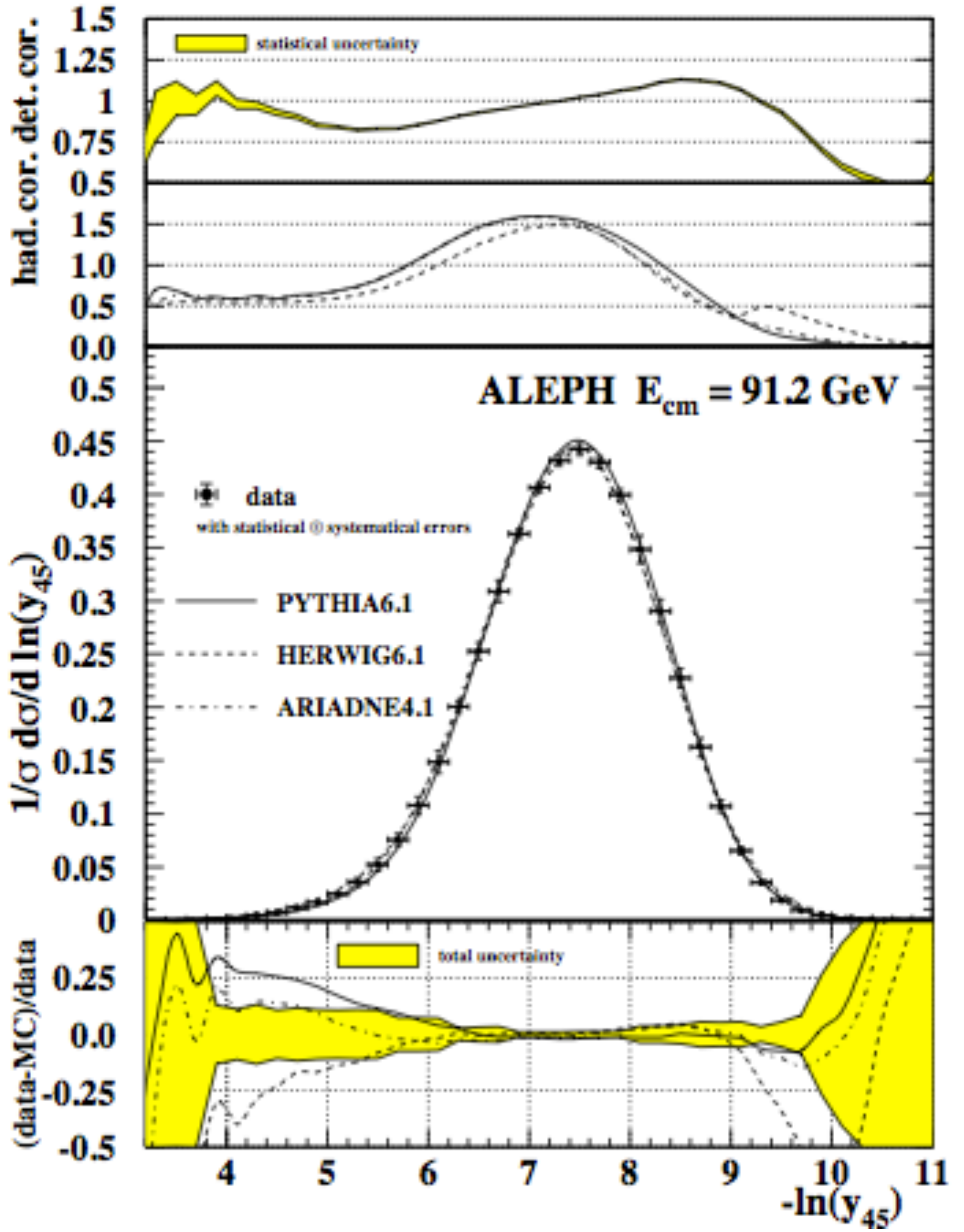
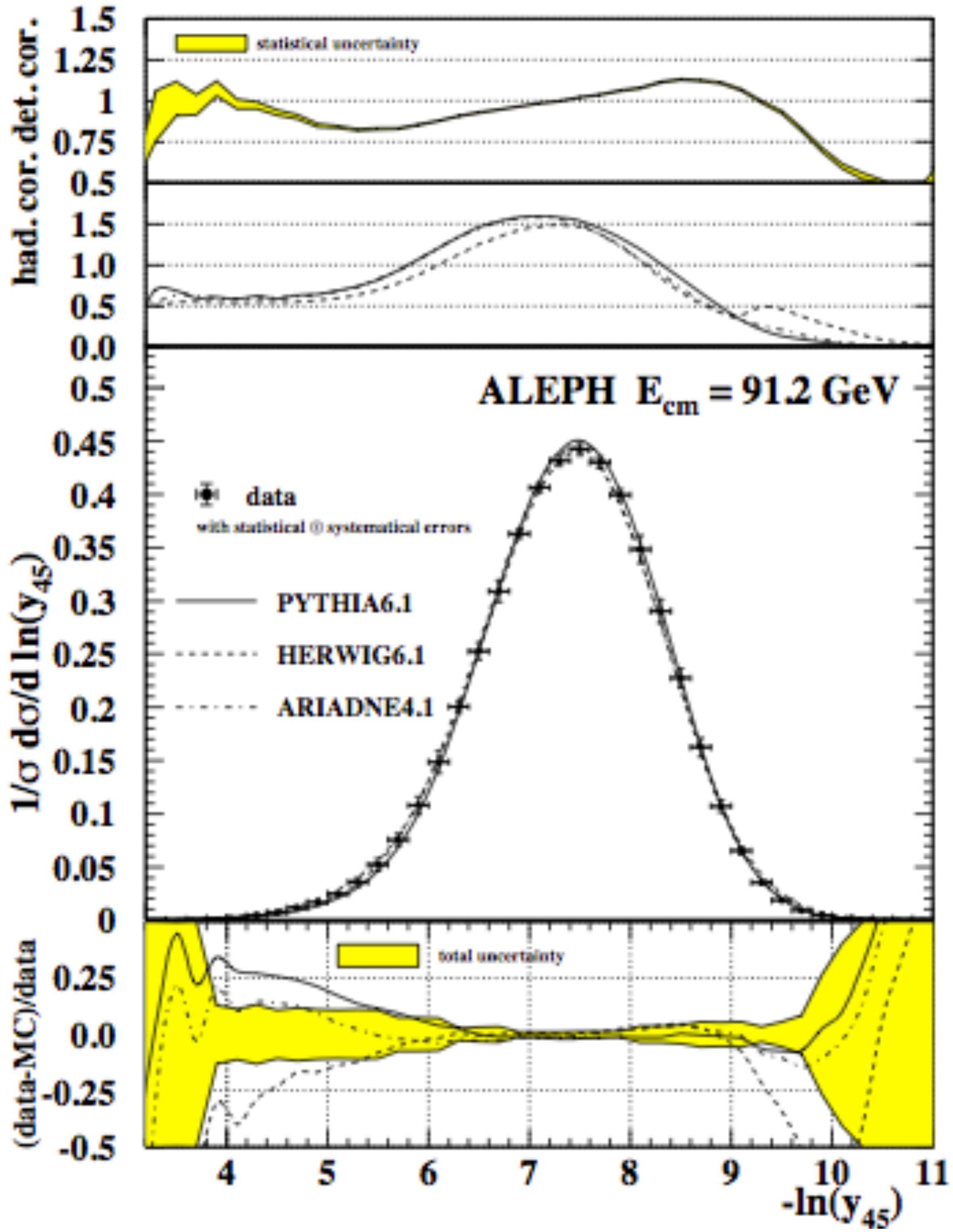


