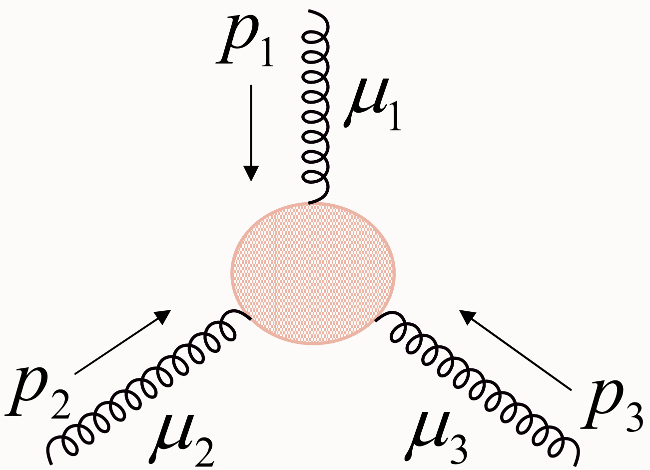
Optimal Renormalization Scales and Schemes for QCD

Stan Brodsky, SLAC

UCLA

February 13, 2007

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$

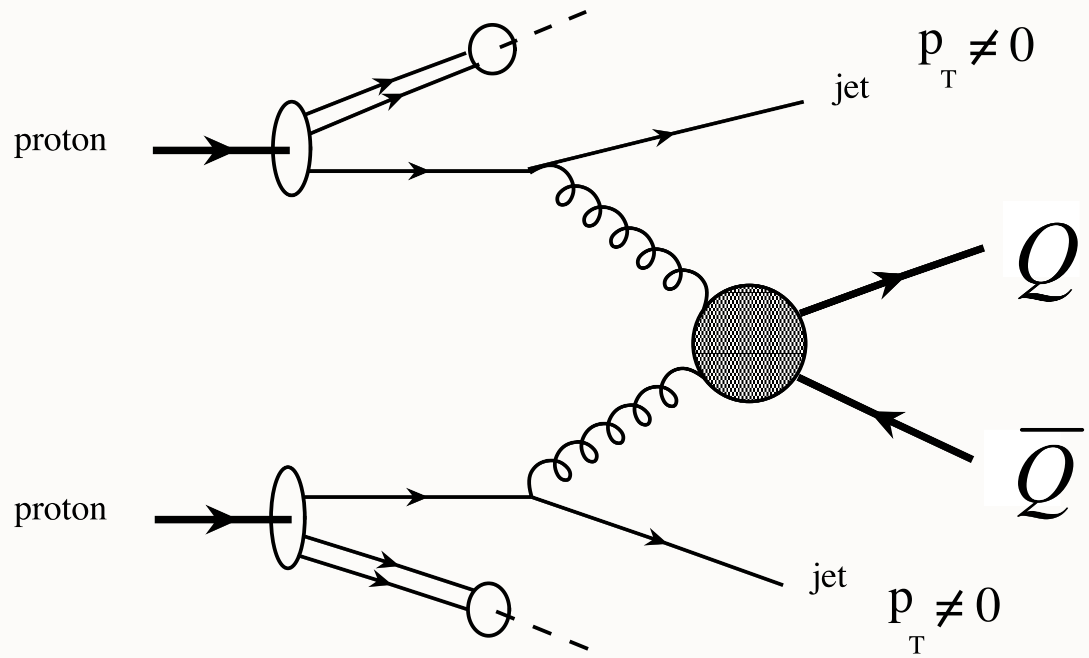


PHYSICAL REVIEW D **74**, 054016 (2006)

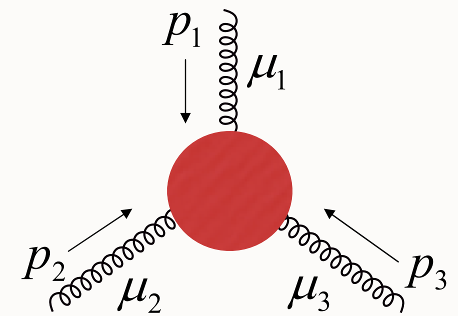
Form factors of the gauge-invariant three-gluon vertex

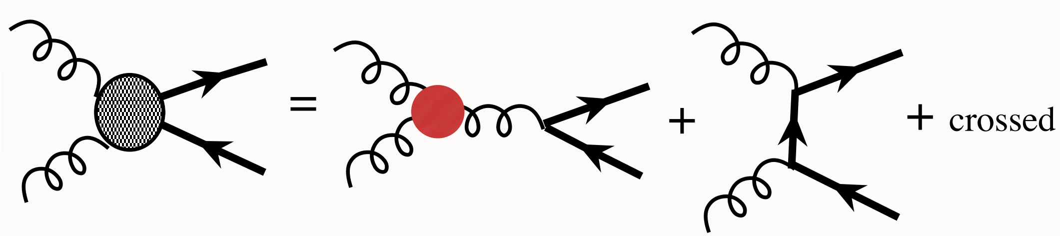
Michael Binger* and Stanley J. Brodsky†

Heavy Quark Hadroproduction



3-gluon coupling depends on 3 physical scales





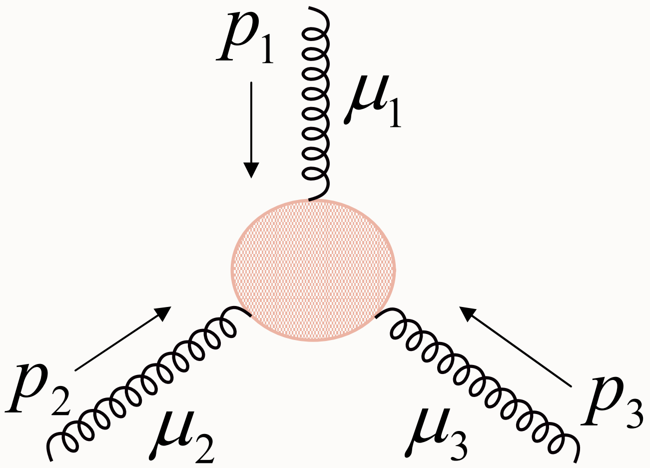
The Renormalization Scale Problem

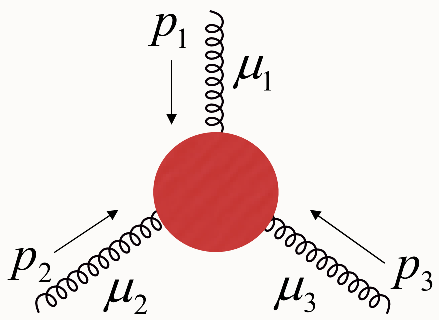
$$\rho(Q^2) = C_0 + C_1\alpha_s(\mu_R) + C_2\alpha_s^2(\mu_R) + \cdots$$

$$\mu_R^2 = CQ^2$$

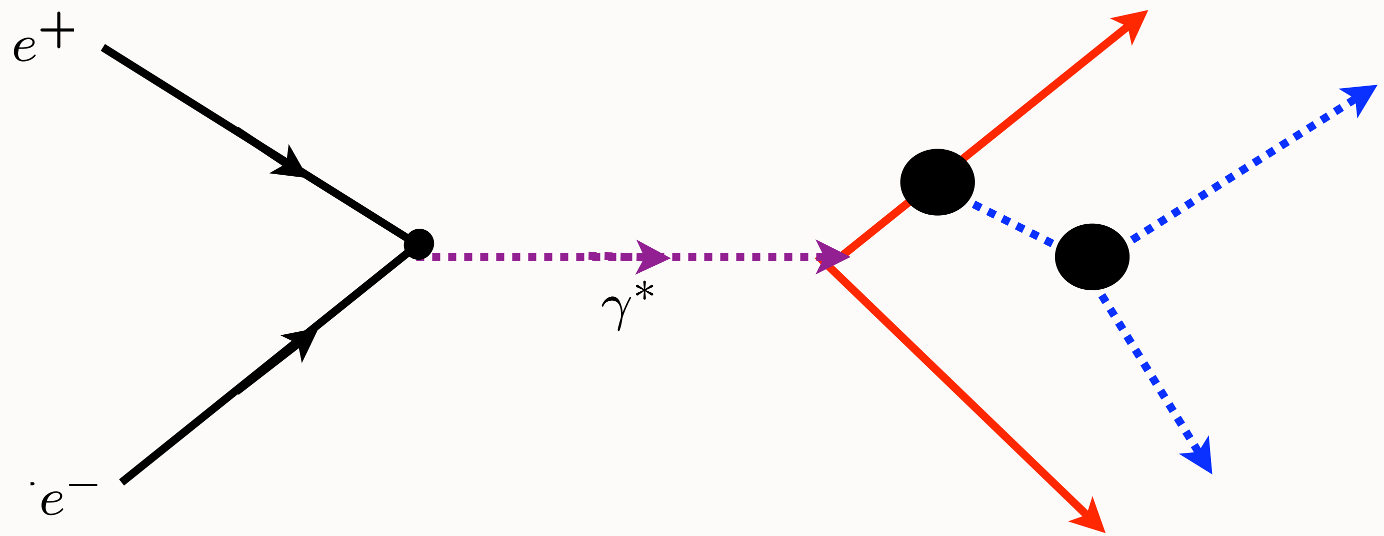
Is there a way to set the renormalization scale μ_R *?*

What happens if there are multiple physical scales ?





$$e^+e^- \to \gamma^* \to \text{4jets}$$



Measurement of the strong coupling α_S from the four-jet rate in e^+e^- annihilation using JADE data

J. Schieck[1,a], S. Bethke[1], O. Biebel[2], S. Kluth[1], P.A.M. Fernández[3], C. Pahl[1],
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Eur. Phys. J. C 48, 3–13 (2006)

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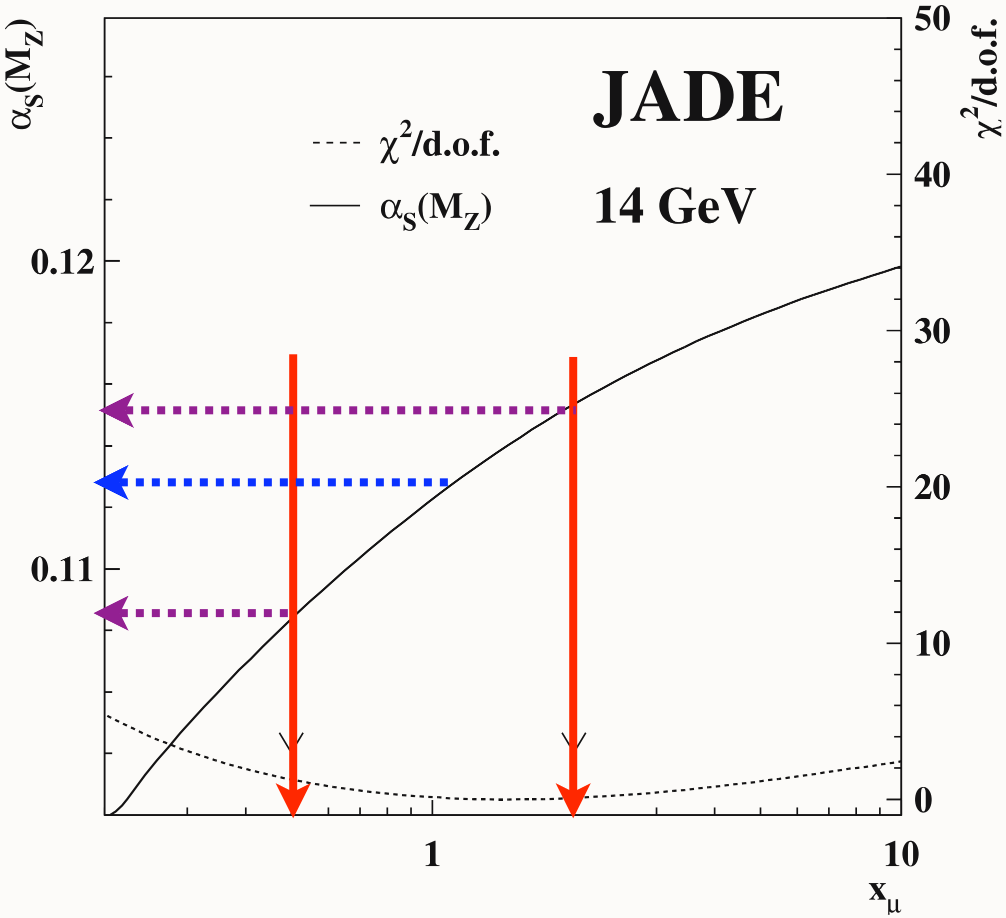


$$x_\mu = \frac{\mu_R}{\sqrt{s}}$$

No PMS

The theoretical uncertainty, associated with missing higher order terms in the theoretical prediction, is assessed by varying the renormalization scale factor x_μ. The predictions of a complete QCD calculation would be independent of x_μ, but a finite-order calculation such as that used here retains some dependence on x_μ. The renormalization scale factor x_μ is set to 0.5 and two. The larger deviation from the default value of α_S is taken as systematic uncertainty.

$\alpha_S(M_{Z^0})$ and the χ^2/d.o.f. of the fit to the four-jet rate as a function of the renormalization scale x_μ for $\sqrt{s}$ = 14 GeV to 43.8 GeV. The arrows indicate the variation of the renormalization scale factor used for the determination of the systematic uncertainties

Conventional wisdom concerning scale setting

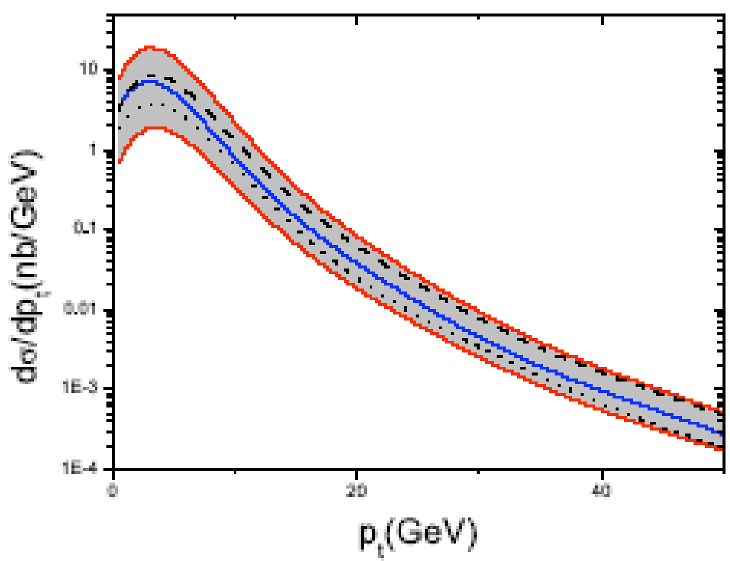
- Renormalization scale can be set to any value; e.g. $\mu_R = Q$
- Sensitivity to renormalization scale disappears at high order
  (only true if mass thresholds are incorporated)
- No optimal scale
- Ignore problem of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking a range
  $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

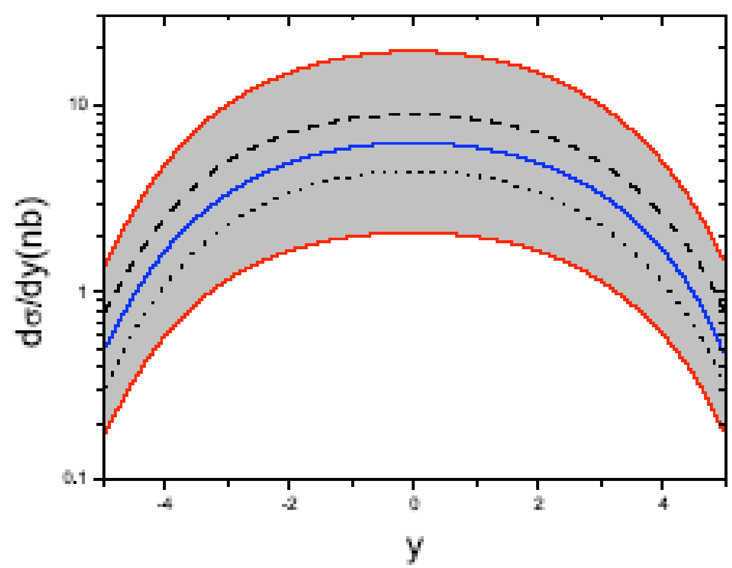
All of these assumptions are fallacious

Chao-Hsi Chang

Uncertainties in P-wave Bc Production due to factorization energy scale

The summed P_t distribution and y distribution of all the P-wave states for different factorization scale μ^2_F and renormalization scale μ^2 at LHC





The upper edge of the band corresponds to $\mu^2_F=4M_{Pt}^2$; $\mu^2=M_{Pt}^2/4$; and the lower edge corresponds to that of $\mu^2_F=M_{Pt}^2/4$; $\mu^2=4M_{Pt}^2$. The solid line, the dotted line and the dashed line corresponds to that of $\mu^2_F=\mu^2=M_{Pt}^2$; $\mu^2_F=\mu^2=4M_{Pt}^2$; $\mu^2_F=\mu^2=M_{Pt}^2/4$.

