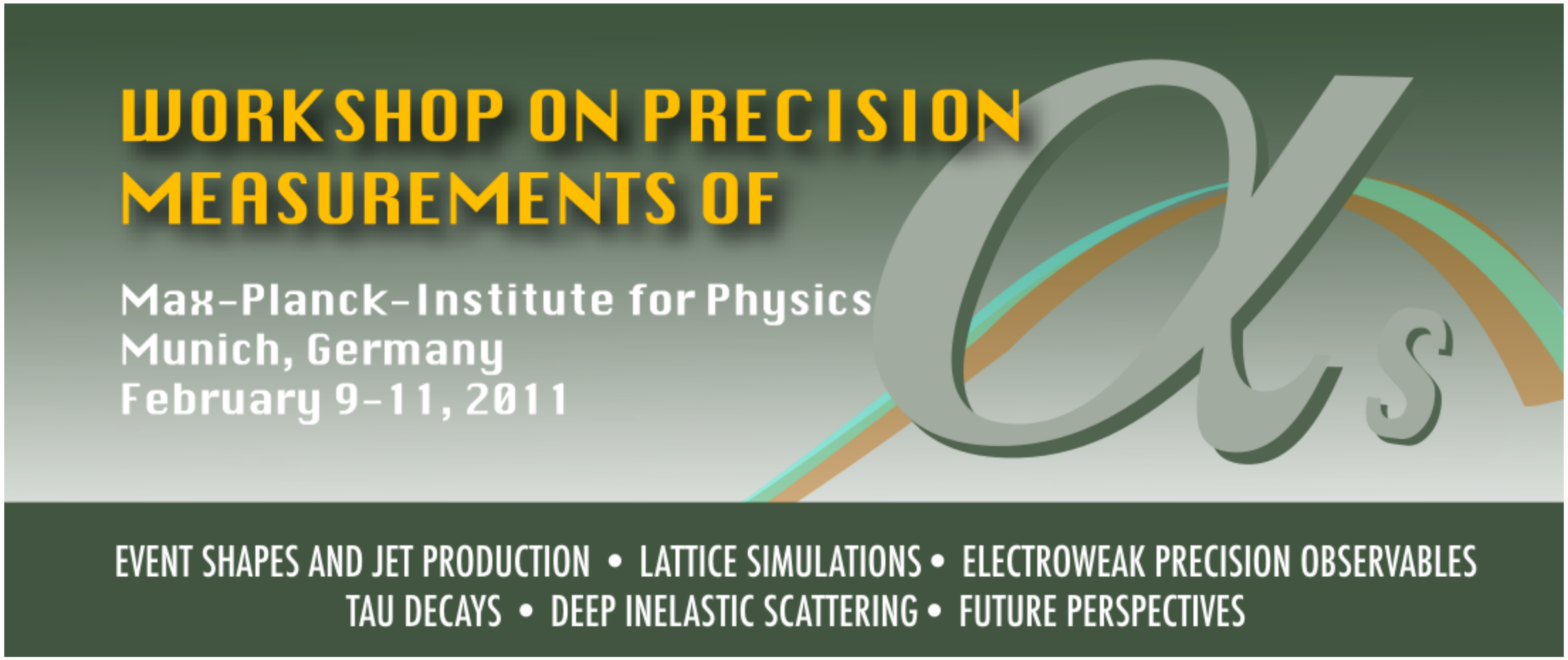
The Principle of Maximal Conformality

Elimination of the QCD Renormalization Scale Ambiguity



Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

with Leonardo Di Giustino

Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders

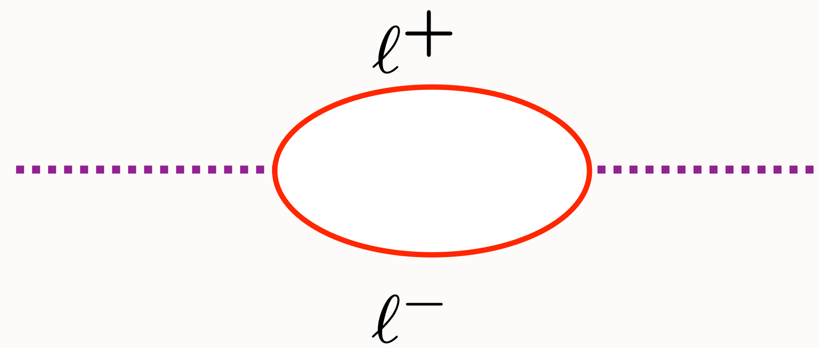
Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\left[\frac{5}{3} - \frac{4m^2}{Q^2} - \left(1 - \frac{2m^2}{Q^2}\right)\sqrt{1 + \frac{4m^2}{Q^2}}\,\log\frac{1+\sqrt{1+\frac{4m^2}{Q^2}}}{\left|1-\sqrt{1+\frac{4m^2}{Q^2}}\right|}\right]$$

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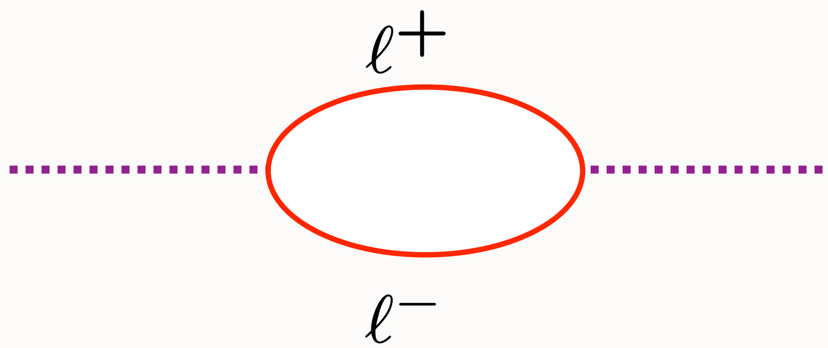

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Analytically continue to timelike t: Complex

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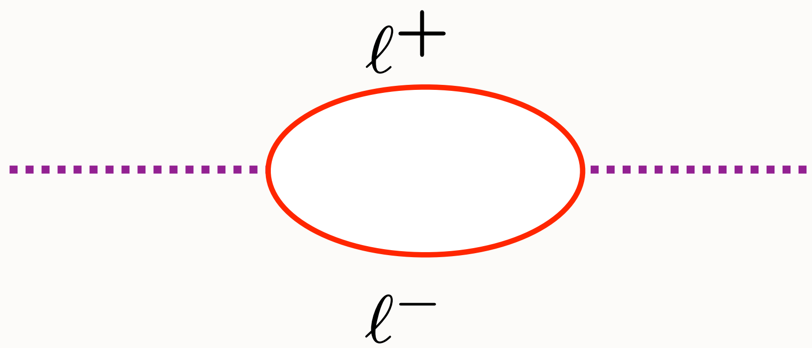

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$Q^2 << 4M^2$ **Serber-Uehling**