Figure provided by H. Stenzel

Comparison with PS



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- All Herwig, Pythia, and Ariadne describe data surprisingly well

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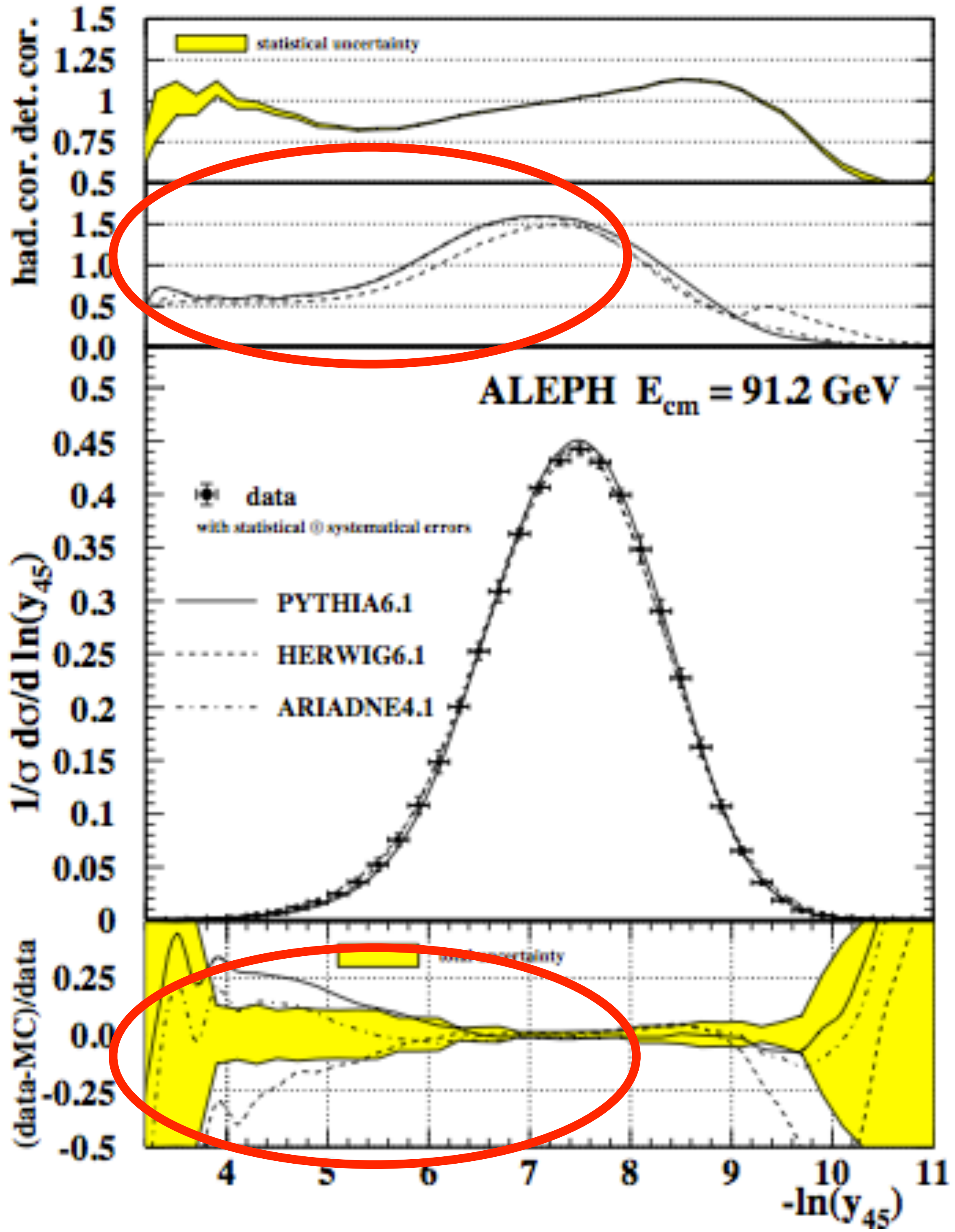