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

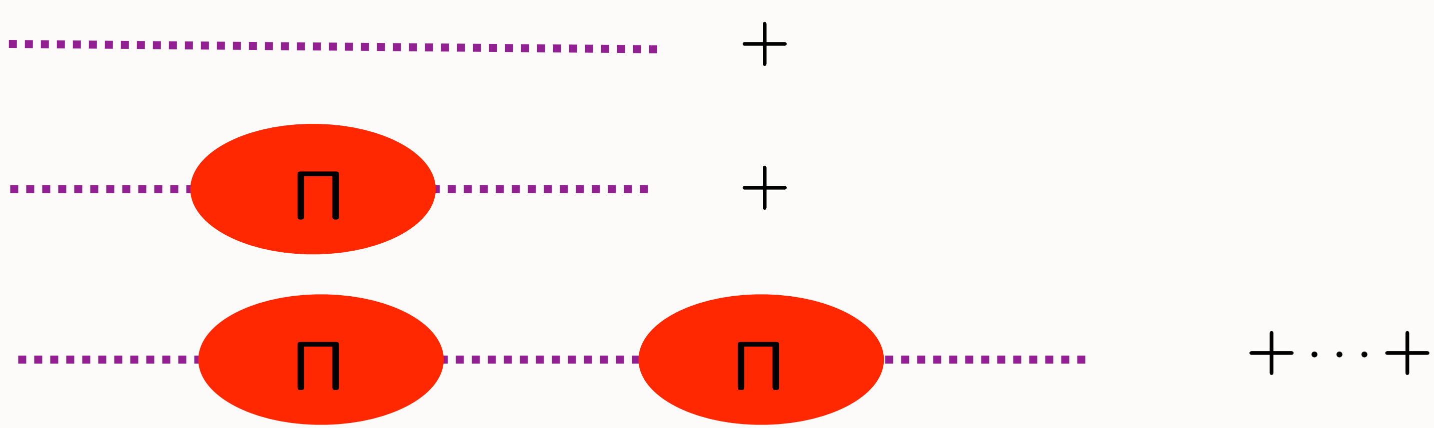
$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

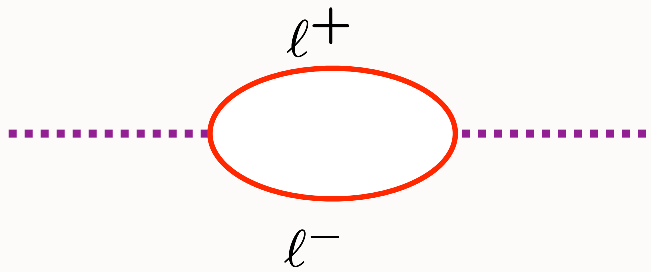
All-orders leptonic loop corrections to dressed photon propagator



$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)}$$

$$\Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}[\frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2})\sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|}]$$

Analytically continue to timelike t: Complex

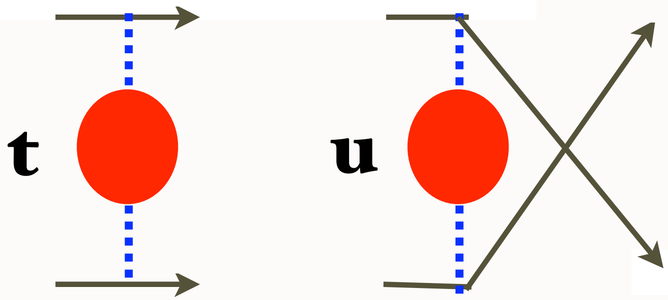
$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi}\frac{Q^2}{m^2}$$ $Q^2 << 4M^2$ **Serber-Uehling**

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\frac{\log Q^2}{m^2}$$ $Q^2 >> 4M^2$ **Landau Pole**

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d \log Q^2} = \frac{4}{3}(\frac{\alpha}{4\pi})^2 n_\ell > 0$$

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- Two separate physical scales.
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one must sum an infinite number of graphs -- but then recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!

$$\beta_{MS}(\alpha) = \sum_{i=1}^{4} \beta_i \left(\frac{\alpha}{4\pi}\right)^{i+1}$$

$$= \frac{4}{3}N\left(\frac{\alpha}{4\pi}\right)^2 + 4N\left(\frac{\alpha}{4\pi}\right)^3 - \left(2N + \frac{44}{9}N^2\right)\left(\frac{\alpha}{4\pi}\right)^4$$

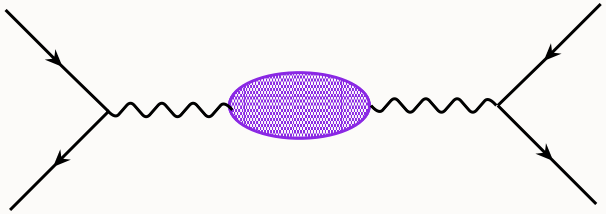
$$-\left\{46N + \left[-\frac{760}{27} + \frac{832}{9}\zeta(3)\right]N^2 + \frac{1232}{243}N^3\right\}\left(\frac{\alpha}{4\pi}\right)^5$$

The analytic four-loop corrections
to the QED β-function in the MS scheme
and to the QED ψ-function.
Total reevaluation

S.G. Gorishny [1], A.L. Kataev, S.A. Larin and L.R. Surguladze [2]
Institute of Nuclear Research, Academy of Sciences of the USSR, SU-117 312 Moscow, USSR

Phys.Lett.B256:81-86,1991

$$e^+e^- \to \mu^+\mu^-$$



$$\mu_R^2 = s$$

Scale of $\alpha(\mu_r)$ unique !

$$M \propto \alpha(s)$$

The QED Effective Charge

- Complex
- Analytic through mass thresholds
- Distinguishes between timelike and spacelike momenta

Analyticity essential !

$$M(e^+e^- \to e^+e^-) \propto \alpha(s)$$

Has correct analytic / unitarity thresholds for $\mathrm{Im}M$ at $s = 4m^2_{\ell^+\ell^-}$

No other scale correct. If one chooses another scale, e.g.,

$$\mu_R^2 = 0.9s,$$

then must resum infinite number of vacuum polarization diagrams.

Recover $\alpha(s)$.

Lessons from QED : Summary

- Effective couplings are complex analytic functions with the correct threshold structure expected from unitarity
- Multiple “renormalization” scales appear
- The scales are unambiguous since they are physical kinematic invariants
- Optimal improvement of perturbation theory

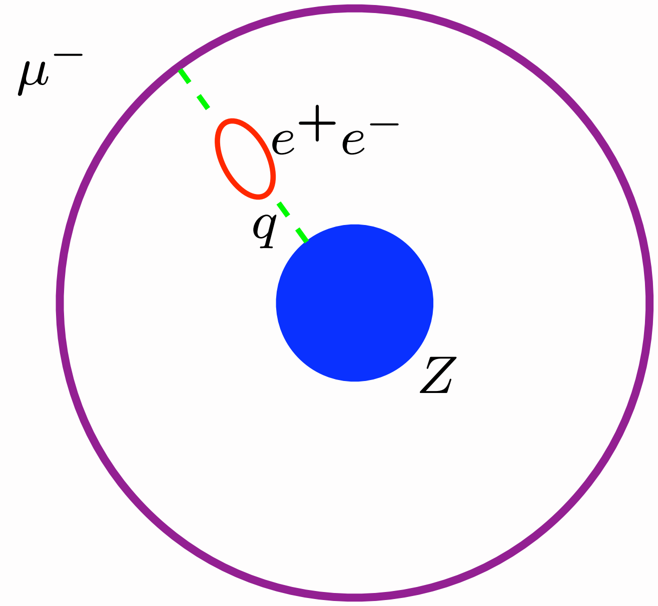
The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling can be defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds
- Examples: muonic atoms, g-2, Lamb Shift

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion

Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

QCD Lagrangian

gluon dynamics — quark kinetic energy + quark-gluon dynamics — mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge — field strength tensor — covariant derivative — quark field

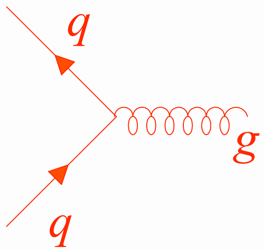
Yang Mills Gauge Principle:
Color Rotation and Phase
Invariance at Every Point of
Space and Time

Scale-Invariant Coupling
Renormalizable
Conformal Template
Asymptotic Freedom
Color Confinement

QCD

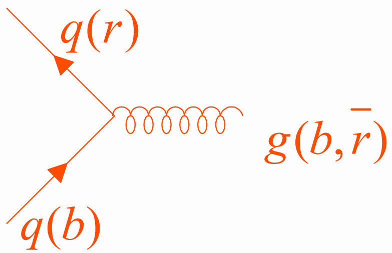
Fundamental Couplings

Only quarks and gluons involve basic vertices: Quark-gluon vertex

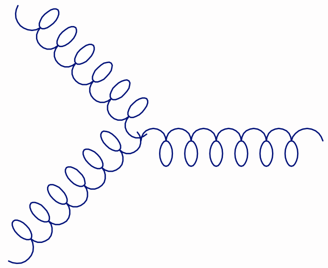


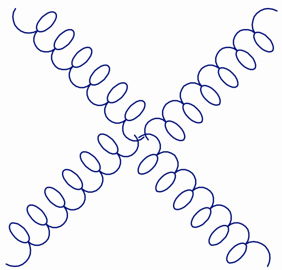
Similar to QED

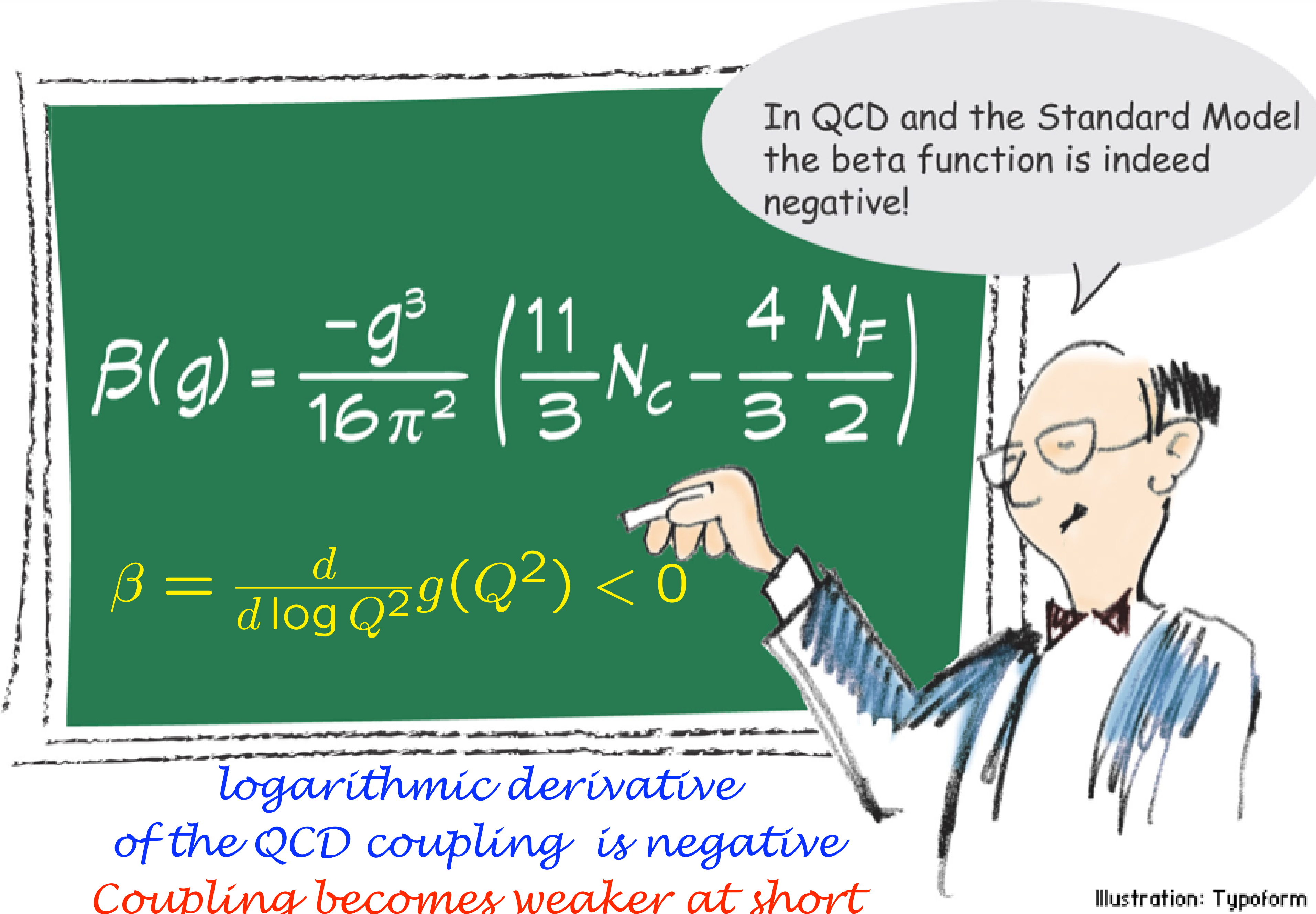
More exactly



Gluon vertices







$$\beta(g) = \frac{-g^3}{16\pi^2}\left(\frac{11}{3}N_c - \frac{4}{3}\frac{N_F}{2}\right)$$

$$\beta = \frac{d}{d\log Q^2} g(Q^2) < 0$$

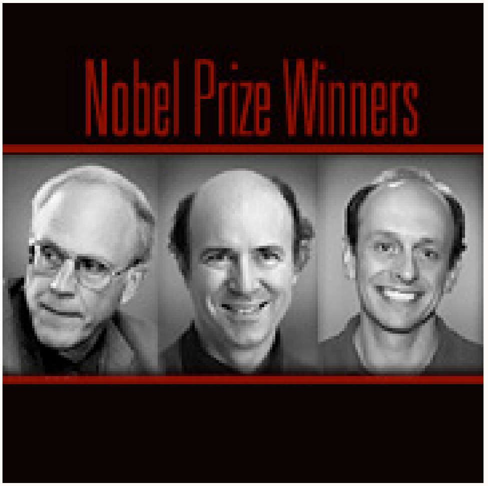
logarithmic derivative of the QCD coupling is negative

Coupling becomes weaker at short distances or high momentum transfer

Gross, Wilczek, Politzer
Khriplovich, `t Hooft

Verification of Asymptotic Freedom

$$\alpha(Q^2) \simeq \frac{4\pi}{\beta_0} \frac{1}{\log Q^2/\Lambda^2_{QCD}}$$



Gross, Wilczek, Politzer
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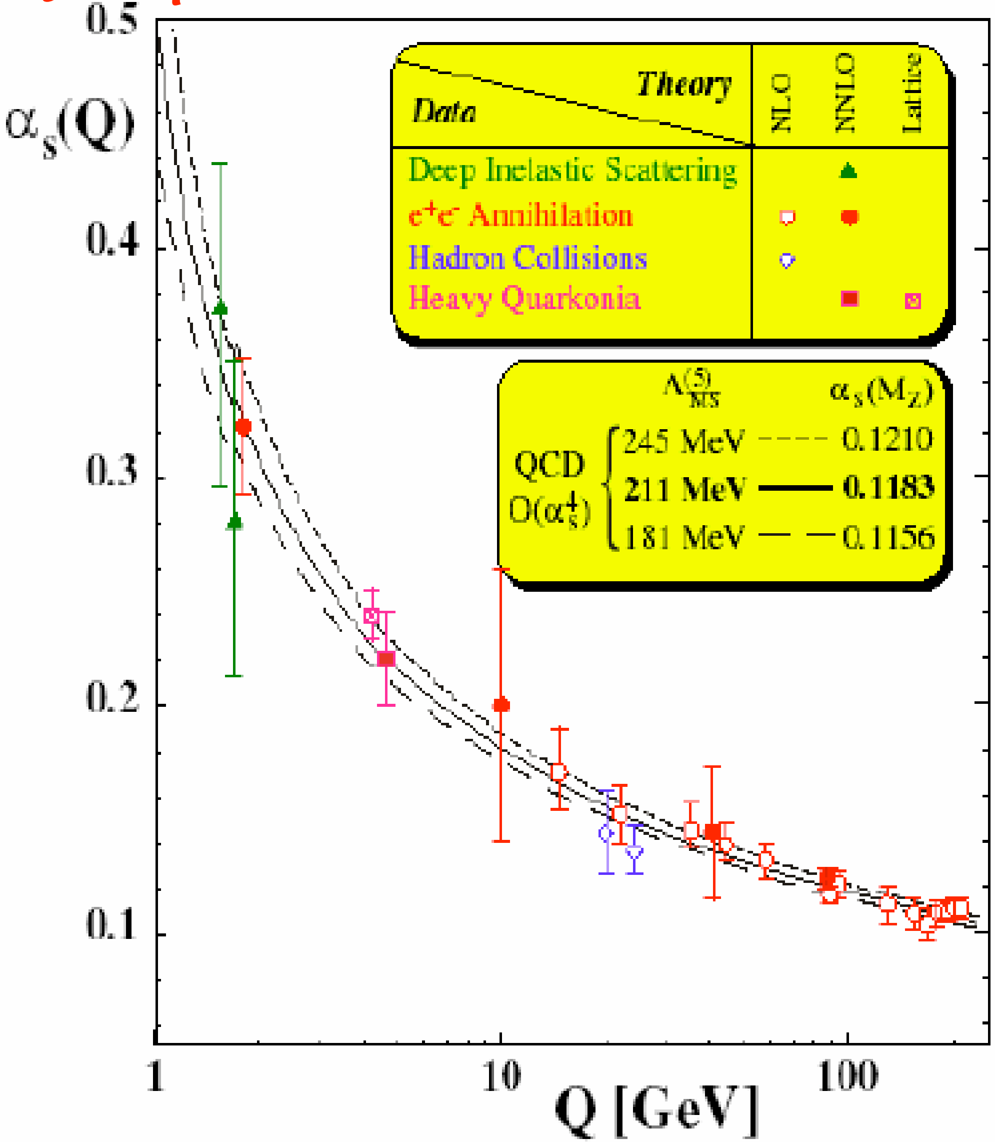
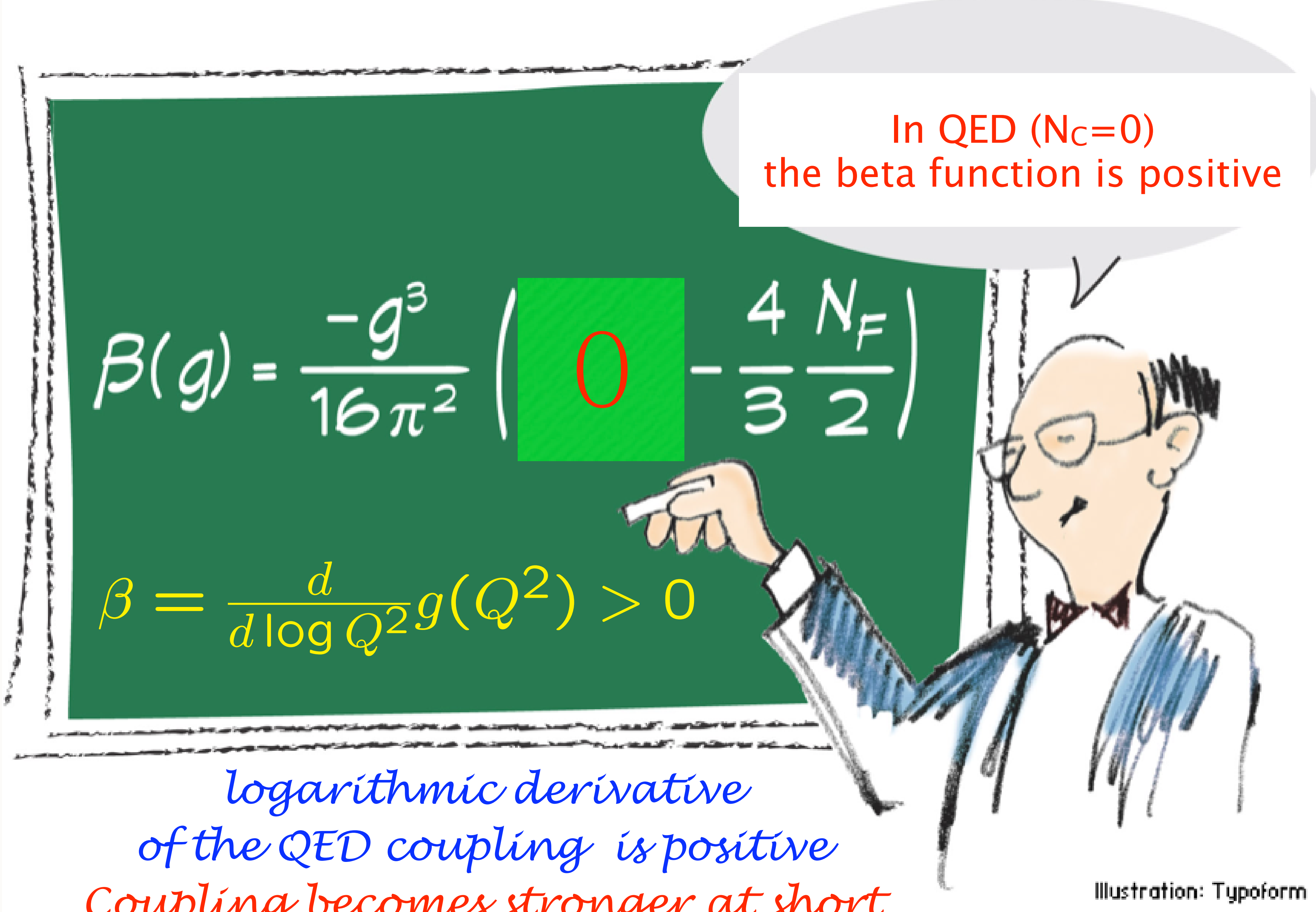


$$\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}$$

proportional to $\alpha_s(Q)$

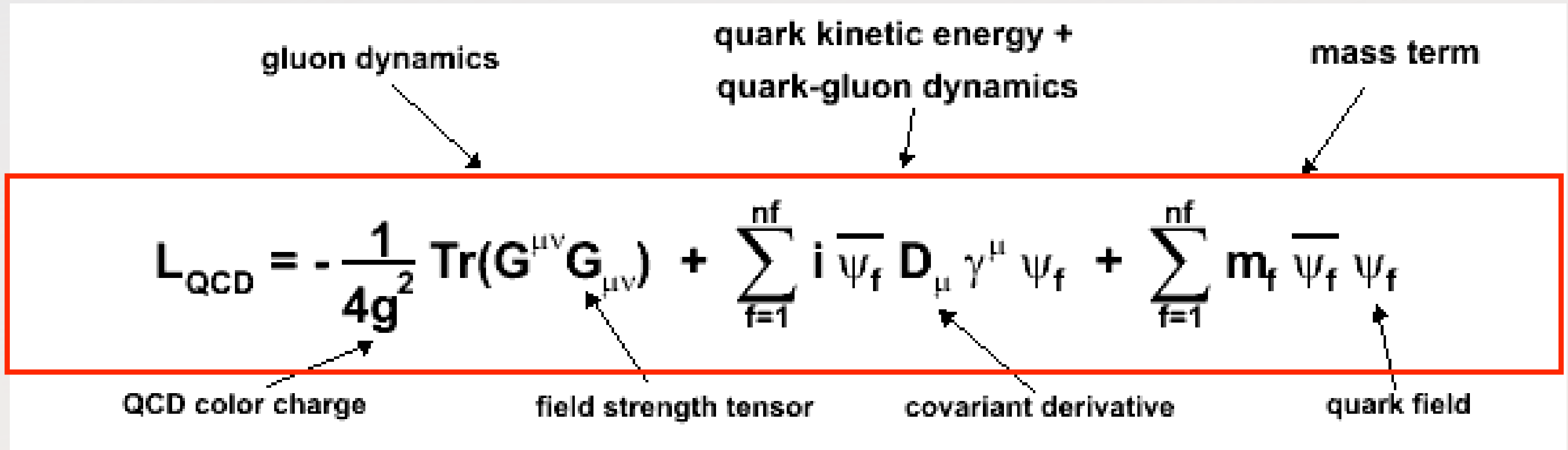
Ratio of rate for $e^+e^- \to q\bar{q}g$ to $e^+e^- \to q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$



$$\beta = \frac{d}{d \log Q^2} g(Q^2) > 0$$

logarithmic derivative
of the QED coupling is positive
Coupling becomes stronger at short distances or high momentum transfer