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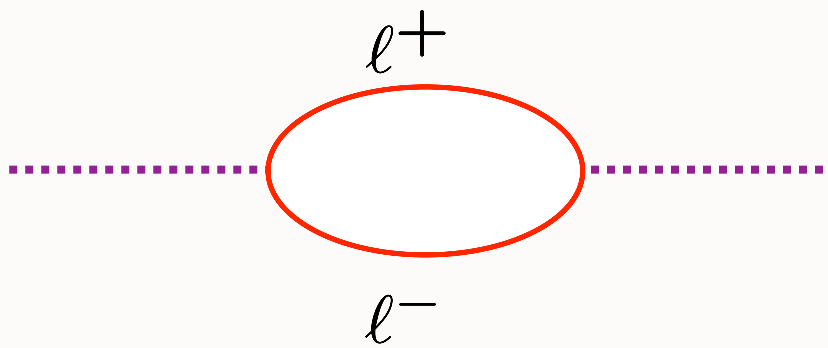

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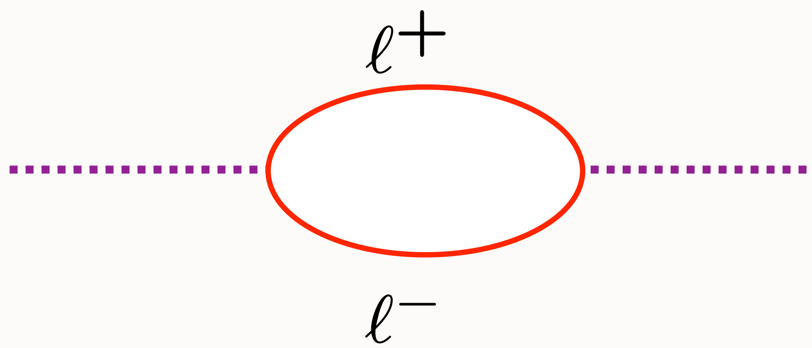

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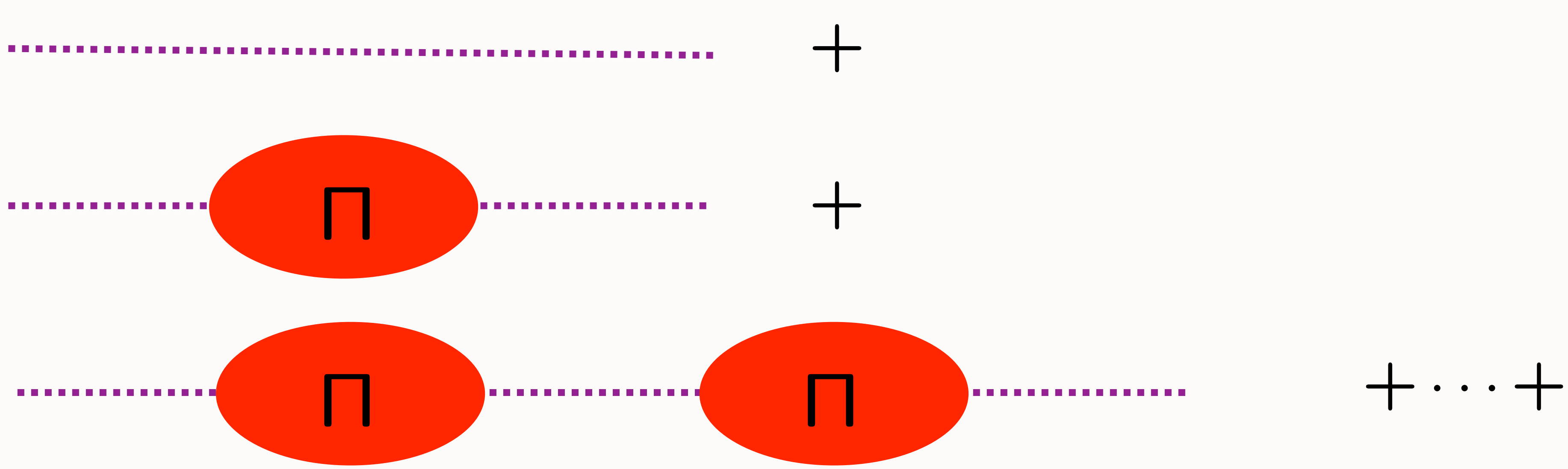
$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3}\left(\frac{\alpha}{4\pi}\right)^2 n_\ell > 0$$

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

All-orders lepton-loop corrections to dressed photon propagator



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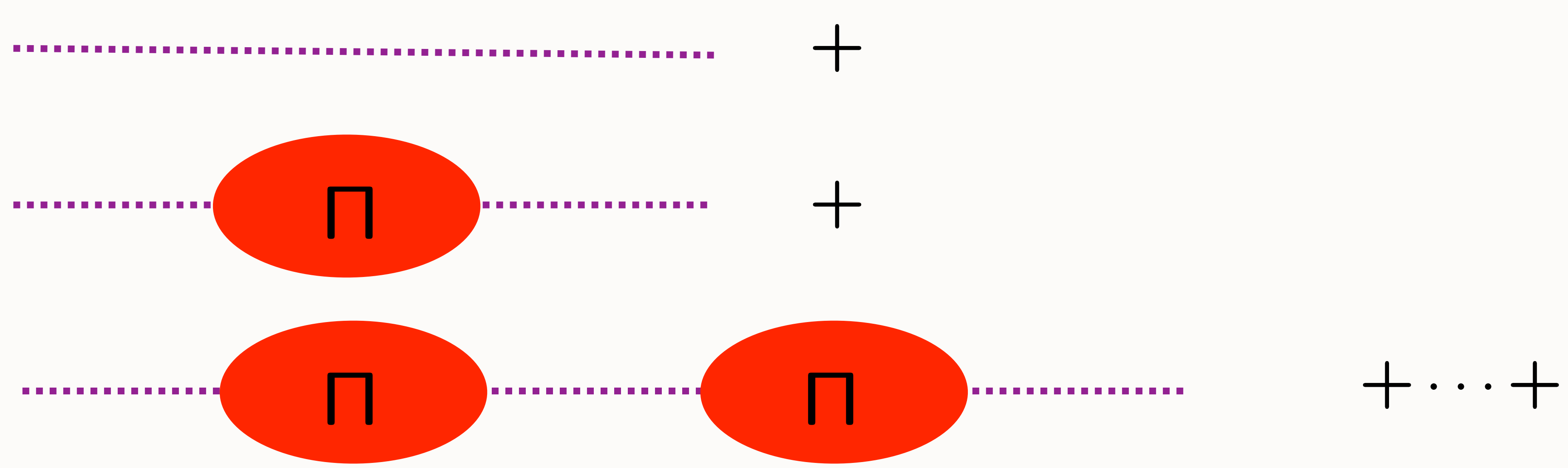

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***Initial* scale t_o is arbitrary -- Variation gives RGE Equations**

Physical renormalization scale t not arbitrary!