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- However, very large hadronization corrections and large differences in their size *(generator dependent!)*

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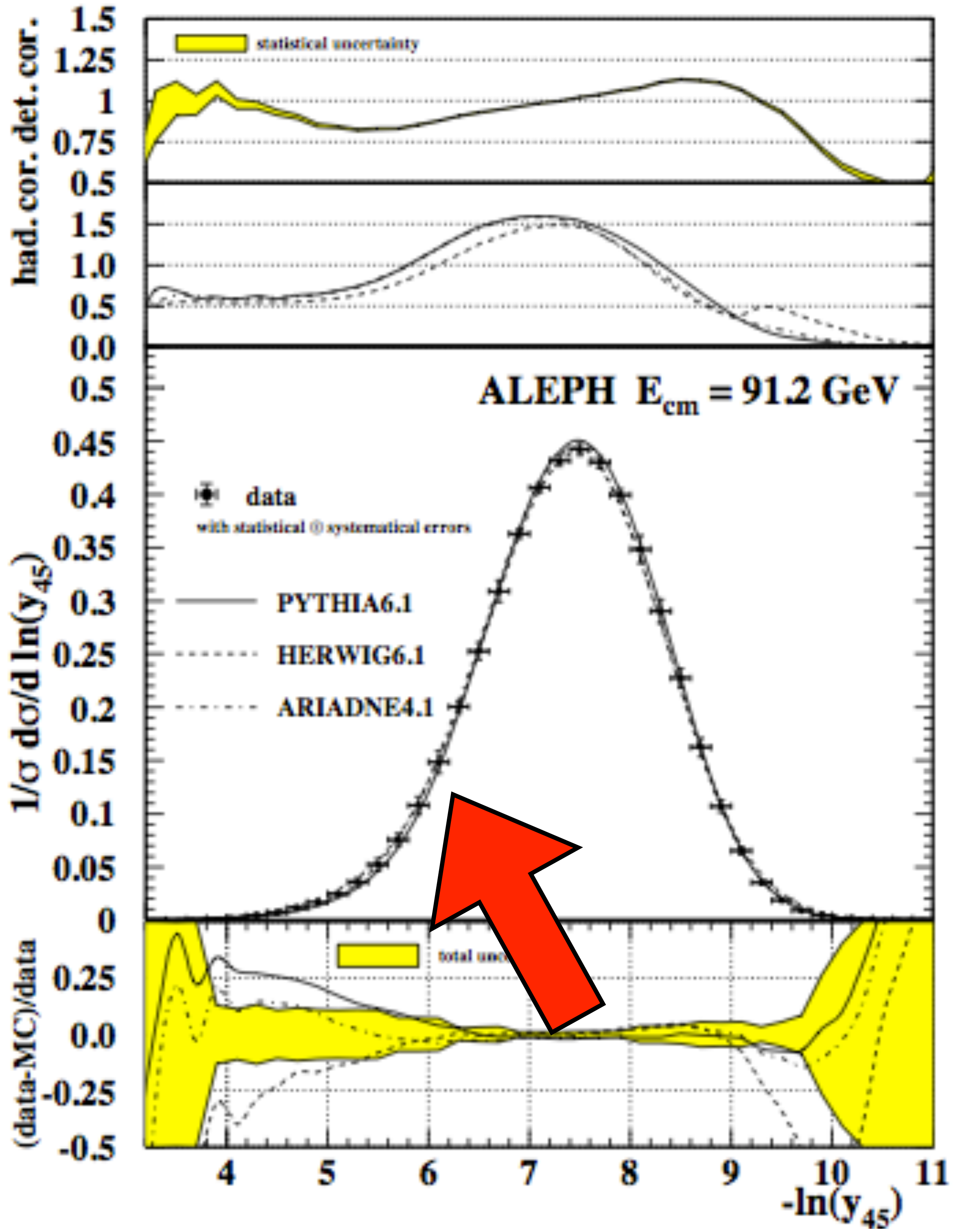

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- All Herwig, Pythia, and Ariadne describe data surprisingly well
- However, very large hadronization corrections and large differences in their size *(generator dependent!)*
- For $-\ln(y_{45}) > 6$ large logarithmic effects (Sudakov peak)
  $\Rightarrow$ can not be described by NLO calculation