QCD Lagrangian



$$L_{QCD} = -\frac{1}{4g^2}\,\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{nf} i\,\overline{\psi_f}\,D_\mu\,\gamma^\mu\,\psi_f + \sum_{f=1}^{nf} m_f\,\overline{\psi_f}\,\psi_f$$

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F/C_F \qquad [C_F = \frac{N_C^2 - 1}{2N_C}]$$

Analytic limit of QCD: Abelian Gauge Theory

P. Huet, sjb

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

IR Fixed Point for QCD?

- Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point! Alkofer, Fischer, von Smekal et al.

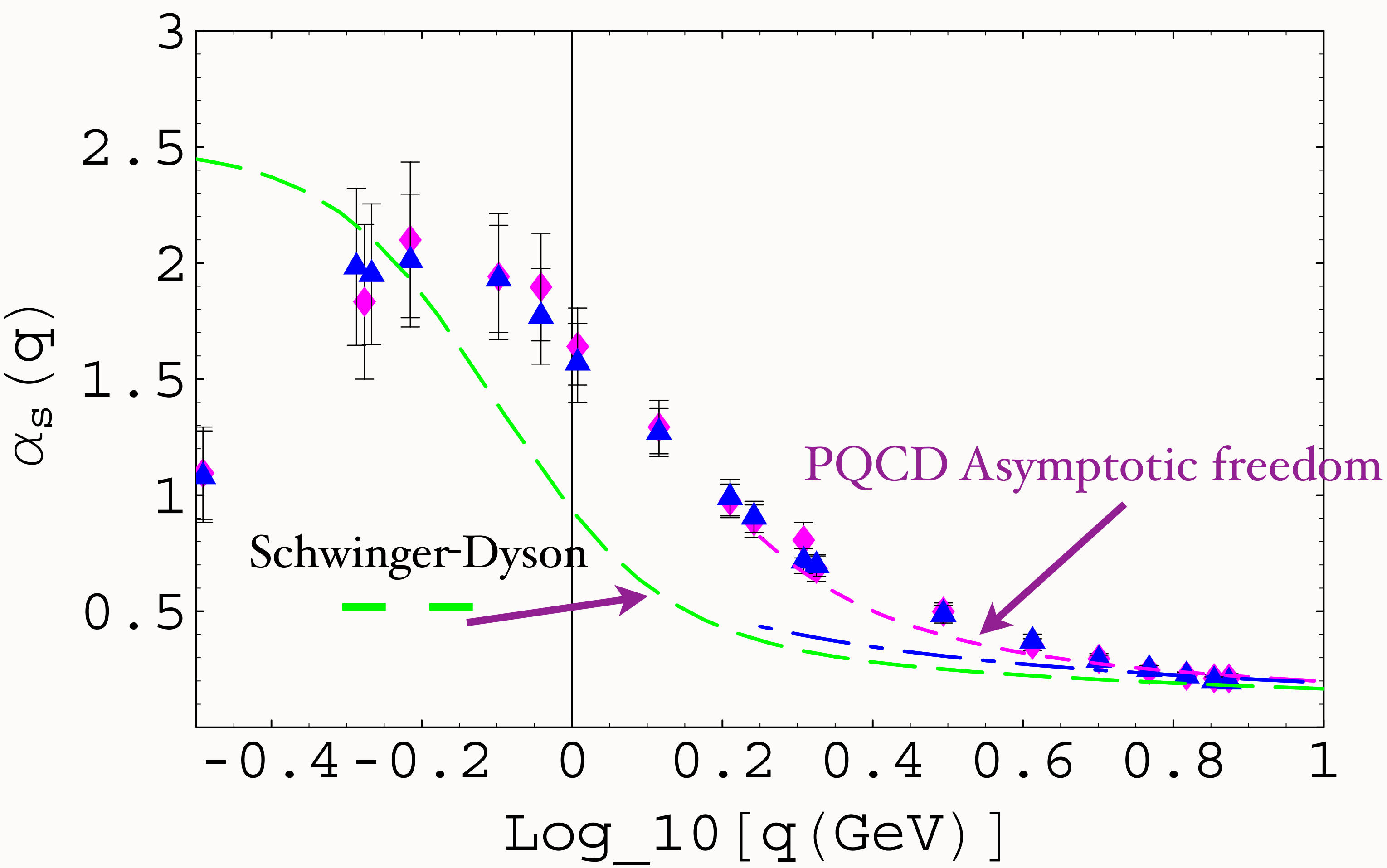
- Lattice Gauge Theory

- Define coupling from observable, indications of IR fixed point for QCD effective charges

- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small Q^2

- Justifies application of AdS/CFT in strong-coupling conformal window

Infrared-Finite QCD Coupling?



Lattice simulation (MILC)

Furui, Nakajima

DSE: Alkofer, Fischer, von Smekal et al.

Define QCD Coupling from Observable

Grunberg

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e \nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d} e \nu) \times [1 + \frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

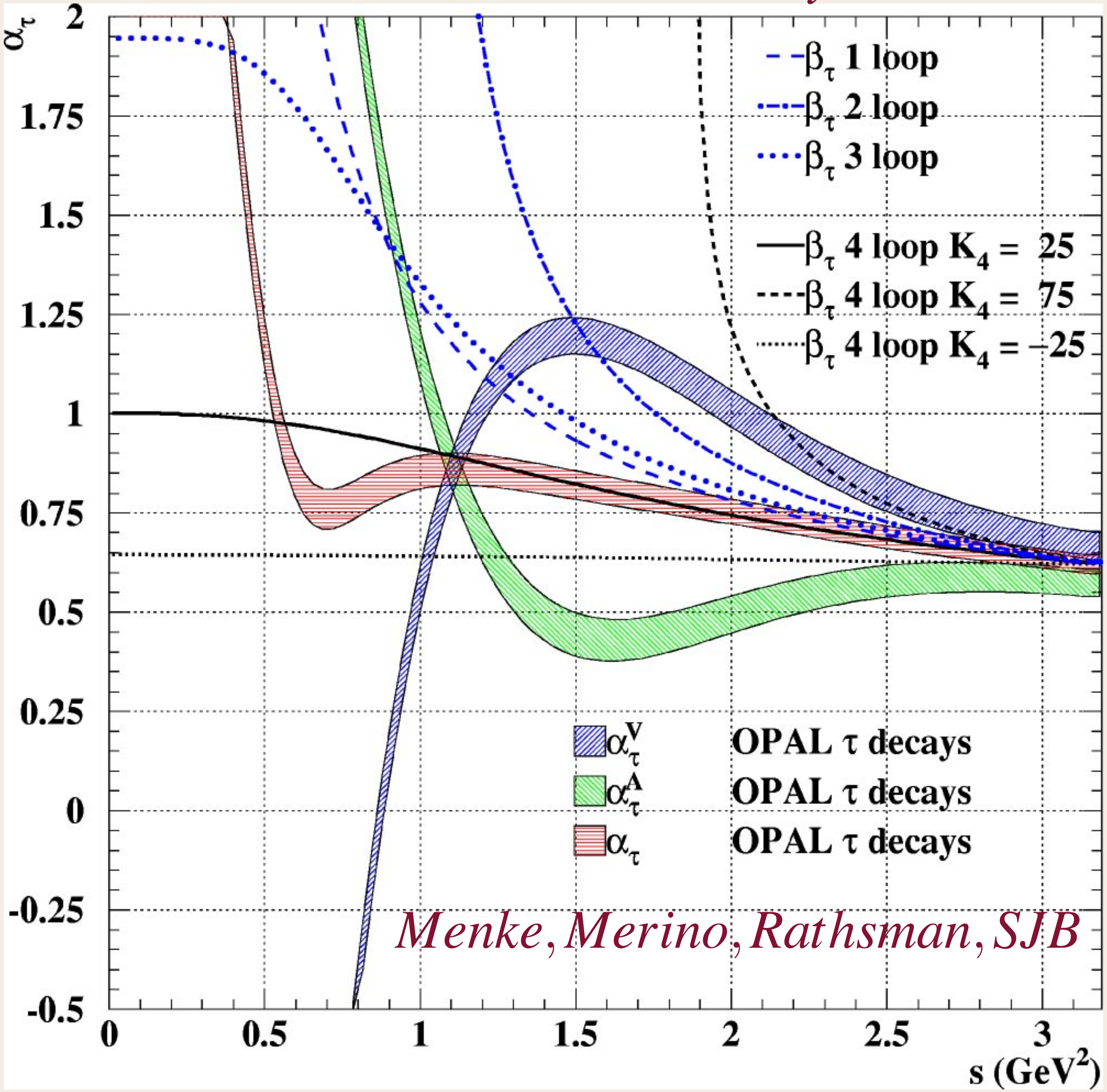
Commensurate scale relations:
Relate observable to observable at commensurate scales

Effective Charges: analytic at quark mass thresholds, finite at small momenta

H.Lu, Rathsman, sjb

Pinch scheme: Cornwall, et al

QCD Effective Coupling from *hadronic τ decay*



Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses
- Eigensolutions of ERBL evolution equation for distribution amplitudes

V. Braun et al;
Frishman, Lepage, Sachrajda, sjb

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation
- Use AdS/CFT

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

BLM Scale Setting

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2}\beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right],$$

Conformal coefficient - independent of β

where

$$Q^* = Q \exp(3A_{VP}),$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B .$$

The term $33A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

Use skeleton expansion:

Gardi, Grunberg, Rathsman, sjb

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$R_{e^+e^-}(Q^2) = 3 \sum_q e_q^{\,2} \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \frac{\alpha_{\overline{MS}}^{\,2}}{\pi^2}(1.98 - 0.115 n_f) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

$$\to 3 \sum_q e_q^{\,2} \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} + \frac{\alpha_{\overline{MS}}^{\,2}(Q^*)}{\pi^2} 0.08 + \cdots \right],$$

Conformal coefficient - independent of β

$Q^* = 0.710Q.$ Notice that $\alpha_R(Q)$ differs from $\alpha_{\overline{MS}}(Q^*)$ by only $0.08\alpha_{\overline{MS}}/\pi$, so that $\alpha_R(Q)$ and $\alpha_{\overline{MS}}(0.71Q)$ are effectively interchangeable (for any value of n_f).

Deep-inelastic scattering. The moments of the nonsinglet structure function $F_2(x,Q^2)$ obey the evolution equation

$$Q^2\frac{d}{dQ^2}\ln M_n(Q^2)$$

$$=-\frac{\gamma_n^{(0)}}{8\pi}\alpha_{\overline{\rm MS}}(Q)\left[1+\frac{\alpha_{\overline{\rm MS}}}{4\pi}\frac{2\beta_0\beta_n+\gamma_n^{(1)}}{\gamma_n^{(0)}}+\cdots\right]$$

$$\to-\frac{\gamma_n^{(0)}}{8\pi}\alpha_{\overline{\rm MS}}(Q_n^*)\left[1-\frac{\alpha_{\overline{\rm MS}}(Q_n^*)}{\pi}C_n+\cdots\right],$$

where, for example,

$$Q_2^*=0.48Q,\quad C_2=0.27\ ,$$

$$Q_{10}^*=0.21Q,\quad C_{10}=1.1\ .$$

For n very large, the effective scale here becomes $Q_n^*\sim Q/\sqrt{n}$

BLM scales for DIS moments

$$V(Q^2) = -\frac{C_F 4\pi\alpha_{\overline{MS}}(Q)}{Q^2}\left[1+\frac{\alpha_{\overline{MS}}}{\pi}\left(\tfrac{5}{12}\beta_0-2\right)+\cdots\right]$$

$$\rightarrow -\frac{C_F 4\pi\alpha_{\overline{MS}}(Q^*)}{Q^2}\left[1-\frac{\alpha_{\overline{MS}}(Q^*)}{\pi}2+\cdots\right],$$

where $Q^* = e^{-5/6}\ Q \cong 0.43Q$. This result shows that the effective scale of the $\overline{\text{MS}}$ scheme should generally be about half of the true momentum transfer occurring in the interaction. In parallel to QED, the effective potential $V(Q^2)$ gives a particularly intuitive scheme for defining the QCD coupling constant

$$V(Q^2) \equiv -\frac{4\pi C_F \alpha_v(Q)}{Q^2}$$

Similar to PT scheme

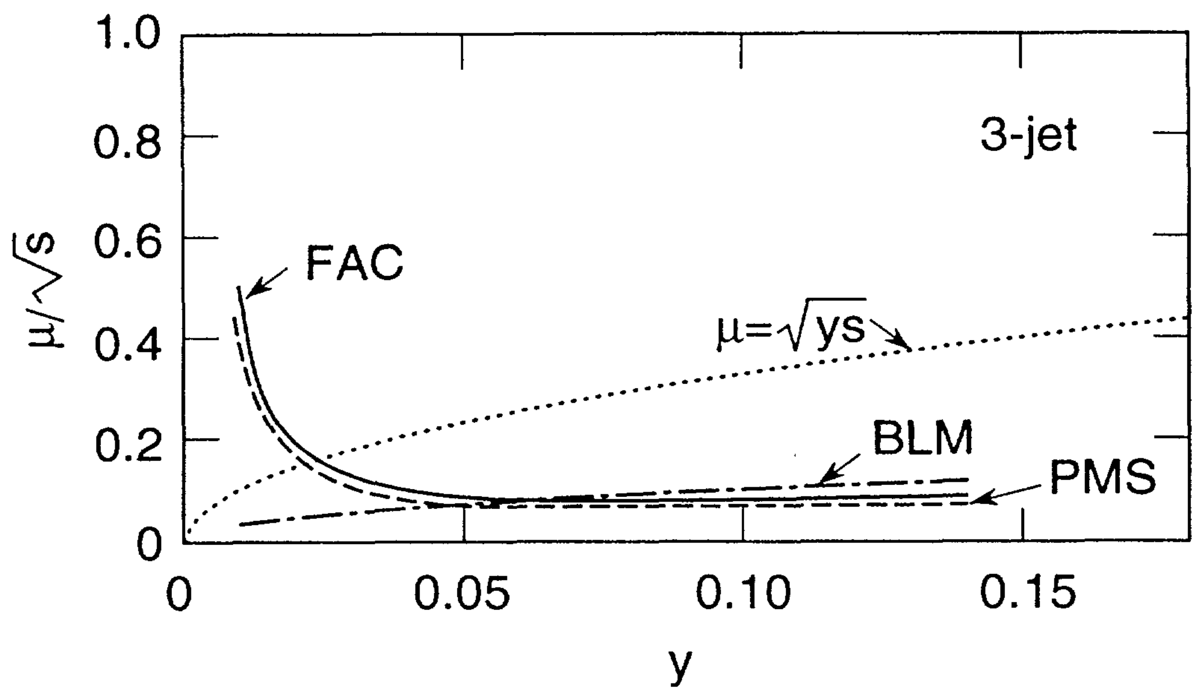
Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb

Phys.Rev.D28:228,1983

- All terms associated with nonzero beta function summed into running coupling
- Identical procedure in QED
- Resulting series identical to conformal series
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants



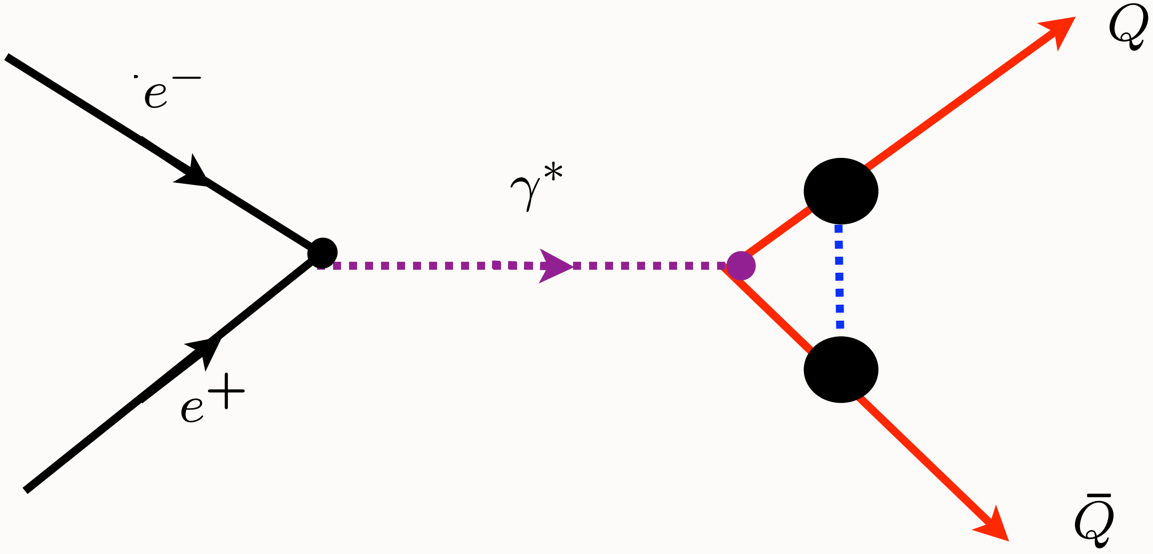
Kramer & Lampe

Three-Jet Rate

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

Rathsman

Other Jet Observables:



$$F_1 + F_2 = 1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta} - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}$$
$$\cong \left(1 - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}\right)\left(1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta}\right)$$

Example of Multiple BLM Scales

Angular distributions of massive quarks and leptons close to threshold.