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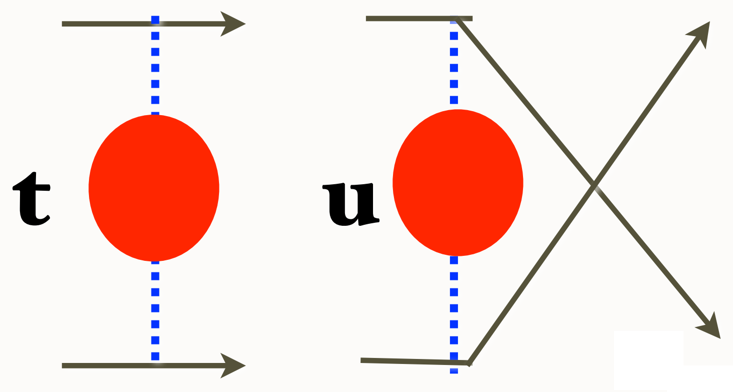

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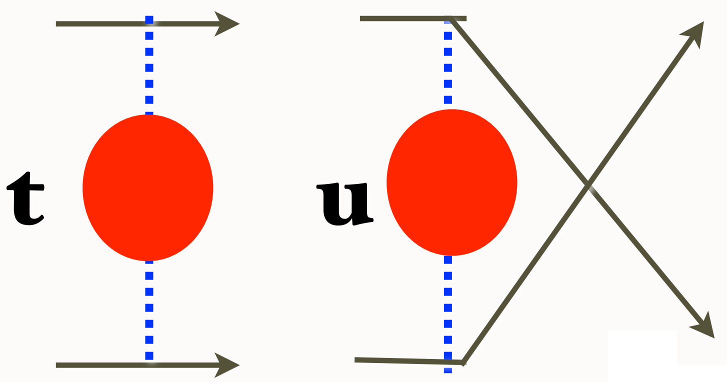

- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!
- If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
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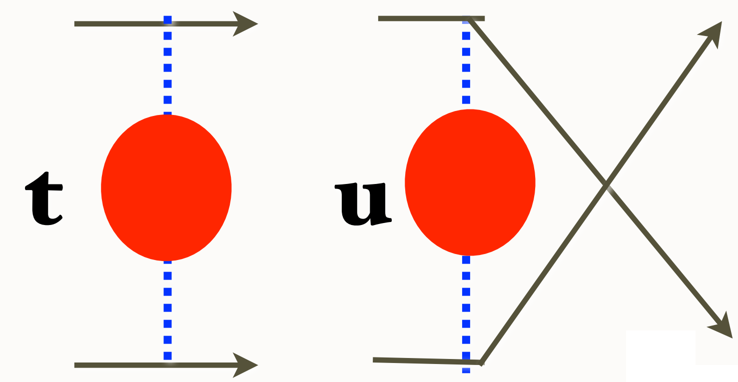

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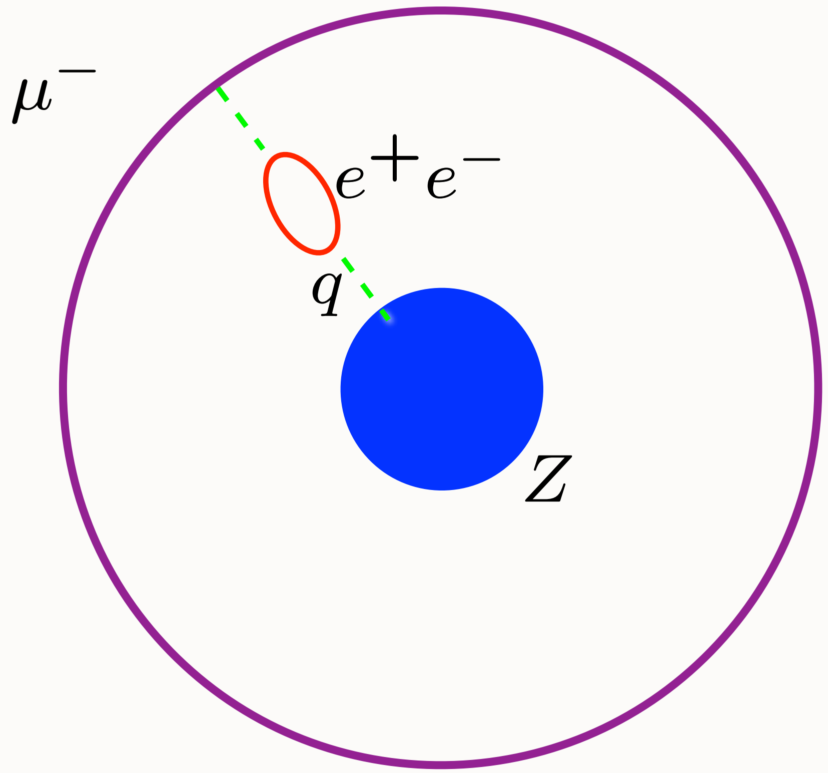

$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

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Scale is unique: Tested to ppm

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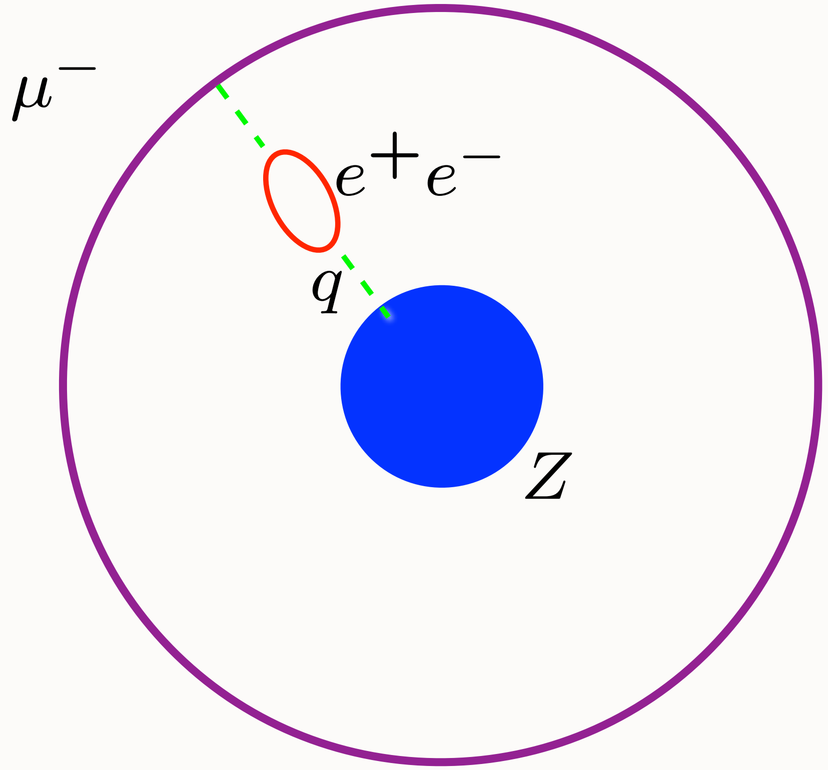