Large Logarithms

- R_5 has been resummed to NLLA accuracy, including all terms

$$\alpha_s^n L^{2n} + \alpha_s^n L^{2n-1}$$

Catani et al. '91

- This resummation is valid for $L \gg 1$ but $\alpha_s L \ll 1$

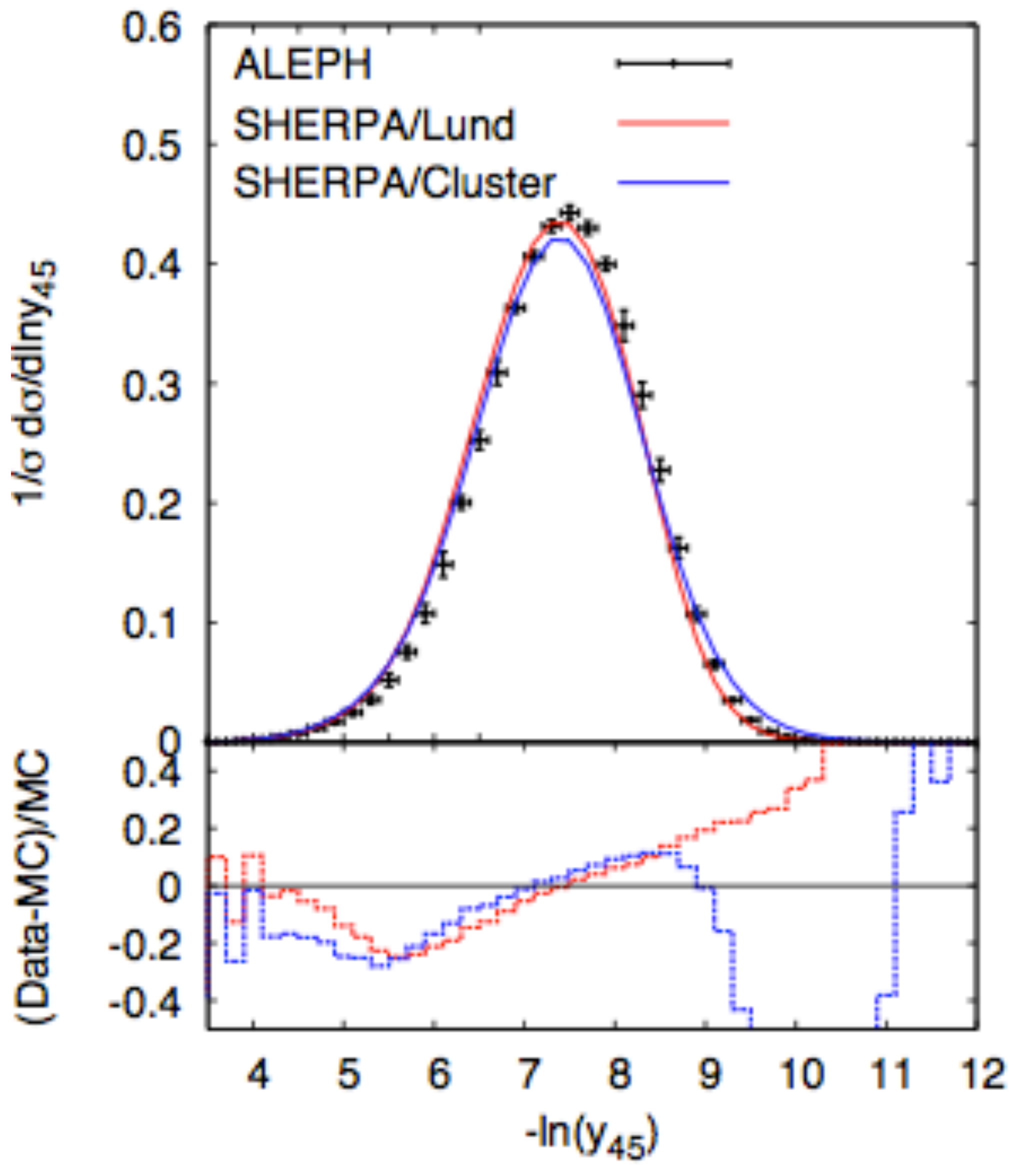
- Since $\alpha_s \sim 0.15$ this means practically $L \gg 5$ but $L \ll 6$

$\Rightarrow$ No suitable range where the approach is valid. We restrict our comparison to data to $L < 6$

NB: a similar bound appears since we neglect the mass of the b-quark. This implies that the resolution parameter must satisfy $sy_{45} > m_b^2$ which at LEP 1 translates to $L < 6$

Comparison with Sherpa

Unlike Pythia/Herwig, Sherpa includes the full LO matrix element for $e^+e^- \longrightarrow 5$ jets and uses the CKKW procedure to match to parton shower