Example of Multiple BLM Scales

Angular distributions of massive quarks and leptons close to threshold.
S.J. Brodsky (**SLAC**) , **A.H. Hoang** (**Karlsruhe U., TTP**) , **Johann H. Kuhn** (**SLAC** & **Karlsruhe U., TTP**) , **T. Teubner** (**Karlsruhe U., TTP**) . **SLAC-PUB-6955, SLAC-PUB-95-6955, TTP-95-26, Jul 1995. 13pp.**
Published in Phys.Lett.B359:355-361,1995.
e-Print Archive: **hep-ph/9508274**

$$A = \frac{|G_m|^2 - (1-\beta^2)|G_e|^2}{|G_m|^2 + (1-\beta^2)|G_e|^2} \qquad A = \frac{\tilde{A}}{1-\tilde{A}}$$

$$\frac{d\,\sigma(e^+e^- \to f\bar{f})}{d\,\Omega} = \frac{\alpha^2 Q_f^2 \beta}{4\,s}\left[\frac{4\,m^2}{s}|G_e|^2 \sin^2\theta + |G_m|^2(1+\cos^2\theta)\right]$$

$$\tilde{A} = \frac{\beta^2}{2}\,\frac{\left(1 - 4\,\frac{\alpha_V(m^2 e^{7/6})}{\pi}\right)}{\left(1 - \frac{16}{3}\,\frac{\alpha_V(m^2 e^{3/4})}{\pi}\right)}\,\frac{1-e^{-x_s}}{1-e^{-x'_s}}\,\frac{\alpha_V(4\,m^2\beta^2/e)}{\alpha_V(4\,m^2\beta^2)}$$

$$x_s = \frac{4\,\pi}{3}\,\frac{\alpha_V(4\,m^2\beta^2)}{\beta}, \qquad x'_s = \frac{4\,\pi}{3}\,\frac{\alpha_V(4\,m^2\beta^2/e)}{\beta}.$$

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right]$$
$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648} f^2\right\}.$$

Eliminate MSbar, Find Amazing Simplification

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx \left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

add Light-by-Light

Lu, Kataev, Gabadadze, Sjb

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)
Analytic matching at quark thresholds
No renormalization scale ambiguity!

* Why is the relation between

α_R and α_{g_1}

so simple?

{ Gabadadze, Kataev, H.J. Lu, SJB }

Consider conformal limit $\beta_0 \Rightarrow 0$, $\beta_1 \rightarrow 0$ ...

$$\text{CSR} \Rightarrow \quad (1+\hat{\alpha}_R)(1-\hat{\alpha}_{g_1}) = 1$$

* Follows from Crewther relation!

$\beta = 0$ chiral

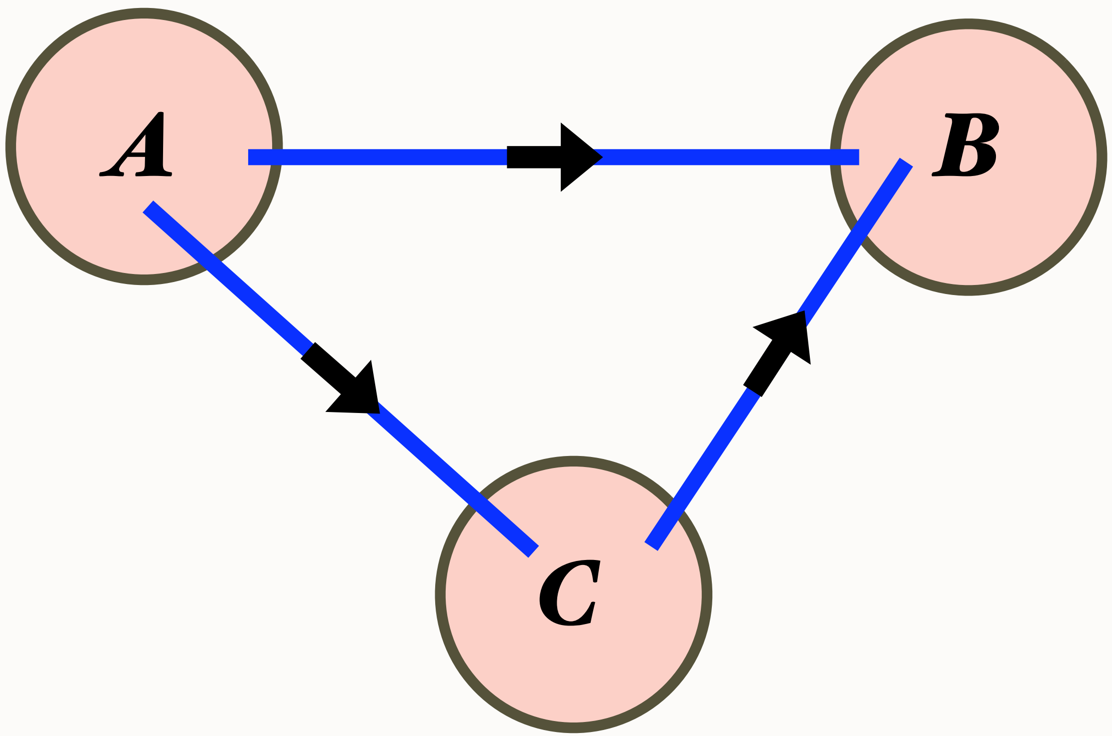
$$3S = K R' = K\left(\tfrac{4}{3} R\right)$$

↑ $\pi^0 \to \gamma\gamma$ (Adler, Bardeen) ↑ β_J GLS ↑ $\hat{R}_{e^+e^-}$ ↑ $\check{R}_{e^+e^-}$

Deviations from Crewther Relation proportional to β

Kataev, Broadhurst

Transitivity Property of Renormalization Group



A ➡ ***C*** ***C*** ➡ ***B*** ***identical to*** ***A*** ➡ ***B***

Relation of observables independent of intermediate scheme C

Commensurate Scale Relation:

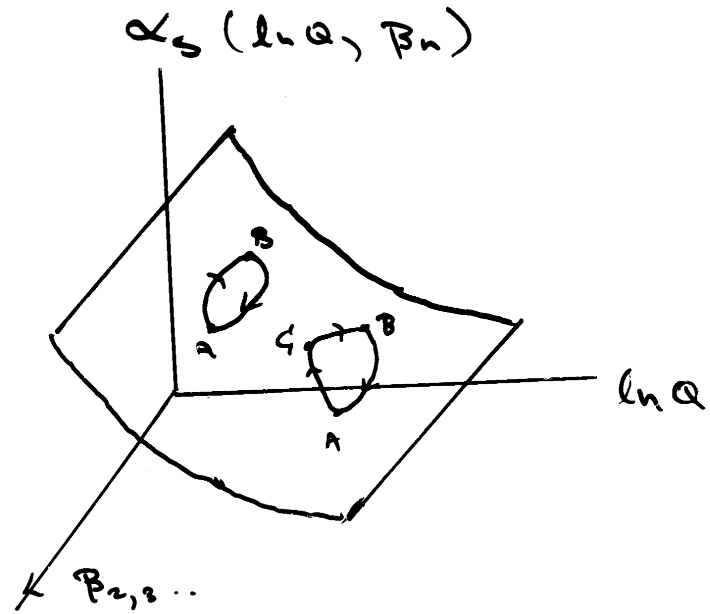
$$* \quad \alpha_B(Q_B) = \alpha_A(Q_A)\left[1 + C^{(1)}_{A/B}\frac{\alpha_A}{\pi} + \cdots\right]$$

↑ conformal coeff.

$$Q_B/Q_A = \lambda_{B/A}$$

Peterman Stückelberg Renormalization "Group"

$\lambda_{B/A} = \lambda_{B/C} / \lambda_{A/C}$	transitive
$\lambda_{B/A} = \lambda^{-1}_{A/B}$	symmetry
$\lambda_{A/A} = I$	identity



Transitivity of the renormalization group implies predictions for a physical observable $\mathcal{O}$ cannot depend on choice of intermediate renormalization scheme,

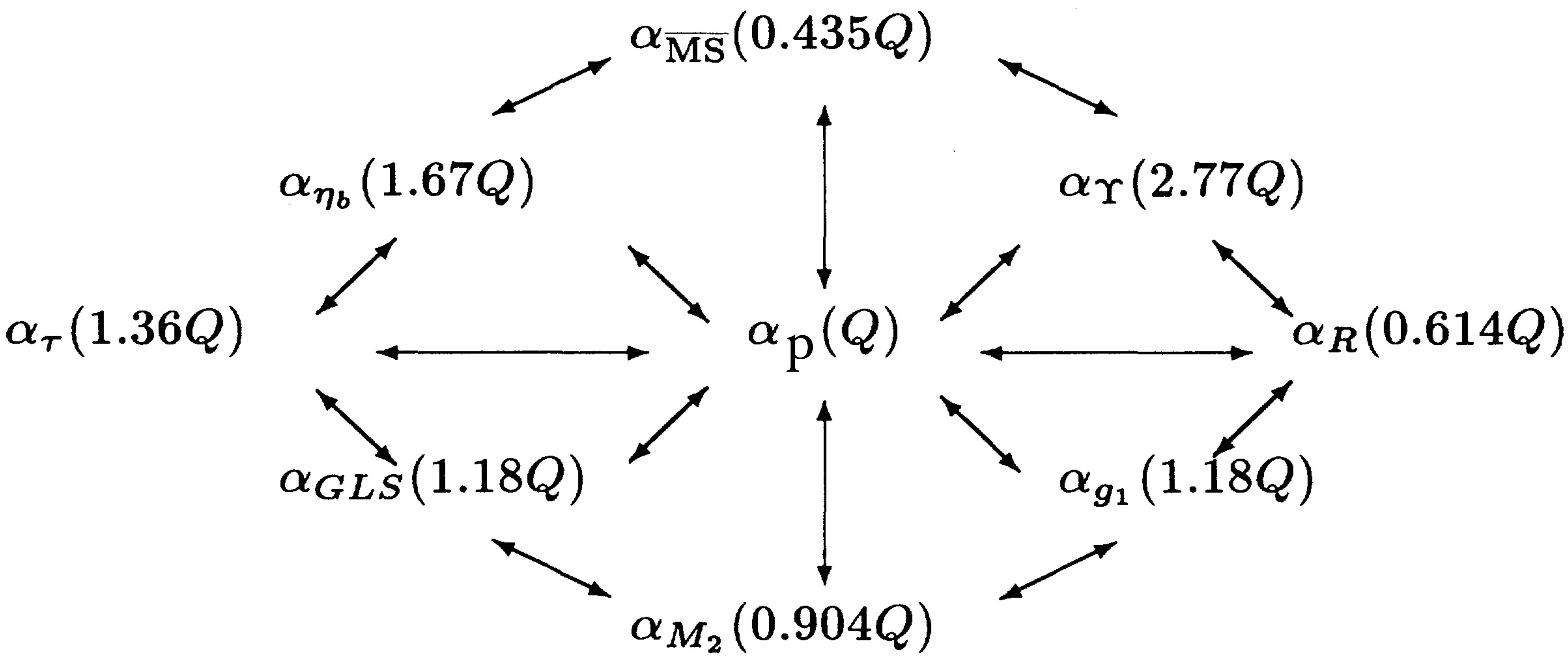
e.g., choice of $\alpha_{\overline{MS}}$ or α_{mom}.

$$\frac{d\mathcal{O}}{d\mu_{\text{scheme}}} = 0$$

not

$$\frac{d\mathcal{O}}{d\mu_{\text{renormalization}}} = 0$$

Leading Order Commensurate Scales



Translation between schemes at LO

Use Physical Scheme to Characterize QCD Coupling

- Use Observable to define QCD coupling or Pinch Scheme
- Analytic: Smooth behavior as one crosses new quark threshold
- New perspective on grand unification

Conformal symmetry: Template for QCD

- Initial approximation to PQCD; then correct for non-zero beta function and quark masses
- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation
- Arguments for Infrared fixed-point for α_s Alhofer, et al.
- Effective Charges: analytic at quark mass thresholds, finite at small momenta
- Eigensolutions of Evolution Equation of distribution amplitudes

Analyticity and Mass Thresholds

$\overline{MS}$ does not have automatic decoupling of heavy particles

→ Must define a set of schemes in each desert region and match

$$\alpha_s^{(f)}(M_Q) = \alpha_s^{(f+1)}(M_Q)$$

- The coupling has discontinuous derivative at the matching point
- At higher orders the coupling itself becomes discontinuous!
- Does not distinguish between spacelike and timelike momenta

"AN ANALYTIC EXTENSION OF THE MS-BAR RENORMALIZATION SCHEME"
S. Brodsky, M. Gill, M. Melles, J. Rathsman. **Phys.Rev.D58:116006,1998**

Unification in Physical Schemes

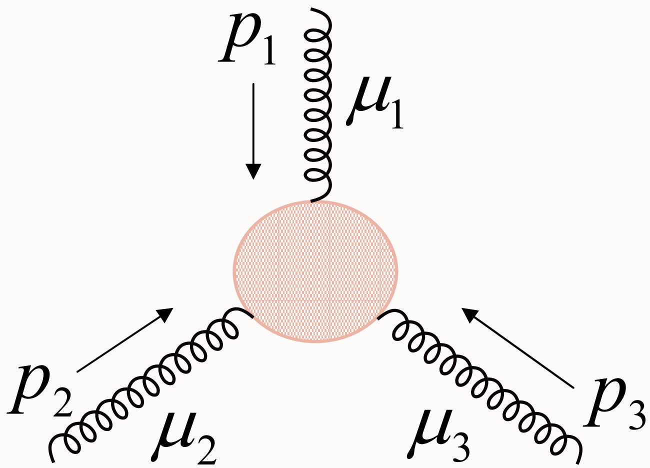
- Smooth analytic threshold behavior with automatic decoupling

- More directly reflects the unification of the forces

- Higher “unification” scale than usual

General Structure of the Three-Gluon Vertex

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



Full calculation, general masses, spin

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3

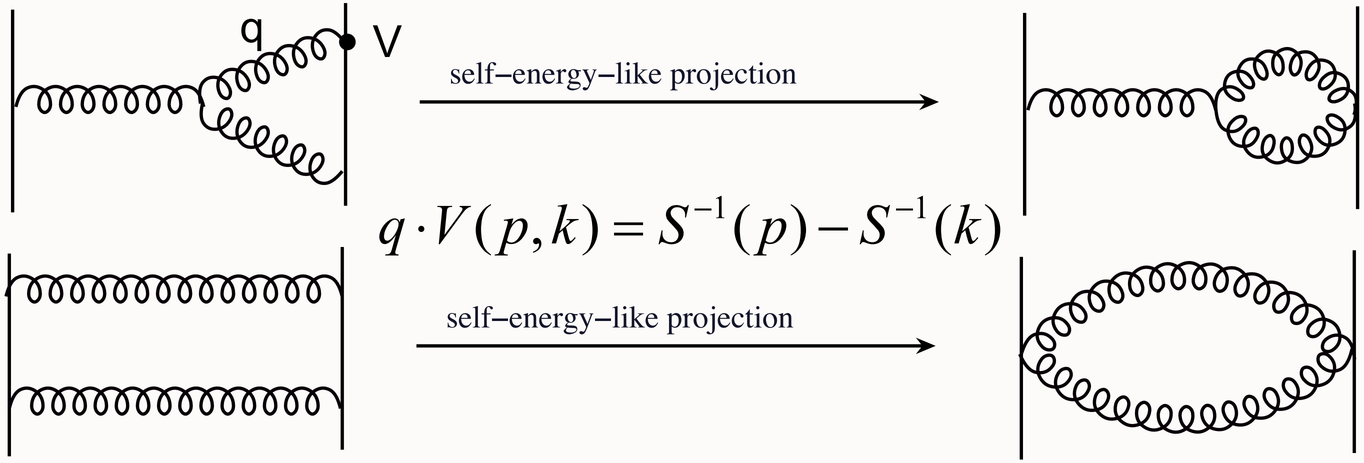
with $p_1 + p_2 + p_3 = 0$

→ 14 basis tensors and form factors

The Pinch Technique

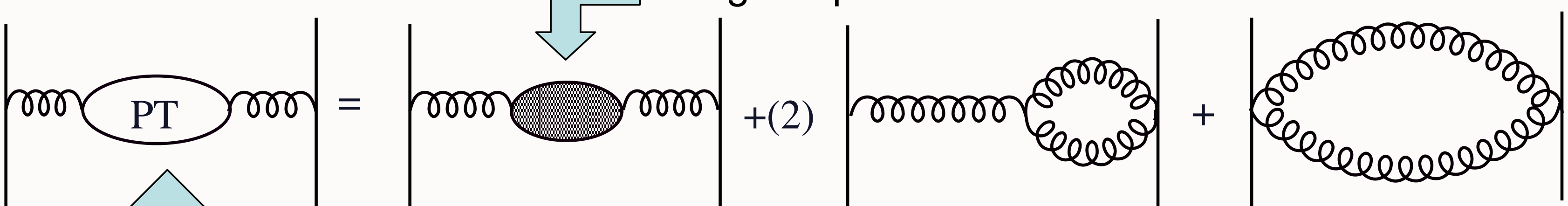
(Cornwall, Papavassiliou)

Natural generalization of QED charge



$$q \cdot V(p,k) = S^{-1}(p) - S^{-1}(k)$$

Gauge-dependent



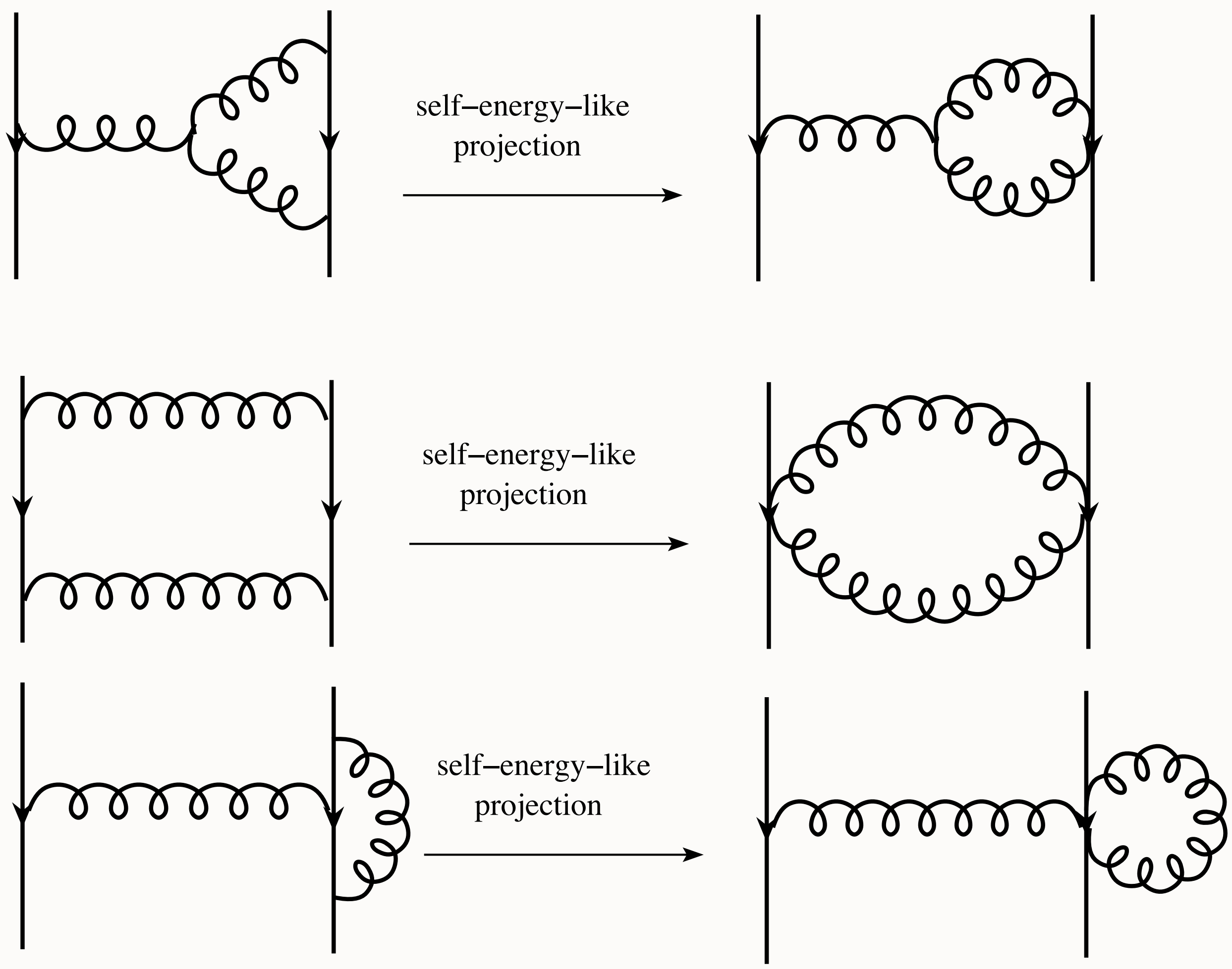
Gauge-invariant gluon self-energy!