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Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

PHYSICAL REVIEW D **74,** 013003 (2006)

Target normal spin asymmetry and charge asymmetry for $e\mu$ elastic scattering and the crossed processes

E. A. Kuraev, V. V. Bytev, and Yu. M. Bystritskiy
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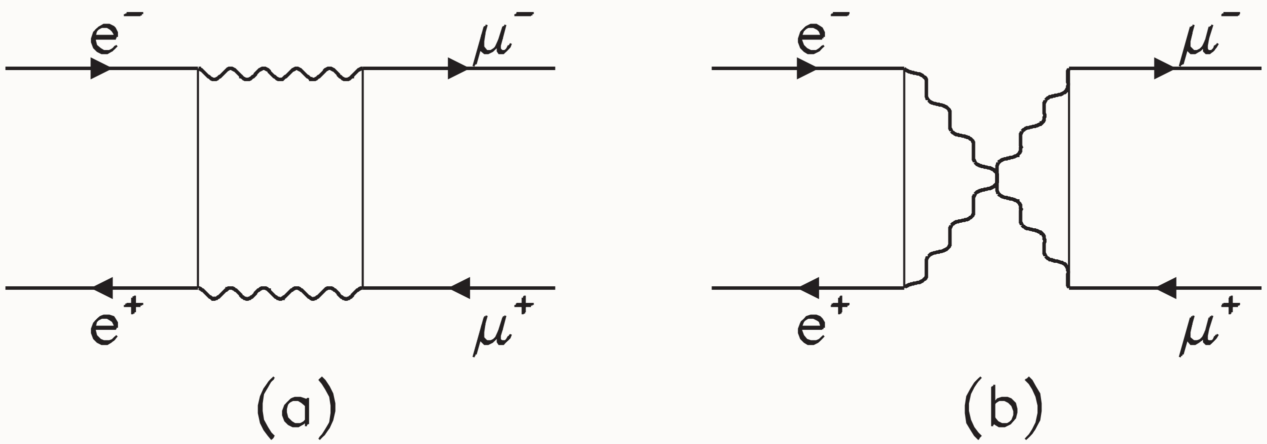
E. Tomasi-Gustafsson
DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France

Physics of conformal series; not associated with renormalization

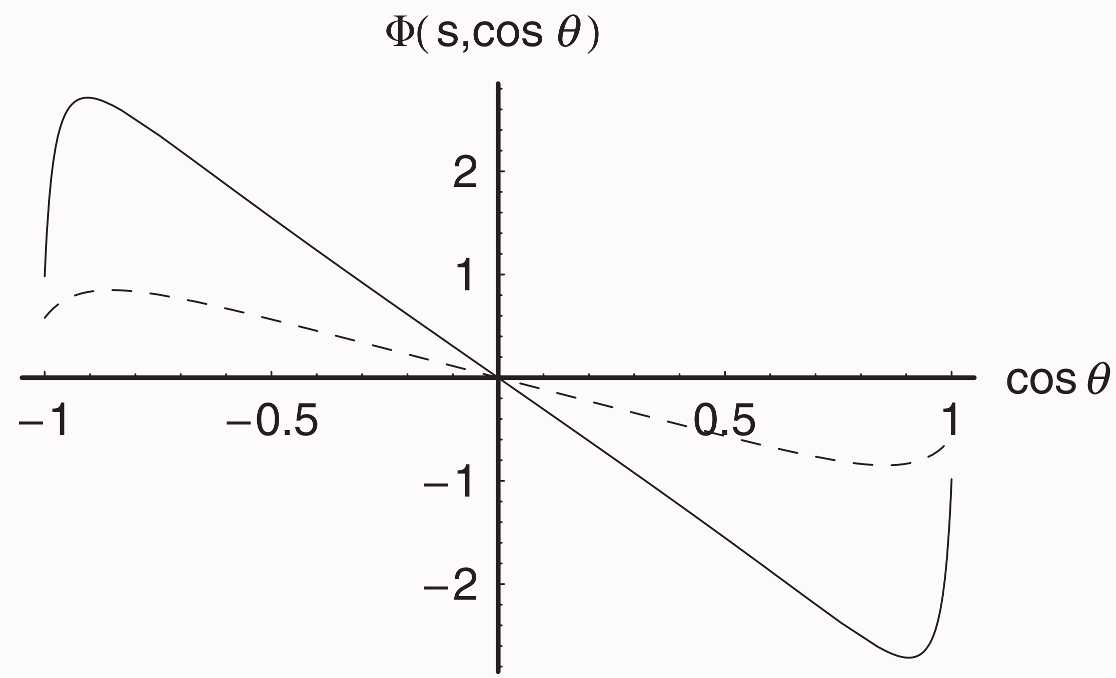


$$A(\theta, \Delta E) = \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma_B(\theta)} = \frac{4\alpha}{\pi}\Upsilon$$

$$\frac{d\sigma_{\rm ann}}{d\Omega} = \frac{\alpha^3\beta}{2\pi s}(2 - \beta^2 + \beta^2 c^2)\Upsilon,$$

$$\Upsilon = 2\ln\frac{1+\beta c}{1-\beta c}\ln\left(\frac{2\Delta E}{m}\right) + \Phi(s, \cos\theta)$$

$$\Phi(s, \cos\theta) = \mathcal{D}_S^{\rm ann} - \frac{\mathcal{D}_V^{\rm ann}}{2 - \beta^2 + \beta^2 c^2}.$$



Relation between scales of the Gell-Mann--Low and $\overline{\text{MS}}$ schemes

$$\log \frac{\mu_0^2}{m_\ell^2} = 6 \int_0^1 x(1-x) \log \frac{m_\ell^2 + Q_0^2 x(1-x)}{m_\ell^2}$$

$$\log \frac{\mu_0^2}{m_\ell^2} = \log \frac{Q_0^2}{m_\ell^2} - 5/3$$

$$\boxed{\mu_0^2 = Q_0^2 \, e^{-5/3}} \quad \text{when } Q_0^2 >> m_\ell^2$$

D. S. Hwang, sjb

M. Binger

Can use $\overline{\text{MS}}$ scheme in QED; answers are scheme independent

Analytic extension: coupling is complex for timelike argument

The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds -- number of active leptons set
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion
- Results are scheme independent!