Sherpa predictions kindly provided by S. Hoeche

- Description of data slightly worse than Ariadne/Pythia/Herwig

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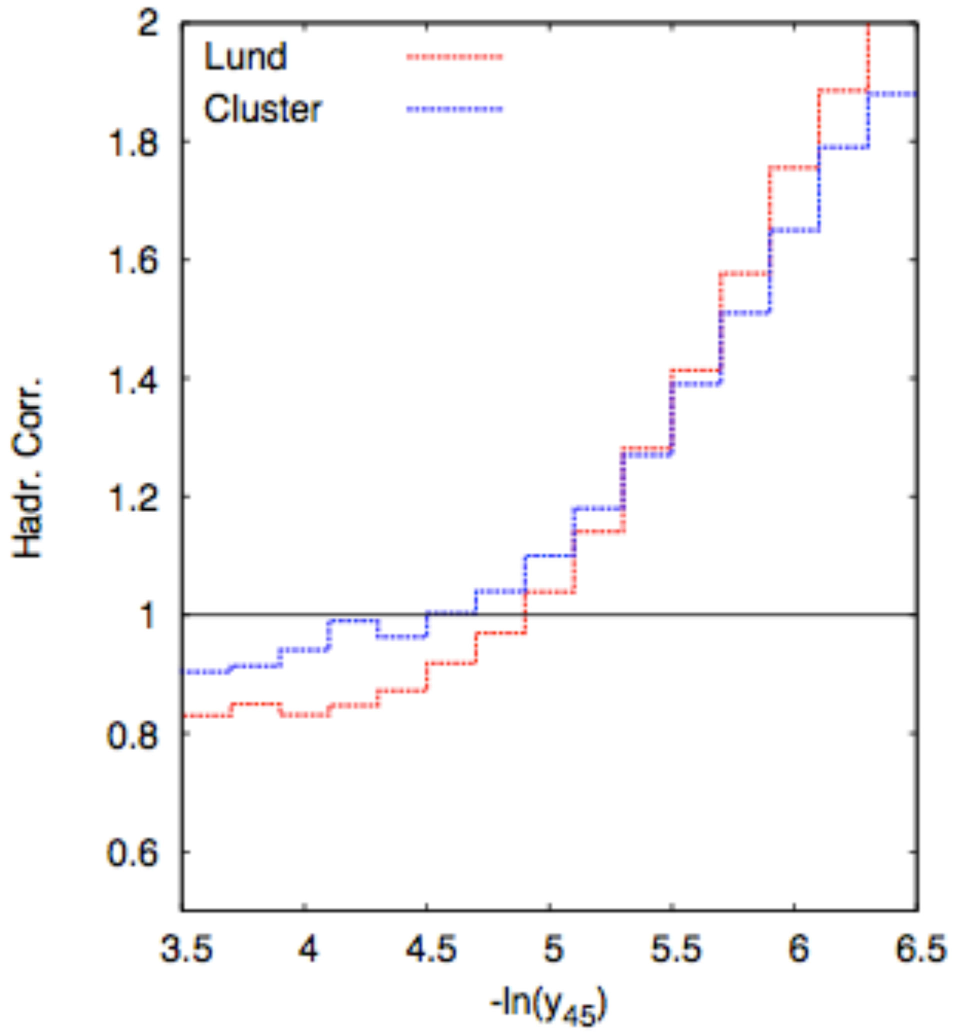