$$< 0|G^{\mu\nu}(x)G^{\sigma\tau}(0)|0 > \qquad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

Pinch Scheme (PT)

- J. M. Cornwall, Phys. Rev. D 26, 345 (1982)
- Equivalent to Background Field Method in Feynman guage
- Effective Lagrangian Scheme of Kennedy & Lynn
- Rearrange Feynman diagrams to satisfy Ward Identities
- Longitudinal momenta from triple-gluon coupling, etc. hit vertices which cancel ("pinch") propagators
- Two-point function: Uniqueness, analyticity, unitarity, optical theorem
- Defines analytic coupling with smooth threshold behavior

Pinch Scheme -- Effective Charge



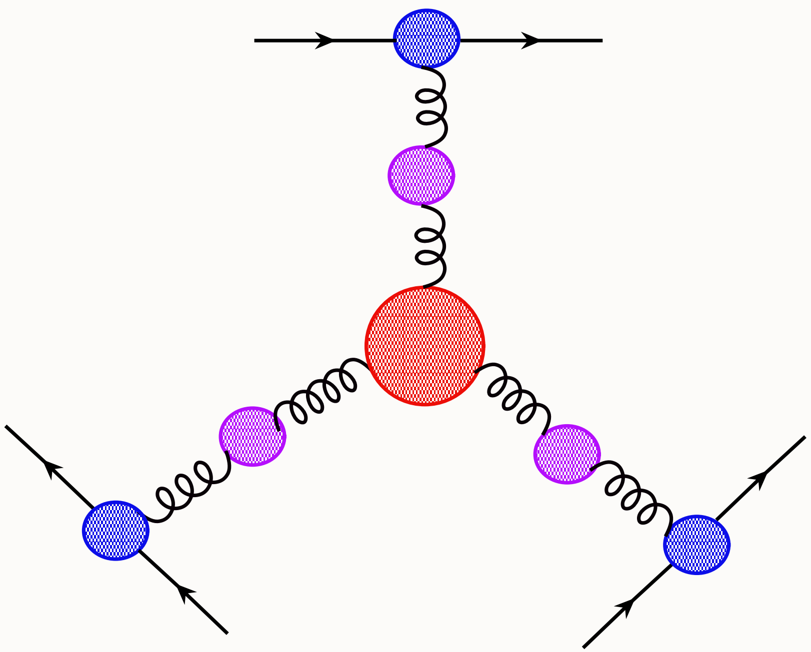
$$< 0|G^{\mu\nu}(x)G^{\sigma\tau}(0)|0 >$$

$$G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

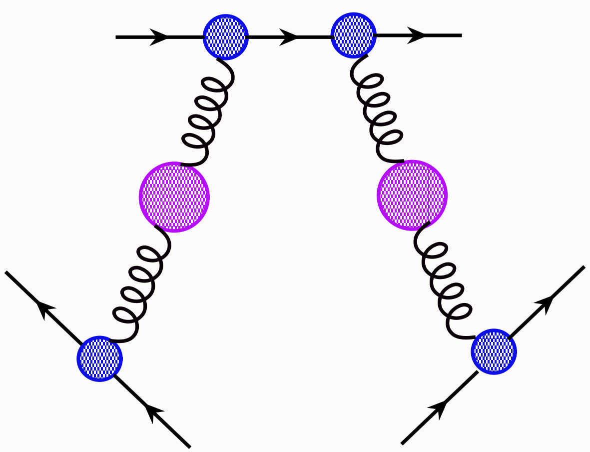
3 Gluon Vertex In Scattering Amplitudes

Pinch-Technique approach :

fully dress with gauge-invariant Green's functions



(A)

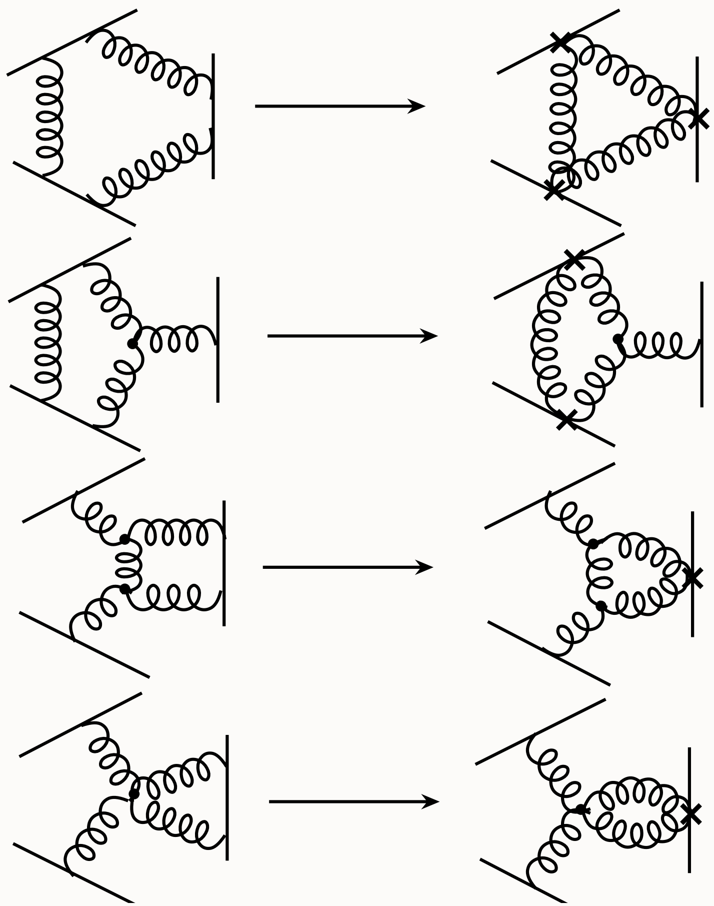


(B)

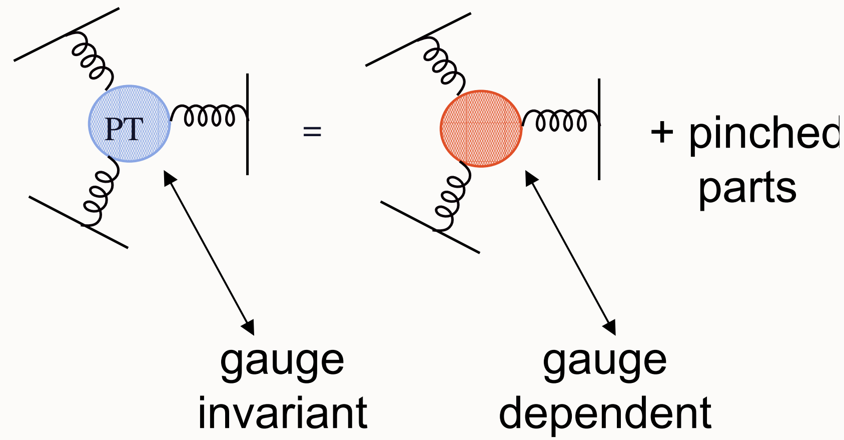
+ perms

The Gauge Invariant Three Gluon Vertex

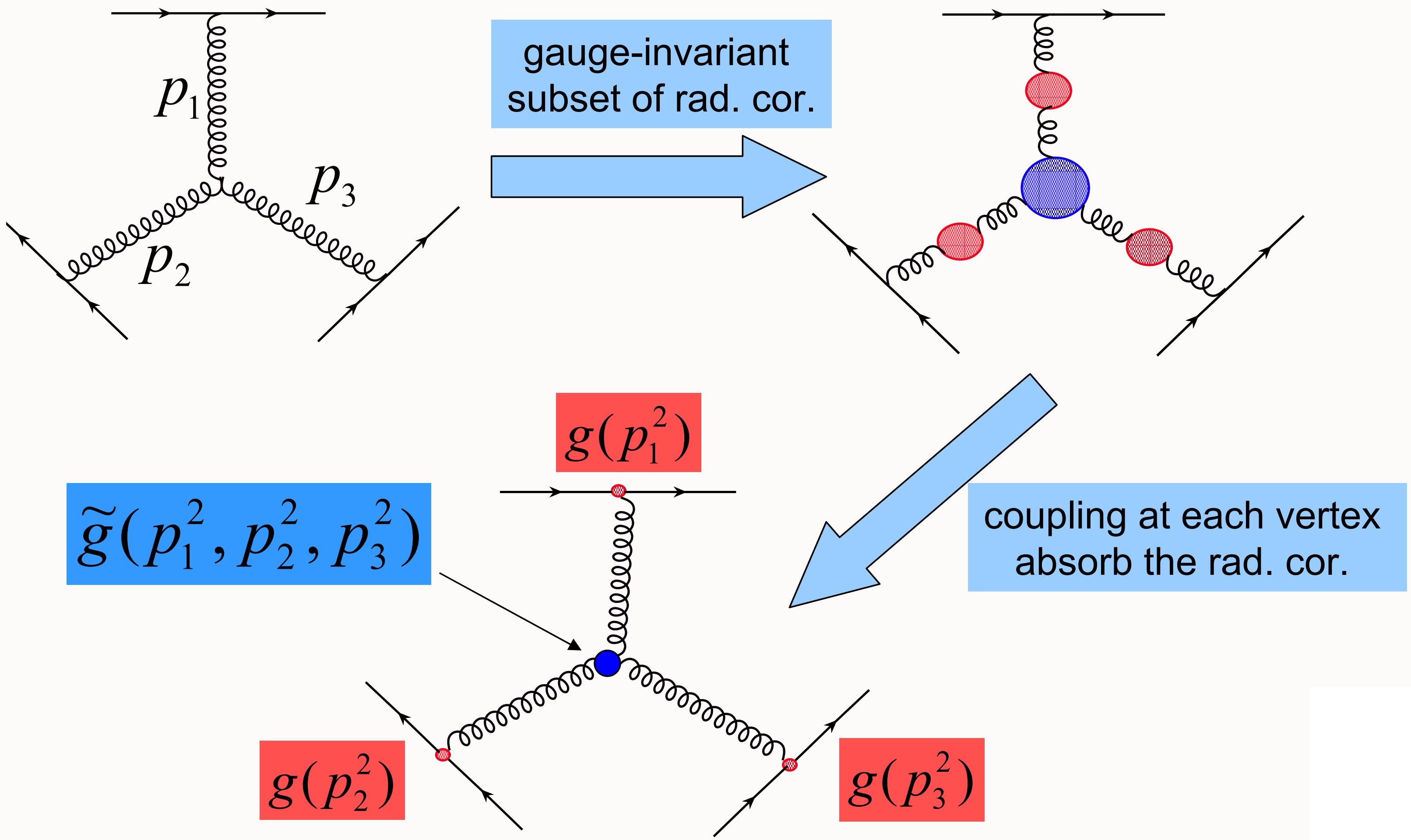
Cornwall and Papavassiliou performed the PT construction :



The "pinched" parts are added to the "regular" 3 gluon vertex



Multi-scale Renormalization of the Three-Gluon Vertex



General Structure of the Three-Gluon Vertex

Simple (QED-like) Ward ID

$$p_3^{\mu_3}\hat{\Gamma}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)=t_{\mu_1\mu_2}(p_2)\left[1+\hat{\Pi}(p_2)\right]-t_{\mu_1\mu_2}(p_1)\left[1+\hat{\Pi}(p_1)\right]$$

where $t_{\mu\nu}(p)=p^2 g_{\mu\nu}-p_\mu p_\nu$

→ One form factor always = 0 (not obvious) → 13 nonzero form factors

3 Gluon Vertex In Scattering Amplitudes

$$Amplitude = color \times vertices \times g(a)g(b)g(c)$$

$$\times g_{bare}\left[(1+A_0)\hat{t}_0 + A_+\hat{t}_+ + A_-\hat{t}_- + H\hat{h}\right]$$

$g_{bare}(1+A_0)$ → $\tilde{g}(a,b,c)$

$A_+\hat{t}_+ + A_-\hat{t}_- + H\hat{h}$ → Other tensors and form factors

$\hat{t}_0$ → Tree level tensor structure :

$$\hat{t}_0 = (p_1 - p_2)^{\mu_3} g^{\mu_1\mu_2} + (p_2 - p_3)^{\mu_1} g^{\mu_2\mu_3} + (p_3 - p_1)^{\mu_2} g^{\mu_3\mu_1}$$

Form factors A_0, A_+, A_-, H depend on these $\begin{cases} a = p_1^2 \\ b = p_2^2 \\ c = p_3^2 \end{cases}$

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Convenient Tensor Bases

Physical $\pm$ Basis

- Written in terms of linear combinations of momenta called "+" and "-" momenta such that $p_+ \cdot V_{ext} = 0$

  by elementary Ward IDs

- Maximum # of FF's vanish when in a physical matrix element

- Good for real scattering problems

LT Basis

- Longitudinal (L) FF's :

$$p_3^{\mu_3} \cdot \hat{\Gamma}^{(L)}_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) \neq 0$$

- Transverse (T) FF's :

$$p_3^{\mu_3} \cdot \hat{\Gamma}^{(T)}_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = 0$$

- Good for theoretical work and solving Ward ID

Complementary in their relation to current conservation (Ward ID's) 24

Form Factors : Supersymmetric Relations

- Any form factor can be decomposed :

$$F = C_A F_G + 2\sum_f T_f F_Q + 2\sum_s T_s F_S$$

G = gluons
Q = quarks
S = scalars

C_A, T_f, T_s are color factors

- Individually, F_G, F_Q, F_S are complicated…

Form Factors : Supersymmetric Relations (Massless)

....but certain linear sums are simple :

$$\Sigma_{QG}(F) \equiv \frac{d-2}{2} F_Q + F_G \longrightarrow 0$$

for 7 of the 13 FF's (in physical basis) $\pm$

Simple N=1 SUSY contribution in d=4

$$F_G + 4F_Q + (10-d)F_S = 0$$

For all FF's !!

N=4 SUSY in d=4 gives 0

These are off-shell generalizations of relations found in SUSY scattering amplitudes by Z. Bern, L.J. Dixon, D.C. Dunbar, and D.A. Kosower (NPB 425,435)

Vanishing contribution of the N=4 supermutiplet in d=4 dimensions

Form Factors : Consequences of Supersymmetric Relations

For any SUSY each of the 13 FF's are $\propto \beta_0$ even though only one FF is directly related to coupling renormalization

$$\beta_0(d) = \frac{7d-6}{2(d-1)} C_A - \frac{2(d-2)}{d-1} \sum_f T_f - \frac{1}{d-1} \sum_f T_s$$

$$\xrightarrow{d\,=\,4} \quad \frac{11}{3} C_A - \frac{4}{3} T_f - \frac{1}{3} T_s$$

⇨ Contributions of gluons, quarks, and scalars have same functional form

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Form Factors Without Supersymmetry (in d=4)

Seven FF's have

$$\Sigma_{QG}(F) = 0 \quad \Rightarrow \quad F = \left(N_c - N_f + \frac{1}{2} N_s \right) F_G$$

FF of tree level tensor

$$A_0 \propto \left(\frac{11}{3} N_c - \frac{2(3d-8)}{3(d-2)} \sum_f T_f - \frac{2}{3(d-2)} \sum_s T_s \right)$$

$$\xrightarrow{d\,=\,4} \quad \left(\frac{11}{3} N_c - \frac{4}{3} T_f - \frac{1}{3} T_s \right) = \beta_0$$

Another FF has $B_0 \propto \left(4N_c - N_f \right)$ $\qquad B_0(S) = 0$

Form Factors : Supersymmetric Relations (Massive)

Equal masses for massive gauge bosons (MG), quarks (MQ), and scalars (MS)

$$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$$

1 d.o.f. “eaten” by MG

Massive gauge boson (MG) inside of loop might be the X and Y gauge bosons of SU(5), for example

External gluons remain unbroken and massless

$$\Sigma_{MQG}(F) \equiv \frac{d-1}{2} F_{MQ} + F_{MG}$$ is simple

Summary of Supersymmetric Relations

Massless	Massive
$F_G + 4F_Q + (10-d)F_S = 0$	$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$
$\Sigma_{QG}(F) \equiv \frac{d-2}{2}F_Q + F_G$	$\Sigma_{MQG}(F) \equiv \frac{d-1}{2}F_{MQ} + F_{MG}$
= simple	= simple

3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi}\beta_0\left(L(a,b,c) - \frac{1}{\varepsilon} + \cdots\right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

L(a,b,c) = 3-scale “log-like” function

L(a,a,a) = log(a)

3 Scale Log-Like Function

$$L(a,b,c) = \frac{1}{K}\left(\alpha\gamma \log a + \alpha\beta \log b + \beta\gamma \log c - abc\bar{J}(a,b,c)\right) + \Omega$$

$$K = \alpha\beta + \beta\gamma + \gamma\alpha$$

Master triangle integral can be written in terms of Clausen functions

$$Cl_2(\theta) = \mathrm{Im}\,Li_2\left(e^{i\theta}\right)$$

$$\alpha = p_1 \cdot p_2 = \frac{1}{2}(c-a-b)$$

$$\beta = p_2 \cdot p_3 = \frac{1}{2}(a-b-c)$$

$$\gamma = p_3 \cdot p_1 = \frac{1}{2}(b-c-a)$$

$$a = p_1^2$$

$$b = p_2^2$$

$$c = p_3^2$$

$$\Omega \approx 3.125$$

3 Scale Effective Scale

$$L(a,b,c) \equiv \log\left(Q^2_{eff}(a,b,c)\right) + i\,\mathrm{Im}\,L(a,b,c)$$

Governs strength of the three-gluon vertex

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\tilde{\alpha}(a,b,c)}$$

Generalization of BLM Scale to 3-Gluon Vertex

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54|a|$$

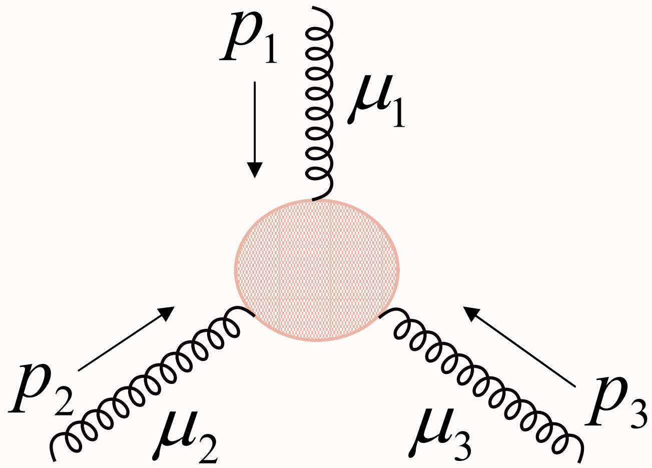
$$Q^2_{eff}(a,a,c) \approx 3.08|c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,-a,c) \approx 22.8|c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8\frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|,|c|$$