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free Conformal Series

Running Coupling Effects

Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

Light by Light Loops

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BLM: Absorb β terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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Institute for Advanced Study, Princeton, New Jersey 08540
*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**

G. Peter Lepage
Institute for Advanced Study, Princeton, New Jersey 08540
*and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q* sets the number of active flavors
- Only n_f dependence required to determine renormalization scale at NLO
- Result is scheme independent! Q* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- **Resulting series identical to conformal series!**
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

BLM Scale Setting

$$\beta_0 = 11 - \frac{2}{3} n_f$$

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2}\beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right] ,$$

where

$$Q^* = Q \exp(3A_{VP}) ,$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B \ .$$

The term $33A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

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n_f dependent coefficient identifies quark loop VP contribution

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Use skeleton expansion:

Gardi, Grunberg, Rathsman, sjb

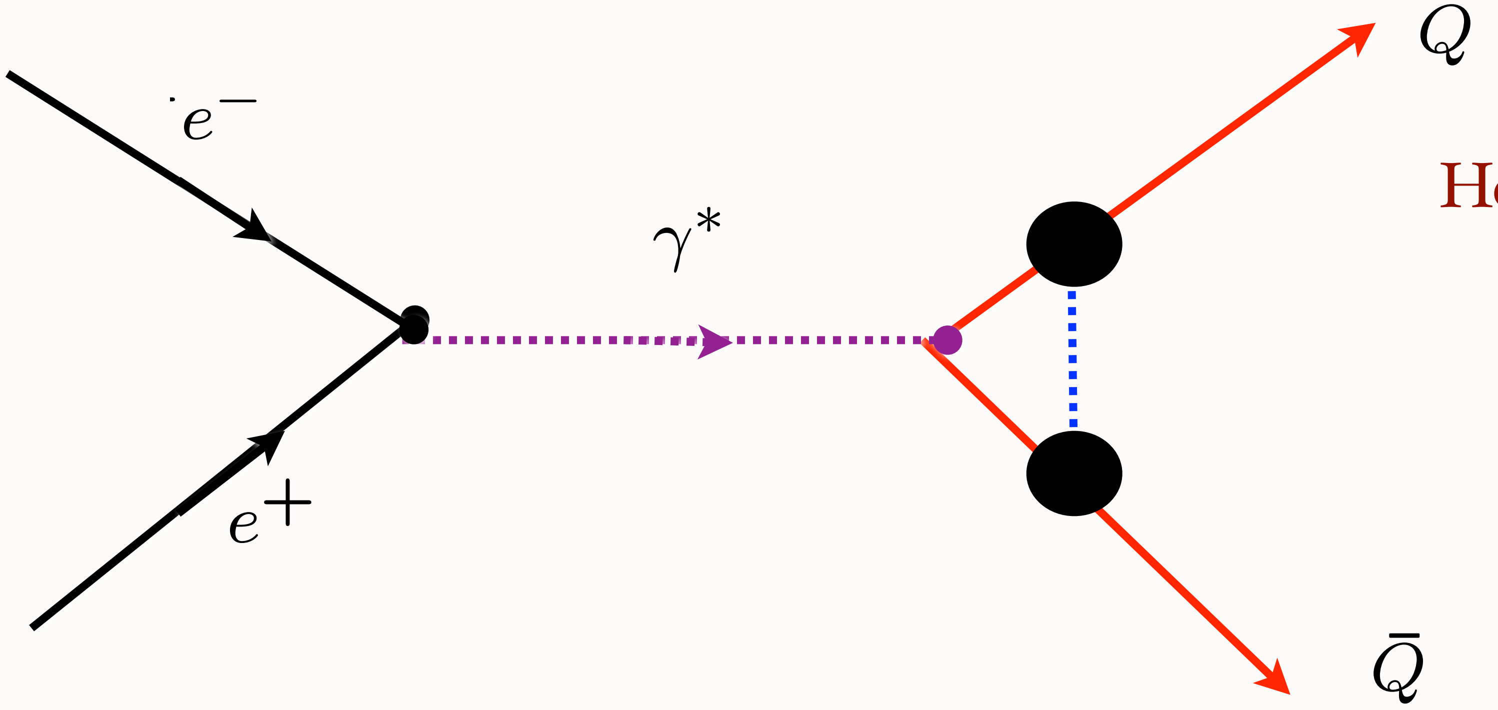
Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED



Hoang, Kuhn, Teubner, sjb

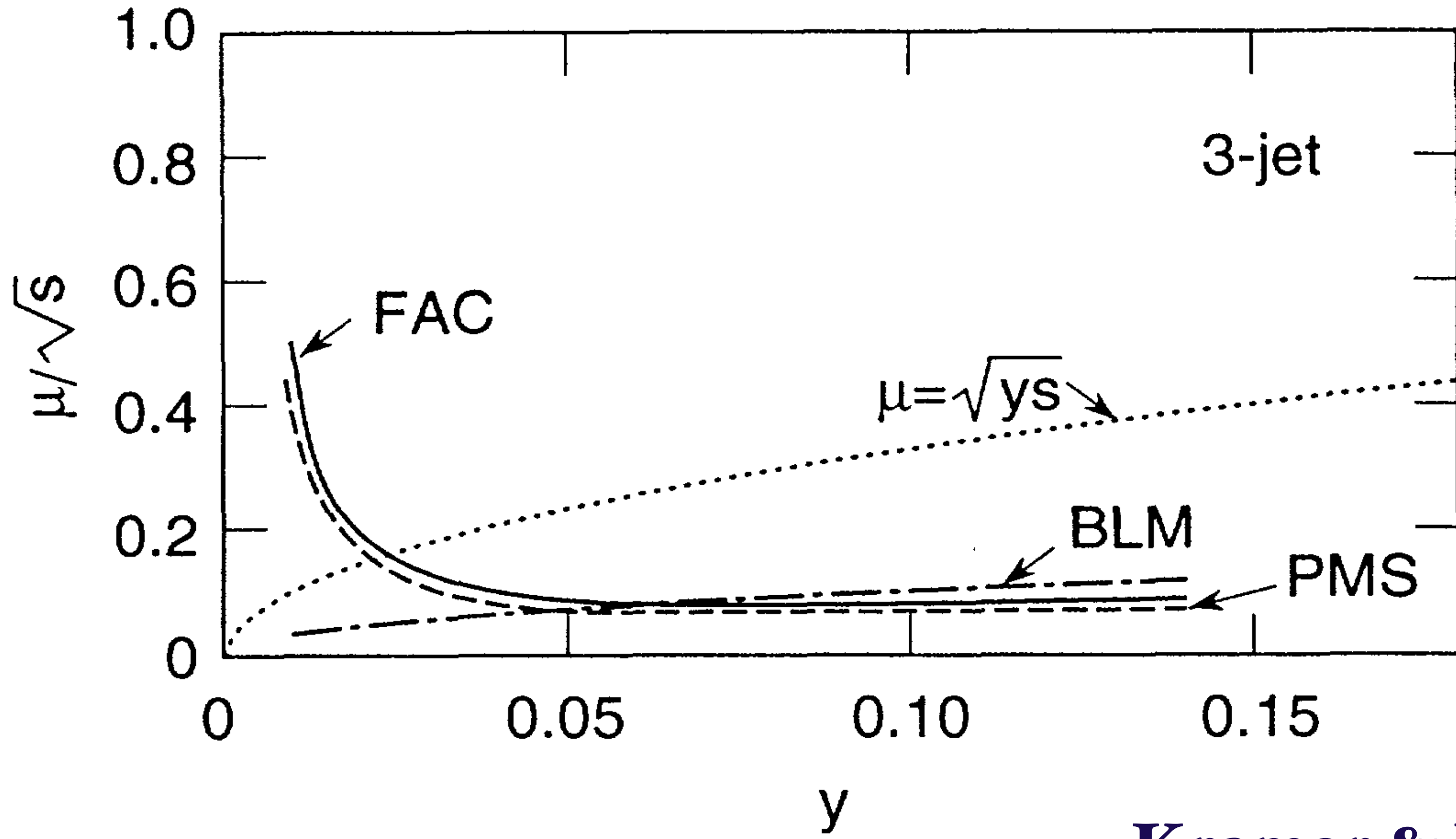
$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(sv^2)}{4v}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Three-jet production in electron-positron annihilation