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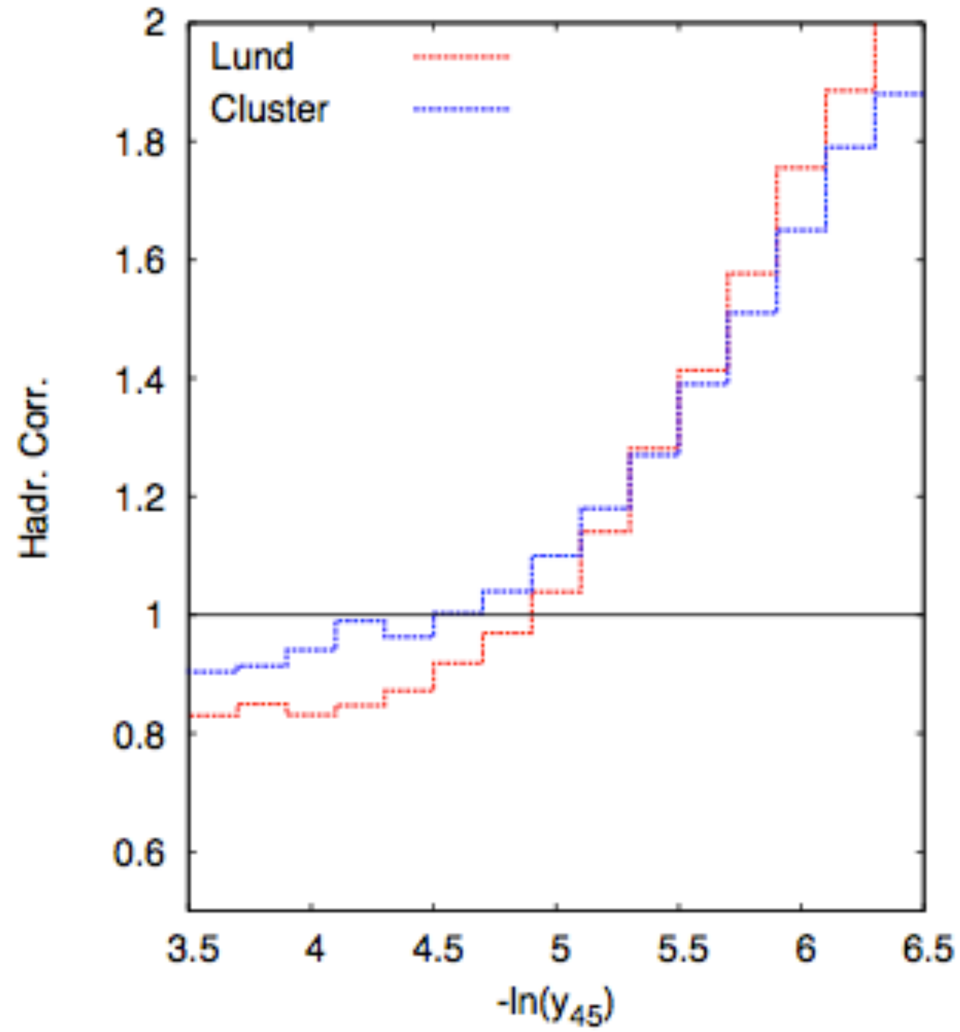
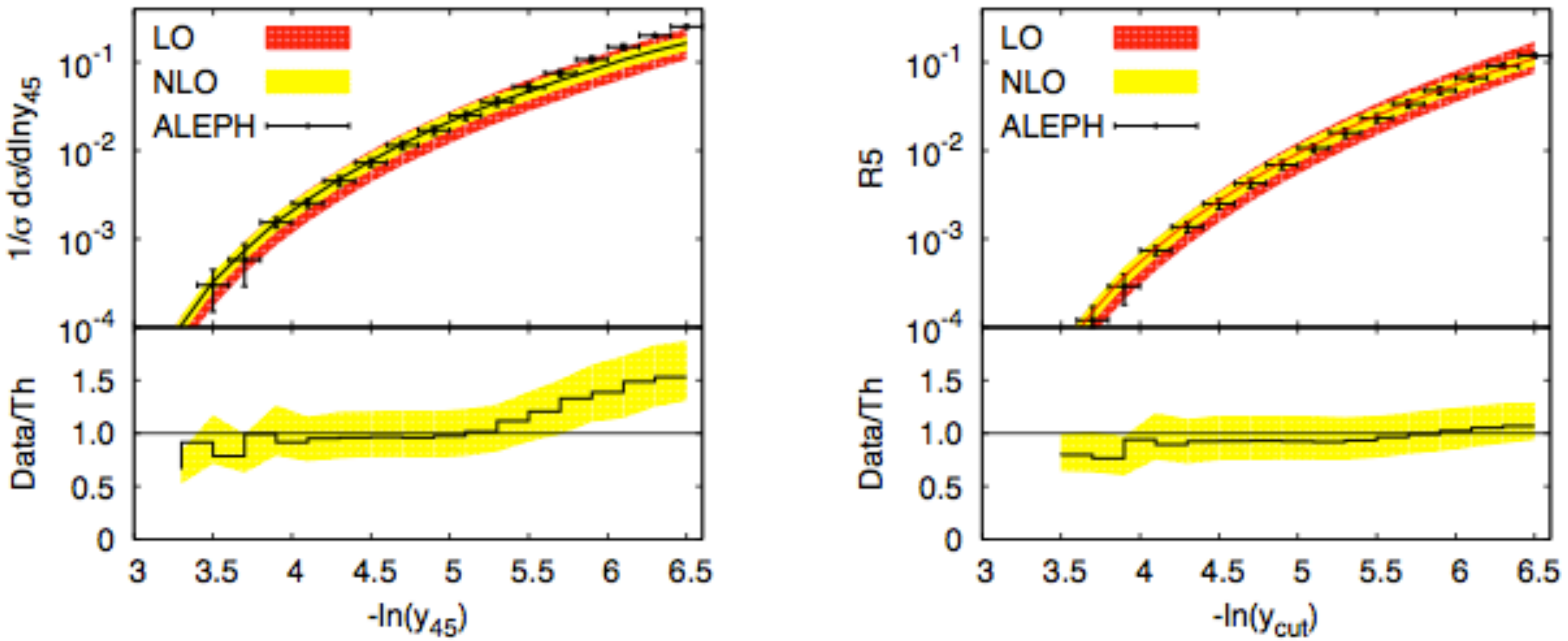


- Description of data slightly worse than Ariadne/Pythia/Herwig
- Size of hadronization corrections depend on hadronization model (Lund or cluster)
- Hadronization corrections are smaller in the range of interest than those of Pythia/Herwig/ Ariadne

We will take hadronization corrections from Sherpa. Since the LO matrix element is implemented, hadronization is less contaminated by PT effects

Comparison: ALEPH vs. NLO

- use default hadronization model of Sherpa (Cluster) to correct NLO
- use renormalization scale μ = 0.3 M_Z $(k_{\perp\max} \sim \langle\sqrt{y_{23}s}\rangle \sim 0.3M_Z)$



- Good agreement with data
- NLO corrections increase LO predictions by +(10-20)%(*)
- Reduced uncertainties: from [-30%, +45%] at LO to [-20%,+25%] at NLO

(*) if α_s = 0.130 is used at LO, if the world average is used, then corrections are ~ 45-60%

α_s from 5-jets: fit details

- Each bin in y_{45} or R_5 at a given energy is an observable

$$\mathcal{O}_i = [X_i, \sigma_i^{\rm stat}, \sigma_i^{\rm syst}]$$

- Each observable can be used to extract α_s by solving

$$\boxed{T_i H_i = E_i}$$

- The values obtained from the fit can be written as

$$\alpha_s^i = \overline{\alpha}_s^i \pm \delta\alpha_s^{i,\rm stat} \pm \delta\alpha_s^{i,\rm syst} \pm \delta\alpha_s^{i,\rm scale} \pm \delta\alpha_s^{i,\rm hadr}$$

Central value: μ_0 = 0.3 M_Z , use cluster hadronization-model

- $\delta\alpha_s^{\rm i,stat}, \delta\alpha_s^{\rm i,syst}$: solve $\square$ with $E_i = X_i \pm \sigma^{\rm i,stat/syst}$
- $\delta\alpha_s^{\rm i,scale}$: solve $\square$ with μ = [$\mu_0/2$; $2\mu_0$]
- $\delta\alpha_s^{\rm i,hadr}$: solve $\square$ with the Lund model