Surprising dependence on Invariants

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



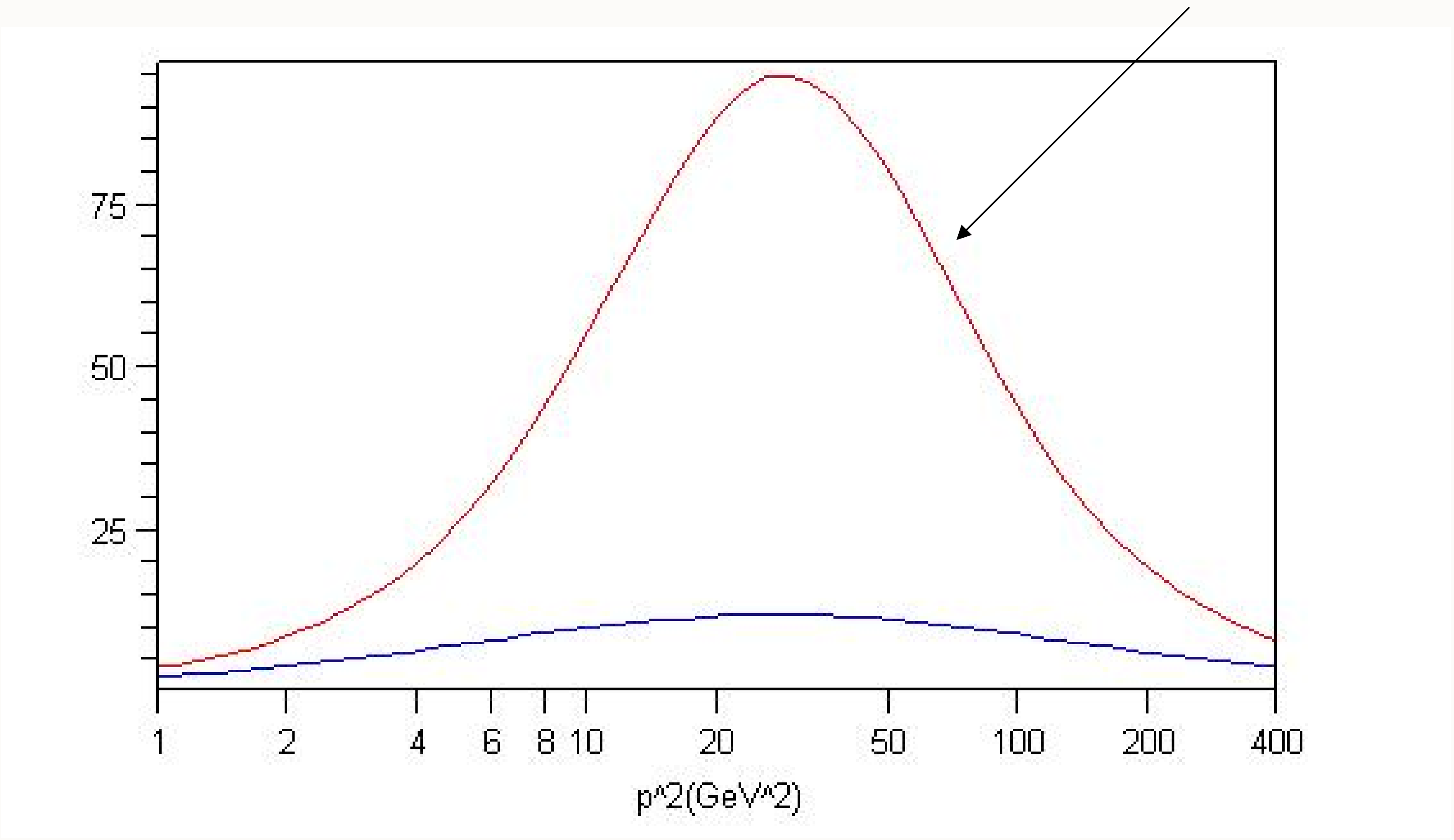
H. J. Lu

$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$

The Effective Scale

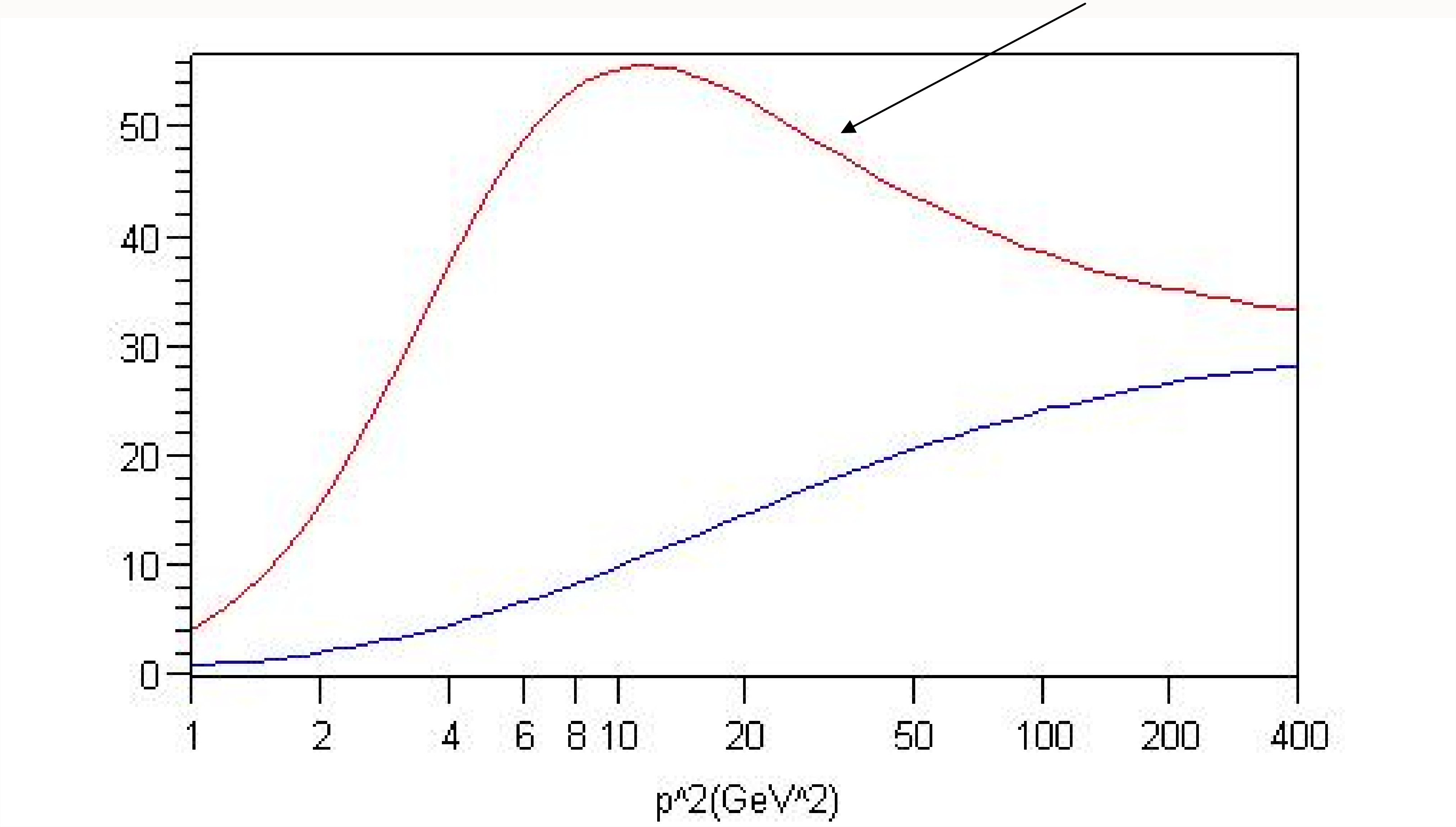
$Q^2_{eff}\left(10\,\mathrm{GeV}^2, 10\,\mathrm{GeV}^2, p^2\right)$

$Q^2_{eff}\left(-10\,\mathrm{GeV}^2, -10\,\mathrm{GeV}^2, p^2\right)$

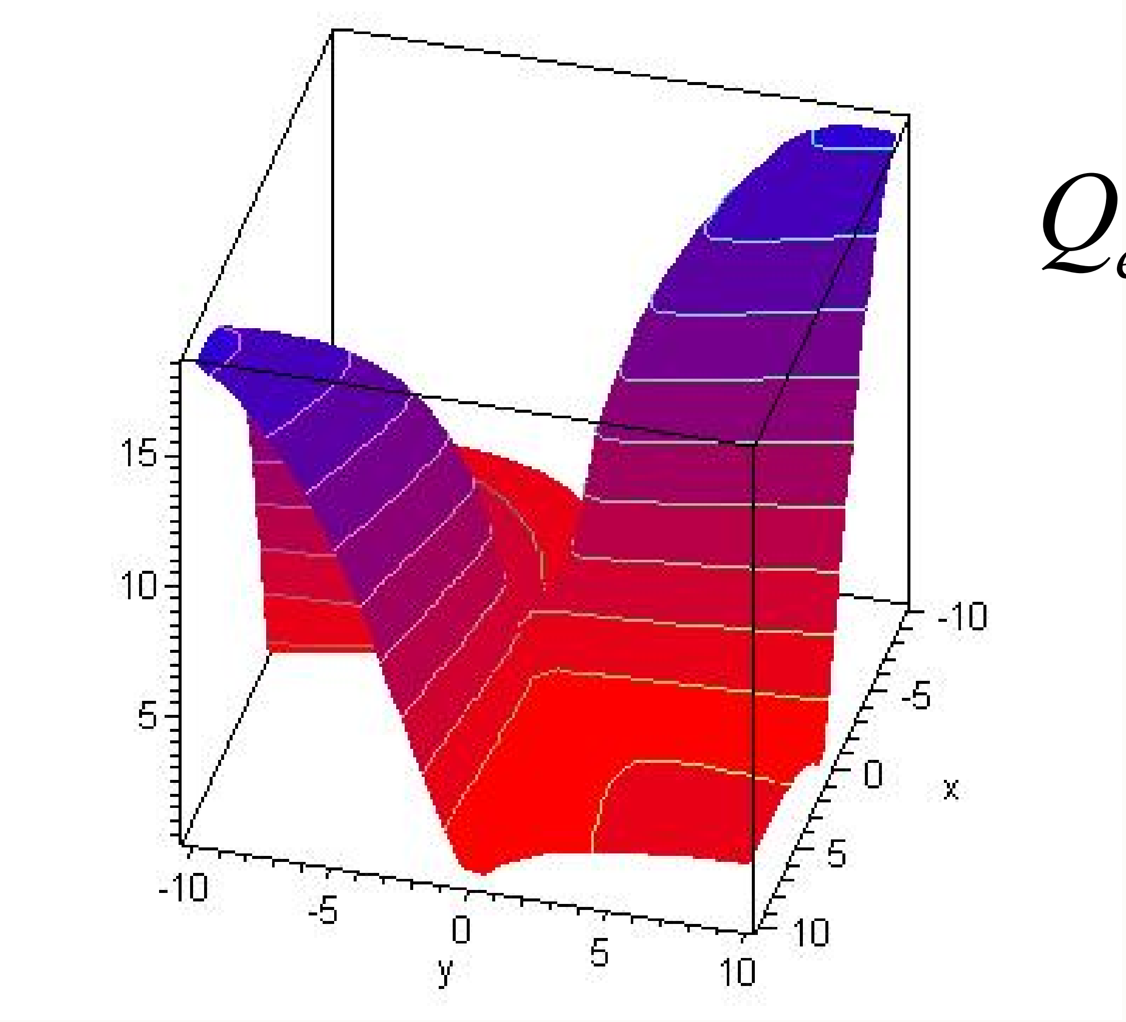


The Effective Scale

$Q^2_{eff}\left(10\ \mathrm{GeV}^2, p^2, p^2\right)$ $Q^2_{eff}\left(-10\ \mathrm{GeV}^2, p^2, p^2\right)$



The Effective Scale



$$Q^2_{eff}(1, x, y)$$

Mass Effects

Calculated for all form factors

SUSY relations $F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$

FF of tree level tensor structure ⟺ Effective Charge

Massive “log-like” function : $L_{MQ}\left(\frac{a}{M^2}, \frac{b}{M^2}, \frac{c}{M^2}\right)$

$$L_{MQ}\left(\frac{a}{M^2}, \frac{b}{M^2}, \frac{c}{M^2}\right) \approx 5.125 \text{ for } M^2 >> |a|,|b|,|c|$$

$$L_{MQ}\left(\frac{a}{M^2}, \frac{b}{M^2}, \frac{c}{M^2}\right) \approx L(a,b,c) - \log M^2 \text{ for } M^2 << |a|,|b|,|c|$$

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Massive Log-Like Function

$$L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)=\frac{1}{K}\left(\alpha\gamma\Lambda(a)+\alpha\beta\Lambda(b)+\beta\gamma\Lambda(c)-abc\overline{J_M}(a,b,c)\right)+\Omega$$

$$+2M^2\left(\frac{\Lambda(a)-2}{a}+\frac{\Lambda(b)-2}{b}+\frac{\Lambda(c)-2}{c}-\overline{J_M}(a,b,c)\right)$$

$$\Lambda(a)=\left\{\begin{array}{l} 2v\tanh^{-1}\left(v^{-1}\right) \\ 2\bar{v}\tan^{-1}\left(\bar{v}^{-1}\right) \\ 2v\tanh^{-1}\left(v\right)-iv\pi \end{array}\right\} \text{ for } \left\{\begin{array}{l} a<0 \\ 0<a<4M^2 \\ a>4M^2 \end{array}\right\}$$

$$v=\sqrt{1-\frac{4M^2}{a}} \qquad \bar{v}=\sqrt{\frac{4M^2}{a}-1}$$

Massive Master Triangle Integral (very complicated)

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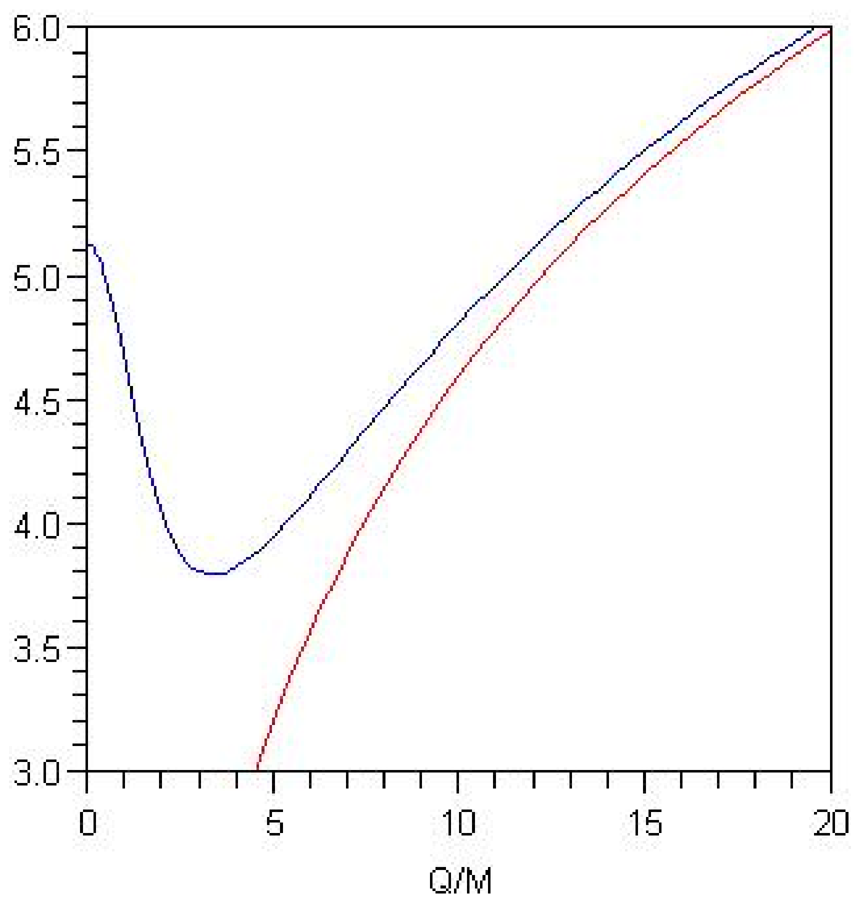
Symmetric Spacelike



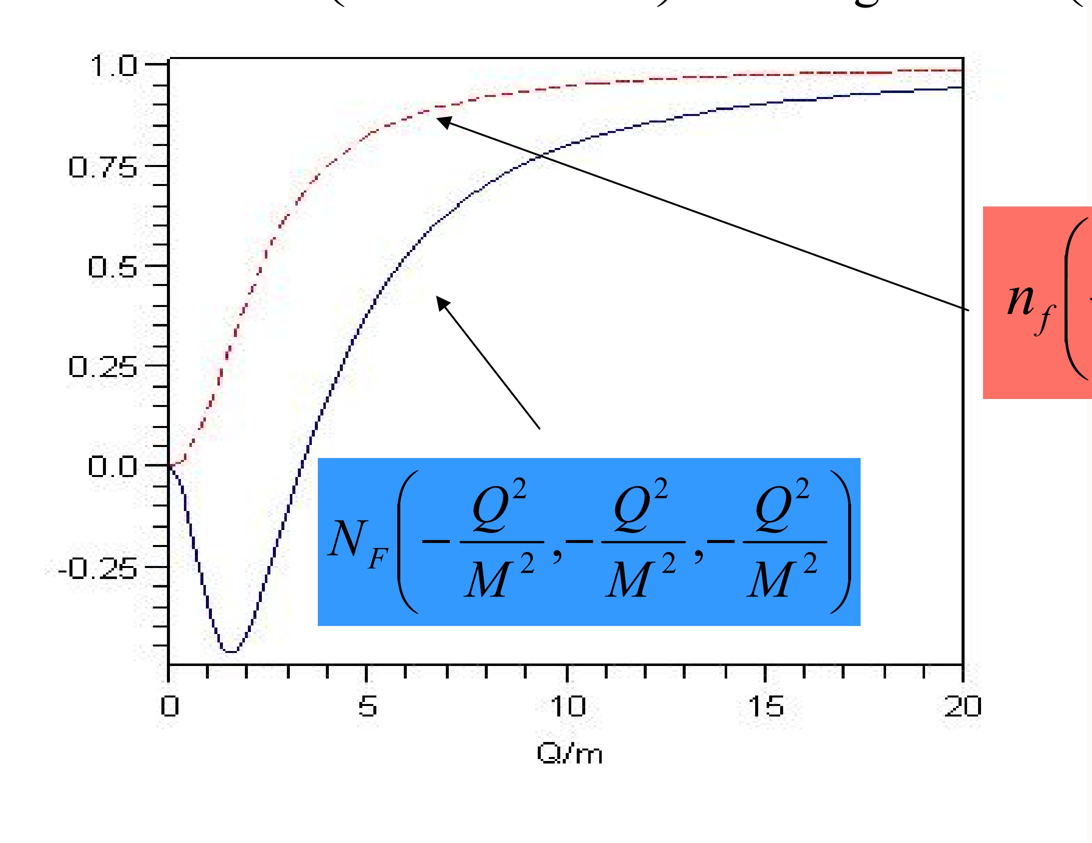
$$L_{MQ}\left(-\frac{Q^2}{M^2},-\frac{Q^2}{M^2},-\frac{Q^2}{M^2}\right)$$

$$\log\left(\frac{Q^2}{M^2}\right)$$

48

Effective Number of Flavors

$$N_F\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right) = -\frac{d}{d\log M^2} L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)$$