Jet Invariant mass squared:

$$\mathcal{M}^2 = ys$$

BLM scale is gluon jet virtuality

Kramer & Lampe

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

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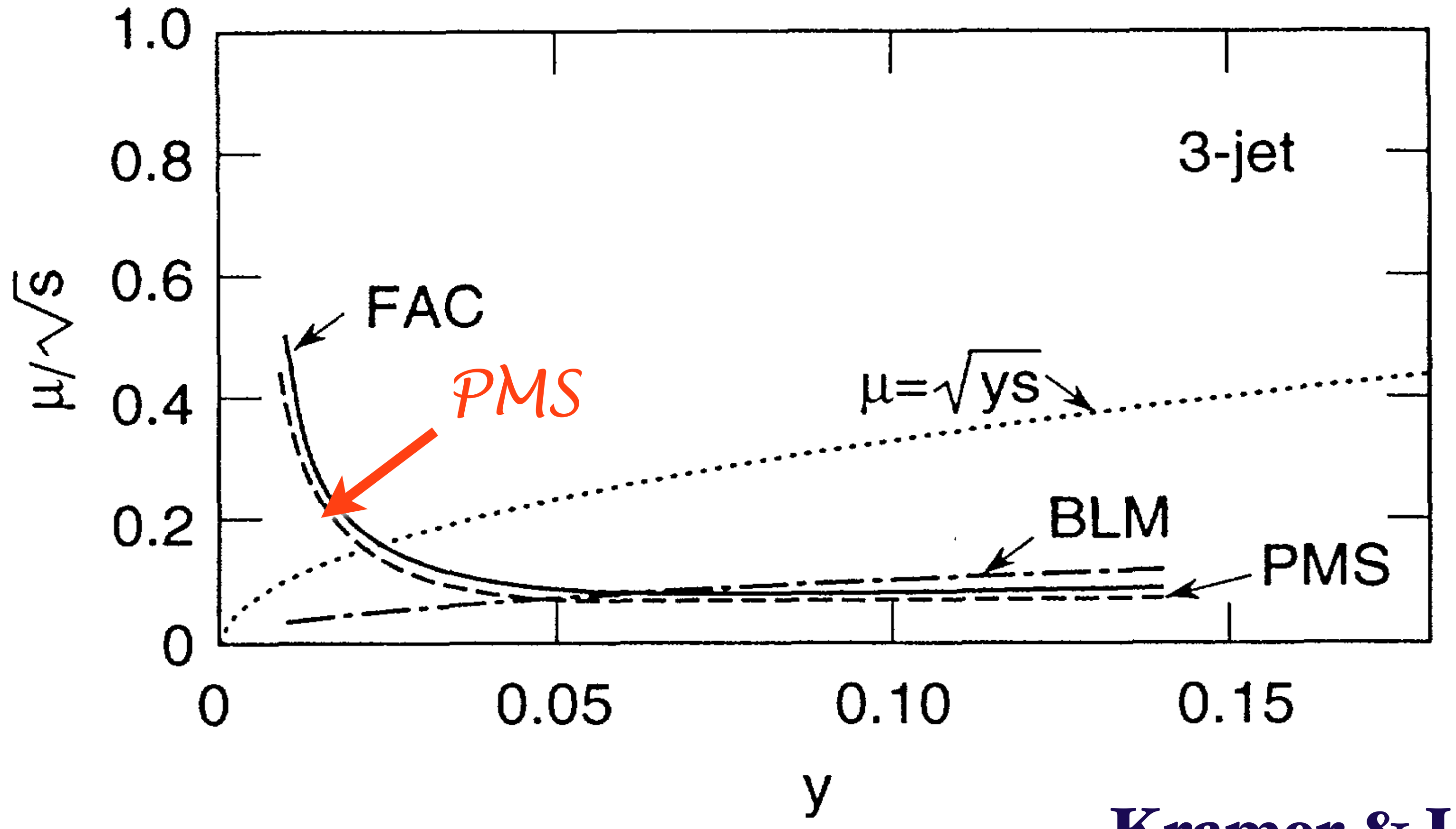