The result of this procedure is a set of values of α_s with corresponding errors that need to be combined

α_s from 5-jets: fit details

Define a covariance matrix as the sum of the individual covariance matrices:

$$V = V^{\mathrm{stat}} + V^{\mathrm{syst}} + V^{\mathrm{scale}} + V^{\mathrm{hadr}}$$

$$V_{ij} = \delta_{ij}\left(\delta\alpha_s^i\right)^2 + (1-\delta_{ij})\,\min\left\{(\delta\alpha_s^i)^2, (\delta\alpha_s^i)^2, C_{ij}\delta\alpha_s^j\delta\alpha_s^j\right\}$$

where C_{ij} describes the correlation, for which we assume

- statistical errors: uncorrelated at different Q. At fixed Q: y_{45} bins uncorrelated, R_5 fully correlated, y_{45} and R_5 are correlated for $y_{cut} < y_{45}$ (we compute the correlation)
- systematic errors: fully correlated at fixed Q, but uncorrelated at different Q (detectors were re-calibrated)
- perturbative errors: fully correlated for all observables and Q, except for the LEP I/LEP II correlation that we neglect
- hadronization errors: assumed to be uncorrelated

α_s from 5-jets: fit details

Finally, we compute the weights*

$$w_i = \sum_{j=1}^{N} (V^{-1})_{ij} \Big/ \sum_{k,l=1}^{N} (V^{-1})_{kl}$$

and use them to estimate the average value of the coupling and its error

$$\alpha_s = \sum_{i=1}^{N} w_i \bar{\alpha}_s^i \qquad \sigma^2(\alpha_s) = \sum_{i,j=1}^{N} w_i V_{ij} w_j$$

We take ALEPH data for R_5 and $1/\sigma_{tot}\, d\sigma/dy_{45}$ as measured at LEP I ($Q=M_Z$) and LEP II (Q = 183, 189, 200, 206 GeV) **

(*) For the central value, we neglect off diagonal entries in V^{scale} and V^{hadr}

(**) Similar to other precision studies at LEP we do not take data with Q < 183 GeV. Data for $1/\sigma_{tot}\, d\sigma/dy45$ is not available at Q = 200 GeV.

α_s from 5-jets: fit range

Take fit range as large as possibly, where NLO is reliable and data good enough

LEP I:

We take as a fit range

$3.8 < -\mathrm{Ln}(y_{45}) < 5.2$ (7 data points) and $4.0 < -\mathrm{Ln}(y_{cut}) < 5.6$ (8 data poins)

and estimate the error due to the fit range by performing a second fit with larger ranges

$3.4 < -\mathrm{Ln}(y_{45}) < 5.6$ (11 points) and $3.4 < -\mathrm{Ln}(y_{cut}) < 6.0$ (13 points)

LEP II:

Because of worse quality of data at small y, we reduce the fit ranges to

$4.8 < -\mathrm{Ln}(y_{45}) < 6.4$ (2 points/Q) and $2.1 < -\mathrm{Log}_{10}(y_{cut}) < 2.9$ (4 points/Q)

and estimate the error due to the fit range by performing a second fit with ranges

$4.8 < -\mathrm{Ln}(y_{45}) < 5.6$ (1 point/Q) and $2.1 < -\mathrm{Log}_{10}(y_{cut}) < 2.5$ (2 points/Q)

this choice leads to the *largest* change in α_s

α_s from five-jets at LEP 1

	LEP1, hadr. $\sigma_{\text{tot}}^{-1}\text{d}\sigma/\text{d}y_{45}$, R_5	LEP1, no hadr. $\sigma_{\text{tot}}^{-1}\text{d}\sigma/\text{d}y_{45}$, R_5
stat.	$+0.0001$ -0.0002	$+0.0001$ -0.0002
syst.	$+0.0027$ -0.0029	$+0.0027$ -0.0029
pert.	$+0.0062$ -0.0043	$+0.0068$ -0.0047
fit range	$+0.0014$ -0.0014	$+0.0005$ -0.0005
hadr.	$+0.0012$ -0.0012	–
$\alpha_s(M_Z)$	$0.1159^{+0.0070}_{-0.0055}$	$0.1163^{+0.0073}_{-0.0055}$

- high sensitivity: very small statistical error
- agreement between values extracted with and without hadr. corrections

α_s from five-jets at LEP II

Because hadr. effects are so small at LEP 1 we neglect them at LEP II

	LEP2, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$	LEP2, no hadr. R_5	LEP2, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$, R_5
stat.	$+0.0020$ -0.0022	$+0.0022$ -0.0025	$+0.0015$ -0.0016
syst.	$+0.0008$ -0.0009	$+0.0012$ -0.0012	$+0.0008$ -0.0008
pert.	$+0.0049$ -0.0034	$+0.0029$ -0.0020	$+0.0029$ -0.0020
fit range	$+0.0038$ -0.0038	$+0.0030$ -0.0030	$+0.0028$ -0.0028
$\alpha_s(M_Z)$	$0.1188^{+0.0065}_{-0.0056}$	$0.1116^{+0.0048}_{-0.0045}$	$0.1149^{+0.0044}_{-0.0039}$