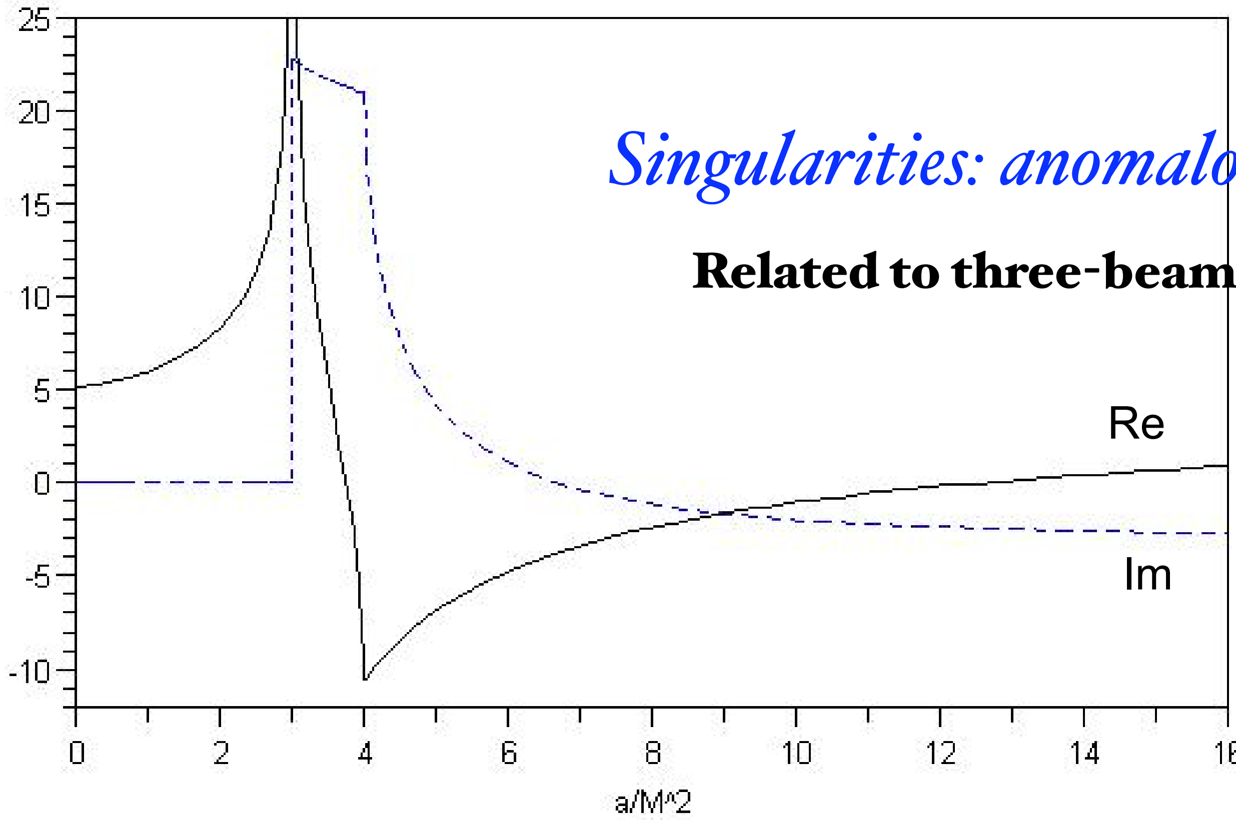
$$N_F\left(-\frac{Q^2}{M^2},-\frac{Q^2}{M^2},-\frac{Q^2}{M^2}\right)$$

$$n_f\left(\frac{Q^2}{M^2}\right) = -\frac{d}{d\log M^2} L_{1/2}\left(\frac{Q^2}{M^2}\right)$$

$$\approx \frac{1}{1+\frac{M^2}{Q^2}e^{5/3}}$$

Symmetric Timelike

$$L_{MQ}\left(\frac{a}{M^2},\frac{a}{M^2},\frac{a}{M^2}\right)$$



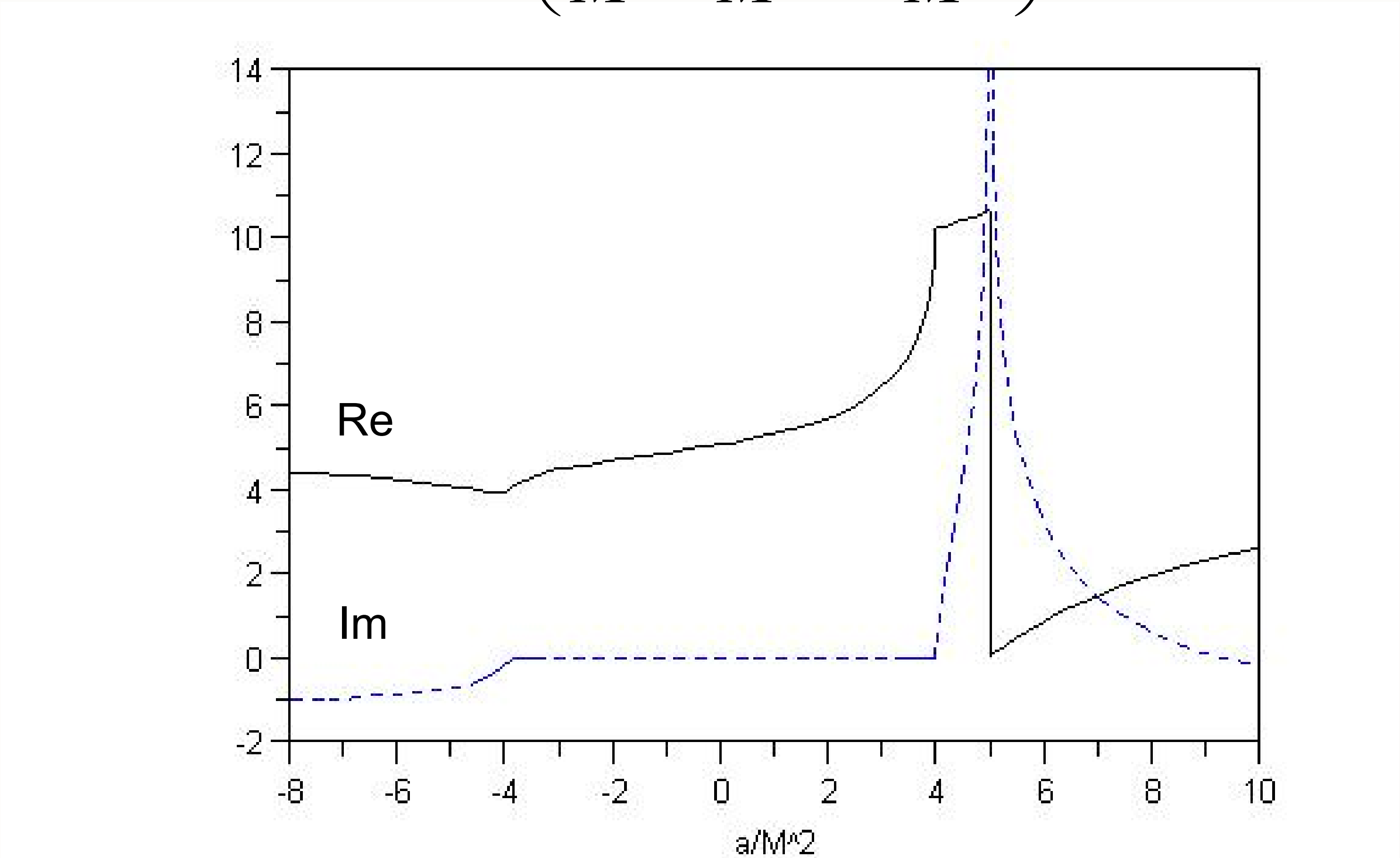
Singularities: anomalous thresholds

Related to three-beam scattering?

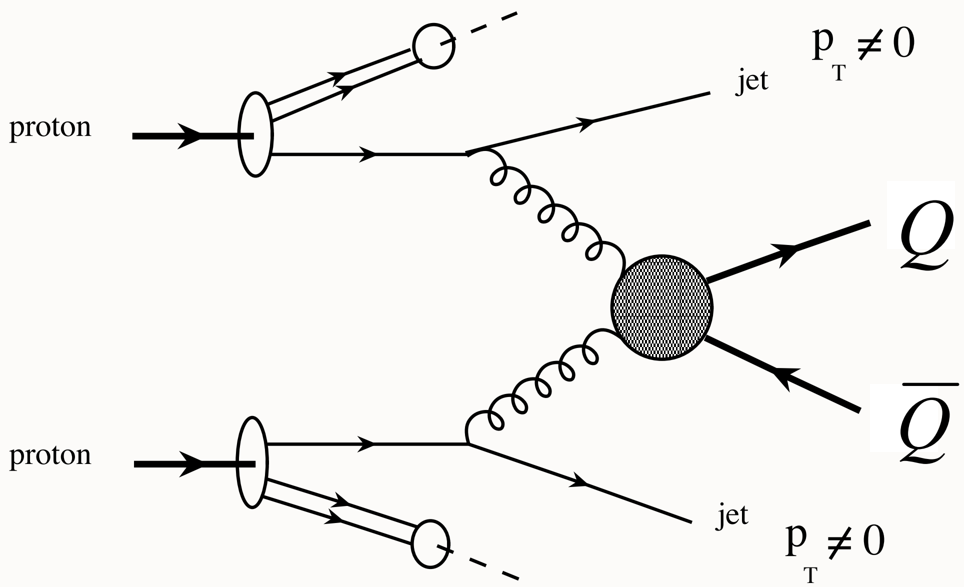
Related to three-beam scattering?

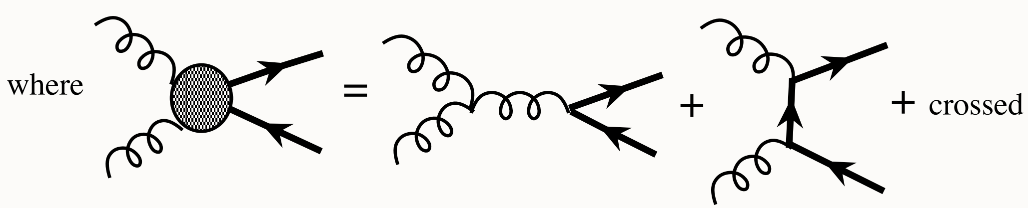
Symmetric Mixed Signature

$$L_{MQ}\left(\frac{a}{M^2},\frac{a}{M^2},-\frac{a}{M^2}\right)$$



Heavy Quark Hadro-production





- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale

  ⇒ much larger cross section than $\overline{MS}$ with scale $\mu_R = M_{Q\overline{Q}}$ or M_Q

- Future : repeat analysis using the full mass-dependent results and include all form factors

Expect that this approach accounts for most of the one-loop corrections

Use Physical Scheme to Characterize QCD Coupling

- Use Observable to define QCD coupling or Pinch Scheme
- Analytic: Smooth behavior as one crosses new quark threshold
- New perspective on grand unification

Binger, Sjb

Unification in Physical Schemes

"PHYSICAL RENORMALIZATION SCHEMES AND GRAND UNIFICATION"
M.B. and Stanley J. Brodsky. **Phys.Rev.D69:095007,2004**

$$\alpha_i(Q) = \frac{\alpha_i(Q_0)}{1+\hat{\Pi}_i(Q)-\hat{\Pi}_i(Q_0)}$$

i=1,2,3

$$\hat{\Pi}_i(Q) = \frac{\alpha_i}{4\pi}\sum_p \beta_i^{(p)}\left(L_{s(p)}(Q^2/m_p^2)+\cdots\right)$$

"log-like" function:

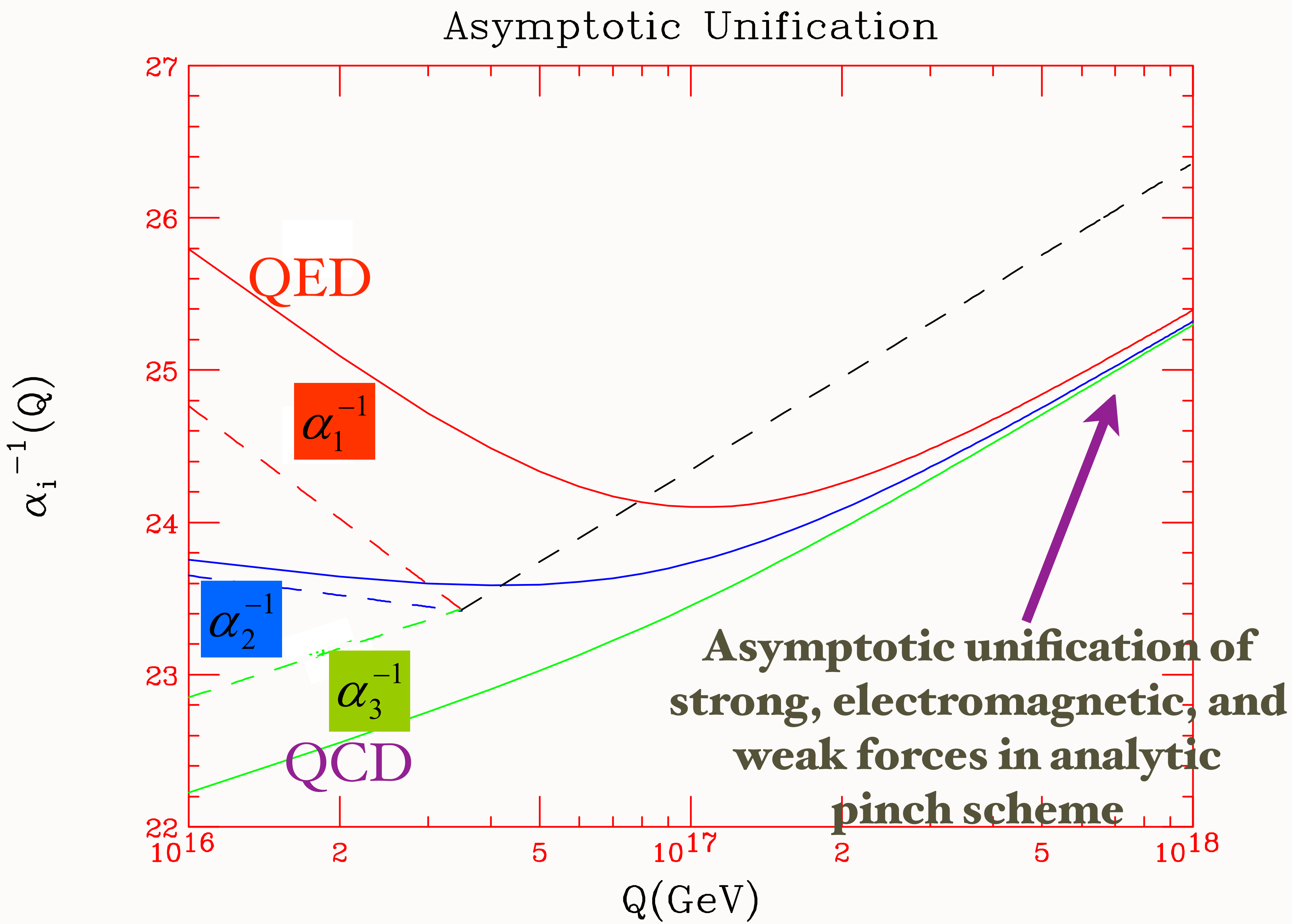
$$L_{s(p)} \approx \log(e^{\eta_p} + Q^2/m_p^2)$$