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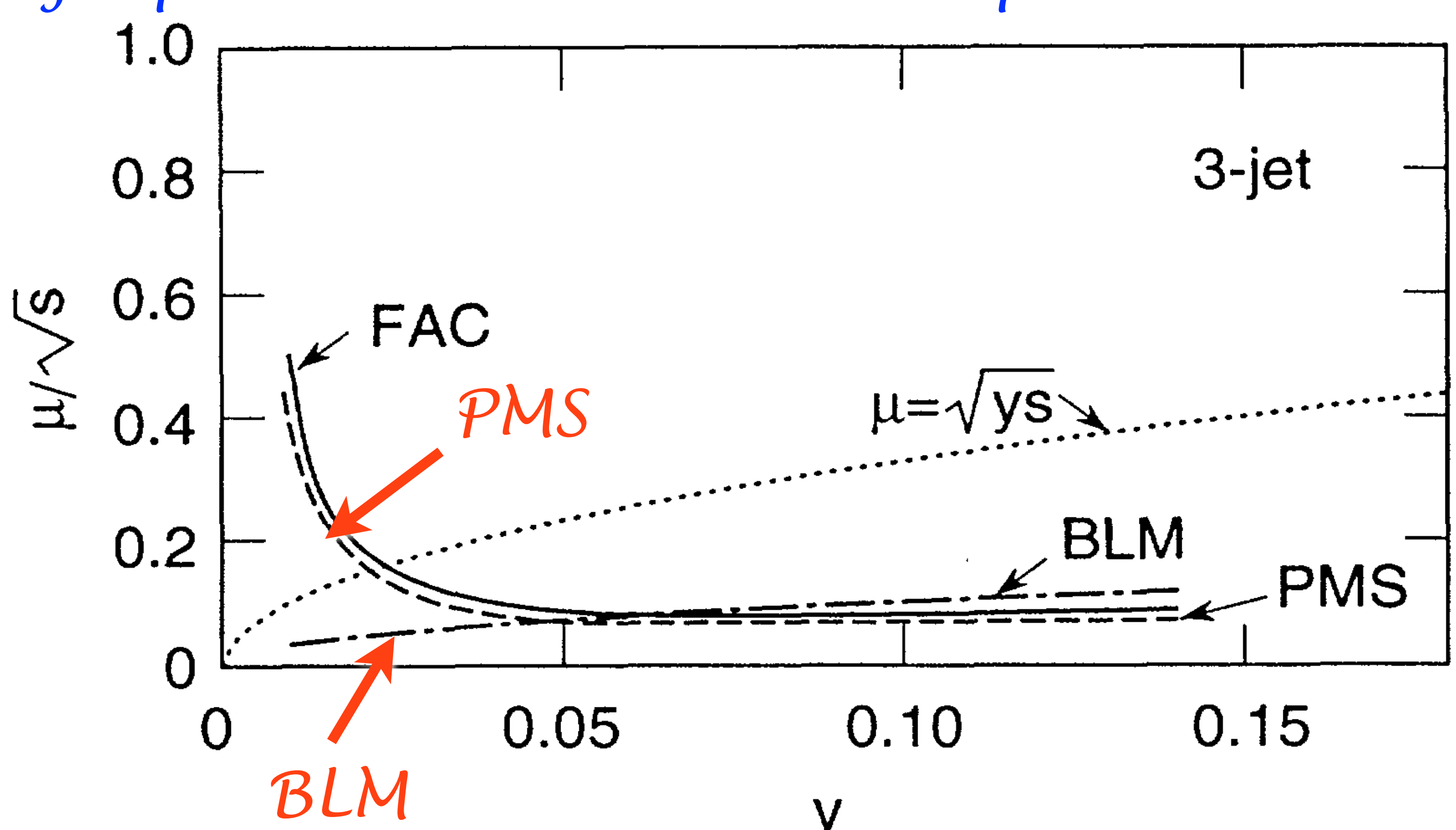

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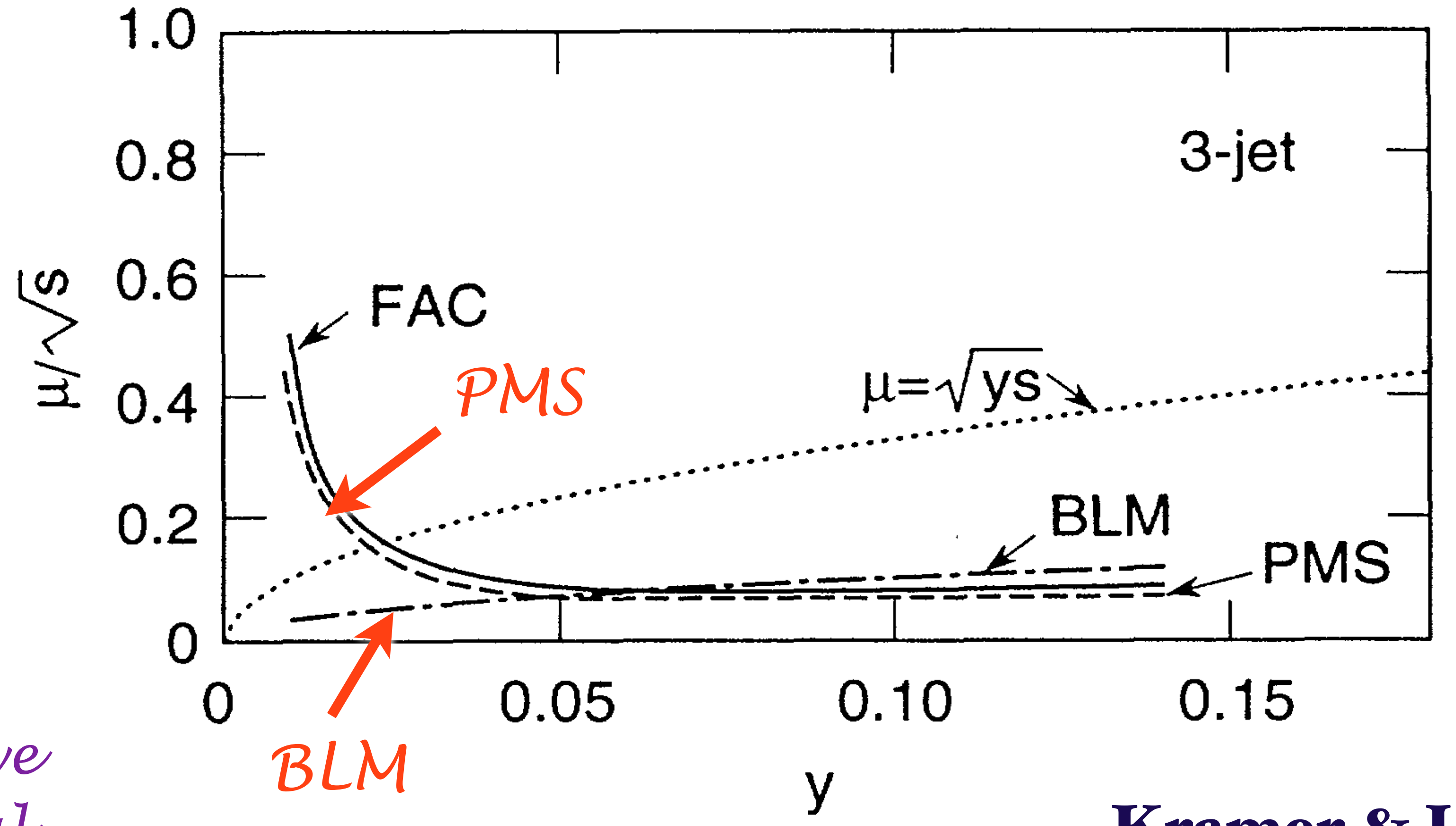