Combined α_s from five-jets

Combining α_s from ALEPH data for R_5 and $1/\sigma_{tot}\, d\sigma/dy_{45}$ at LEP I and LEP II we obtain the value of the coupling from five-jet observables

$$\alpha_s(M_Z) = 0.1152^{+0.0037}_{-0.0032}$$

This value compares well with other determinations, and is compatible with the world average (but it is on the lower side)

Performing a simultaneous fit to LEP I and LEP II data we get

$$\alpha_s(M_Z) = 0.1154^{+0.0042}_{-0.0037}$$

in good agreement with the value above

Conclusions

NLO corrections to R_5 and $1/\sigma_{tot}\, d\sigma/dy_{45}$ at LEP I, LEP II are moderate (~+10-20%) and scale uncertainties are reduced by a factor 2 to ~ ±20%

Hadronization corrections from Pythia/Herwig are large and uncertain ⇒ need to use an event generator with correct matrix elements to extract hadronization. *This is an important message for LHC jet-physics too.*

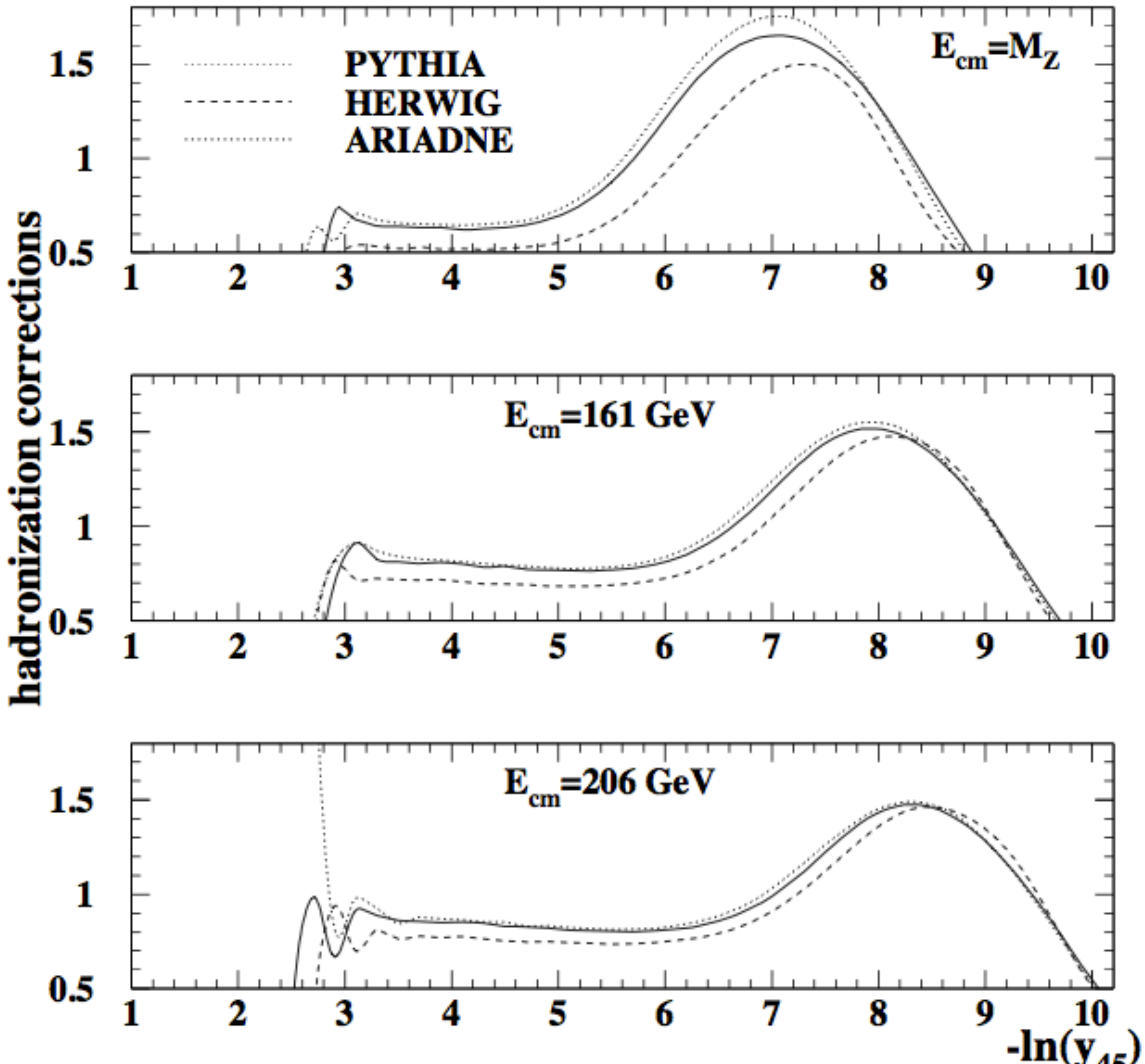
With hadronization corrections from Sherpa, combining α_s from ALEPH data for R_5 and $1/\sigma_{tot}\, d\sigma/dy_{45}$ at LEP I and LEP II we obtain *the value of the coupling from five-jet observables*

$$\alpha_s(M_Z) = 0.1152^{+0.0037}_{-0.0032}$$

There is room for improving this number: resumming large logarithms, with a more detailed knowledge of correlation of systematics and a more sophisticated treatment of theoretical errors

Extra slides

Hadronization at different Q



Hadronization for y_{23}, y_{34}, y_{45}