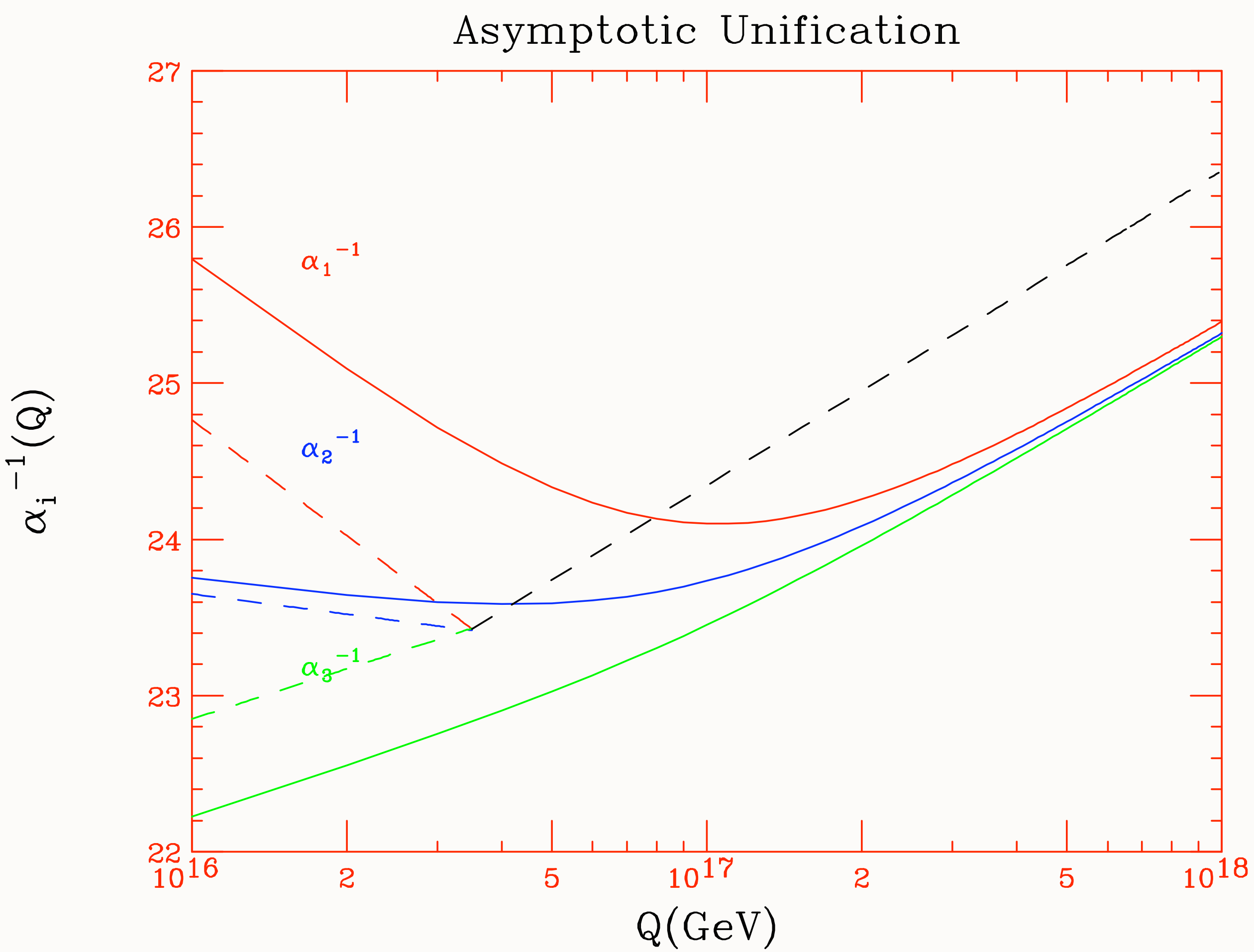
$$\eta_p = 8/3,\, 5/3,\, 40/21$$

For spin s(p) = 0, ½, and 1

→ Elegant and natural formalism for all threshold effects

Binger, sjb

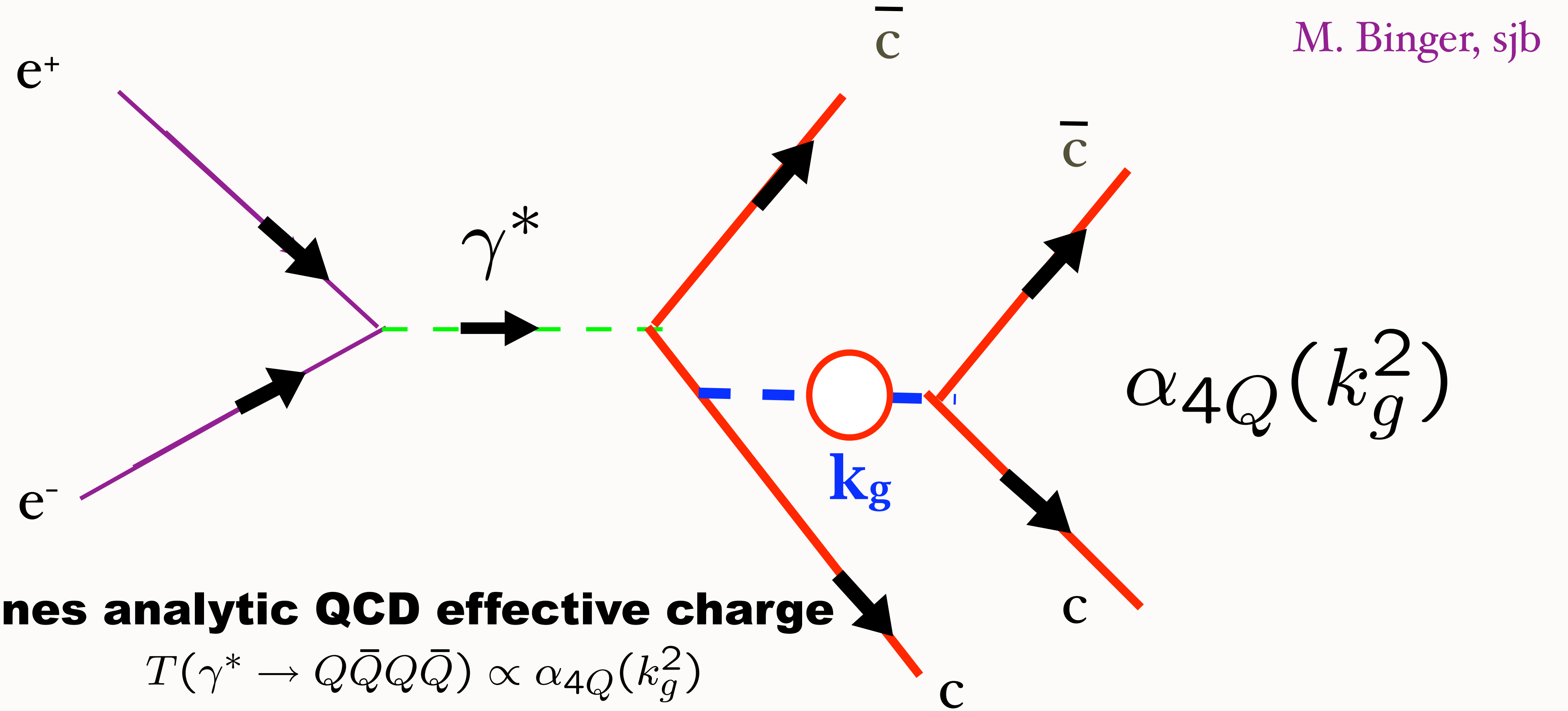

Asymptotic Unification
27
26
25
24
23
22
10^{16}
2
5
10^{17}
2
5
10^{18}
Q(GeV)
$\alpha_i^{-1}(Q)$
QED
α_1^{-1}
α_2^{-1}
α_3^{-1}
QCD
Asymptotic unification of strong, electromagnetic, and weak forces in analytic pinch scheme



Binger, sjb

Asymptotic Unification. The solid lines are the analytic $\overline{PT}$ effective couplings, while the dashed lines are the $\overline{DR}$ couplings. For illustrative purposes, $\alpha_3(M_Z)$ has been chosen so that unification occurs at a finite scale for $\overline{DR}$ and asymptotically for the $\overline{PT}$ couplings. Here $M_{SUSY} = 200$GeV is the mass of all light superpartners except the wino and gluino which have values $\frac{1}{2}m_{\widetilde{g}} = M_{SUSY} = 2m_{\widetilde{w}}$. For illustrative purposes, we use $SU(5)$.

Production of four heavy-quark jets



Defines analytic QCD effective charge

$$T(\gamma^* \to Q\bar{Q}Q\bar{Q}) \propto \alpha_{4Q}(k_g^2)$$

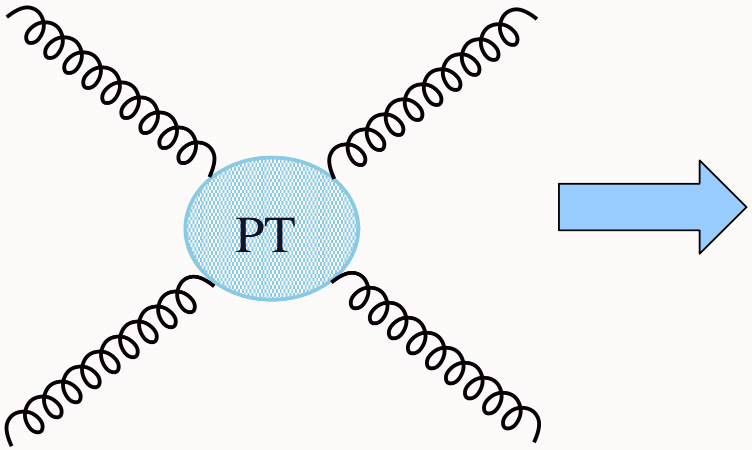
time-like values not same as space-like

coupling similar to "pinch" scheme

complex for time-like argument

Future Directions

Gauge-invariant four gluon vertex

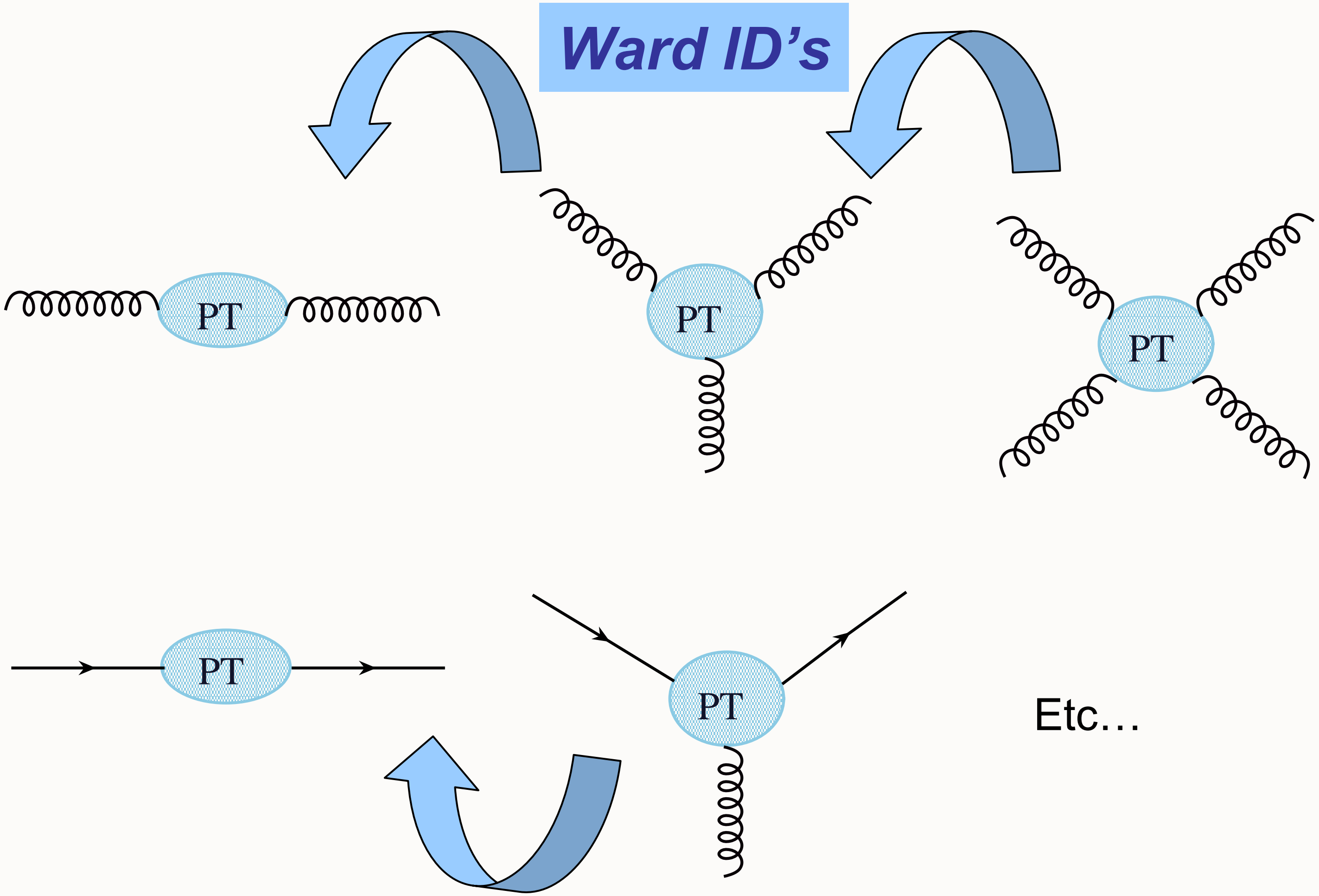


$$L_4(p_1, p_2, p_3, p_4)$$

$$Q^2_{4\,eff}(p_1, p_2, p_3, p_4)$$

Hundreds of form factors!

The Gauge-Invariant Family of Green's Functions



PT Self-Energy at Two-Loops

- Finite terms give relation between

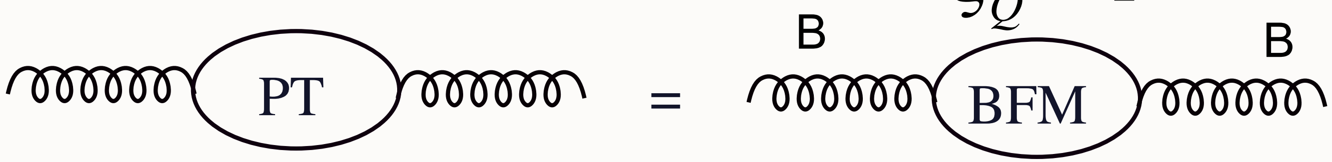
  $$\alpha_{PT}(Q^2) \text{ and } \alpha_{\overline{MS}}(Q^2)$$

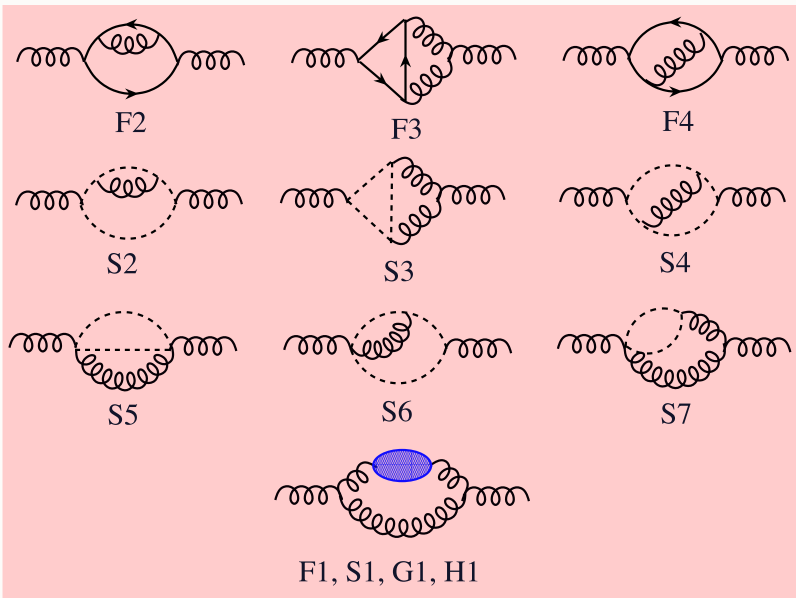
- 3-loop beta function
- 2-loop longitudinal form factors of the three-gluon vertex (via the Ward ID)
- N=4 Supersymmetry gives a non-zero but UV finite contribution

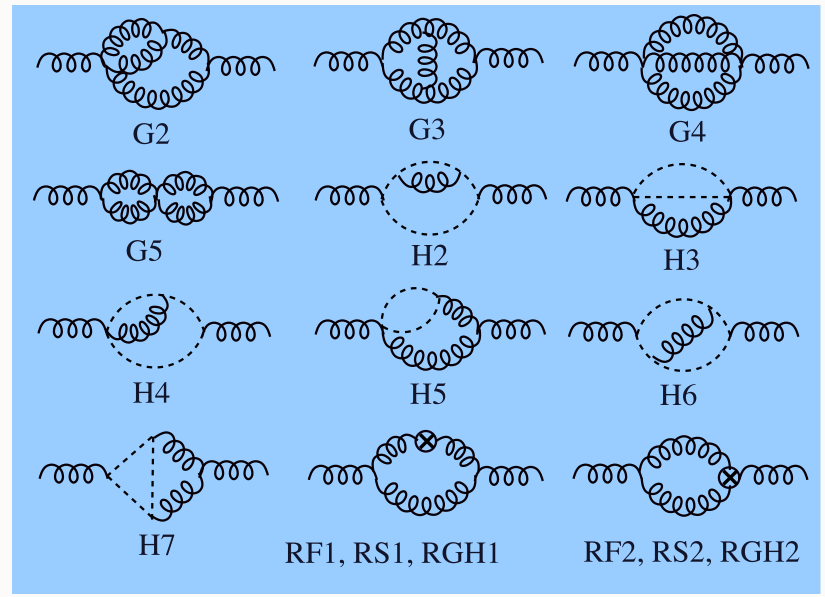
PT Self-Energy at Two-Loops

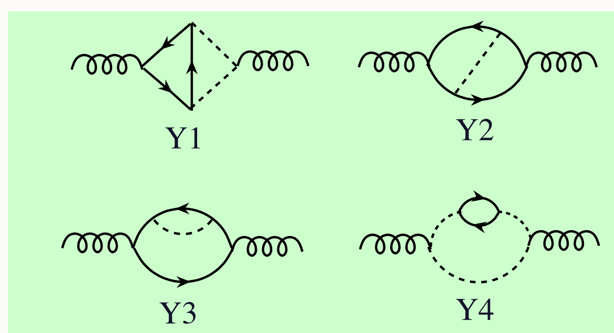
Papavassiliou showed :

$\xi_Q = 1$









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Future Directions

- *Implement in Monte Carlo generator*
- *Gauge-invariant Standard Model triple gauge boson vertices*
- *Schwinger-Dyson Equations*

Summary and Future

- ***Multi-scale analytic*** renormalization based on ***physical, gauge-invariant*** Green's functions

- ***Optimal*** improvement of perturbation theory with ***no scale-ambiguity*** since physical kinematic invariants are the arguments of the (multi-scale) couplings

Conventional renormalization scale-setting method :

- Guess arbitrary renormalization scale and take arbitrary range. Wrong for QED and Precision Electroweak.
- Prediction depends on choice of renormalization scheme
- Variation of result with respect to renormalization scale only sensitive to nonconformal terms; no information on genuine (conformal) higher order terms
- FAC and PMS give unphysical results.
- Renormalization scale not arbitrary: Analytic constraint from flavor thresholds

Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb

Phys.Rev.D28:228,1983

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q* sets the number of active flavors
- Only n_f dependence required to determine renormalization scale at NLO
- Result is scheme independent: Q* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- **Resulting series identical to conformal series!**
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

Use BLM!

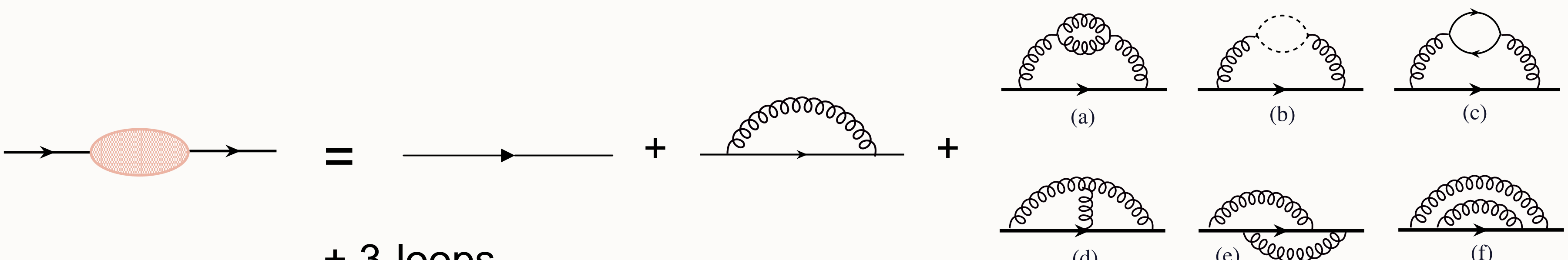
- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit ($N_C = 0$)
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...

On Renormalons and the Structure of Perturbation Theory

Investigate the relation between :

1. Renormalons
2. BLM Scale Fixing
3. Effective Charges Running Inside of Loops

Laboratory : Higher order corrections to the quark propagator



+ 3-loops

(Gray, Broadhurst, Grafe, Schilcher and Chetyrkin, Steinhauser)

Relation between quark pole mass $\overline{MS}$ mass

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On Renormalons and the Structure of Perturbation Theory

BLM Methods

- Predicts 3-loop term with an accuracy of 3-4%
- Conformal term is very small

Not associated with running coupling

→ Expect that almost all of the loop corrections are "associated with" the running coupling

Seems to be very much in contrast to what we found using the RIA

→ Perhaps the success of BLM is not tied to a hypothetical skeleton expansion with running charges inside of loops

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Factorization scale

$$\mu_{\text{factorization}} \neq \mu_{\text{renormalization}}$$

- Arbitrary separation of soft and hard physics
- Dependence on factorization scale not associated with beta function - present even in conformal theory
- Keep factorization scale separate from renormalization scale

$$\frac{d\mathcal{O}}{d\mu_{\text{factorization}}} = 0$$

- Residual dependence when one works in fixed order in perturbation theory.