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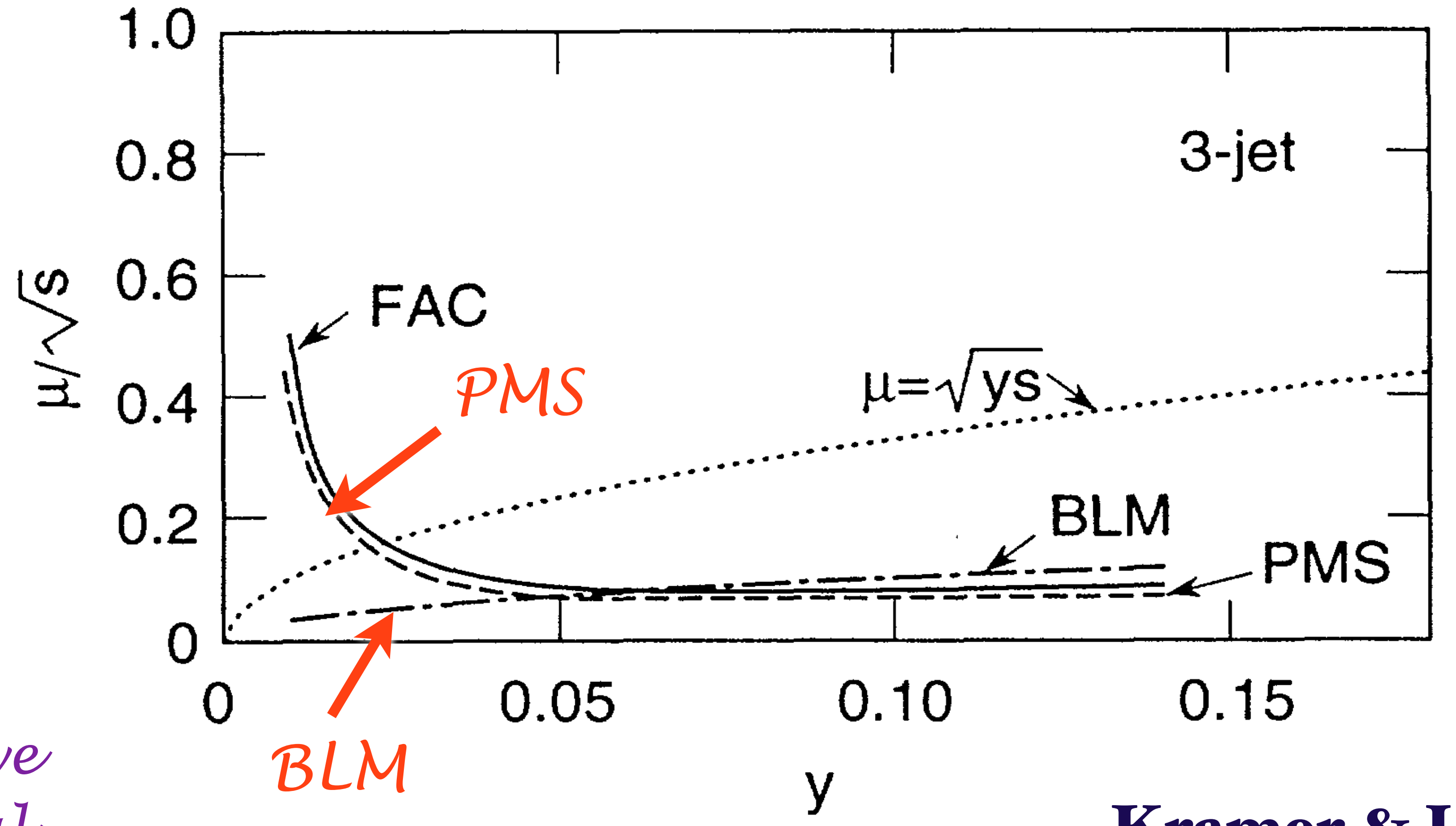

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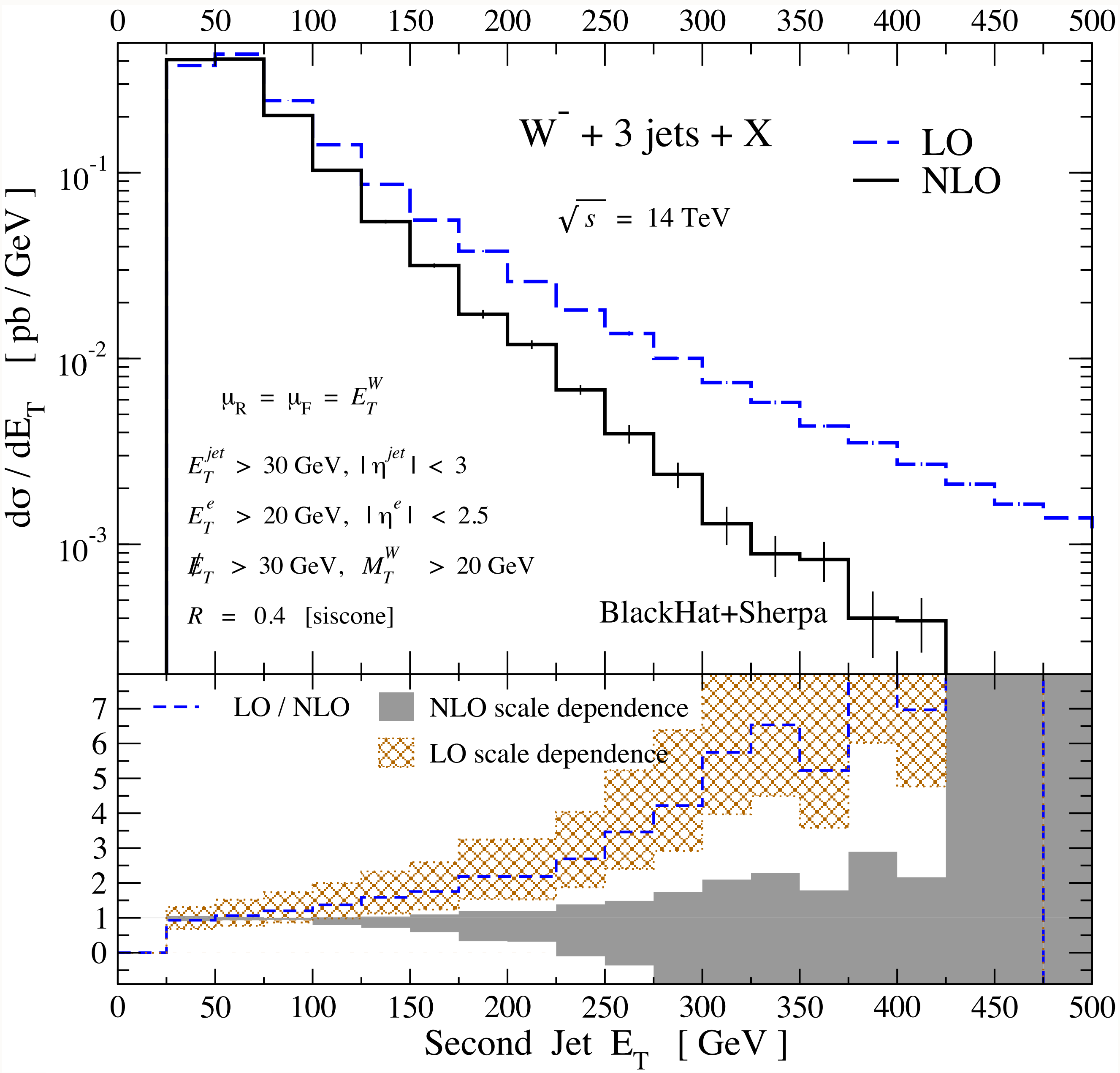
Other Jet Observables using BLM: Rathsman

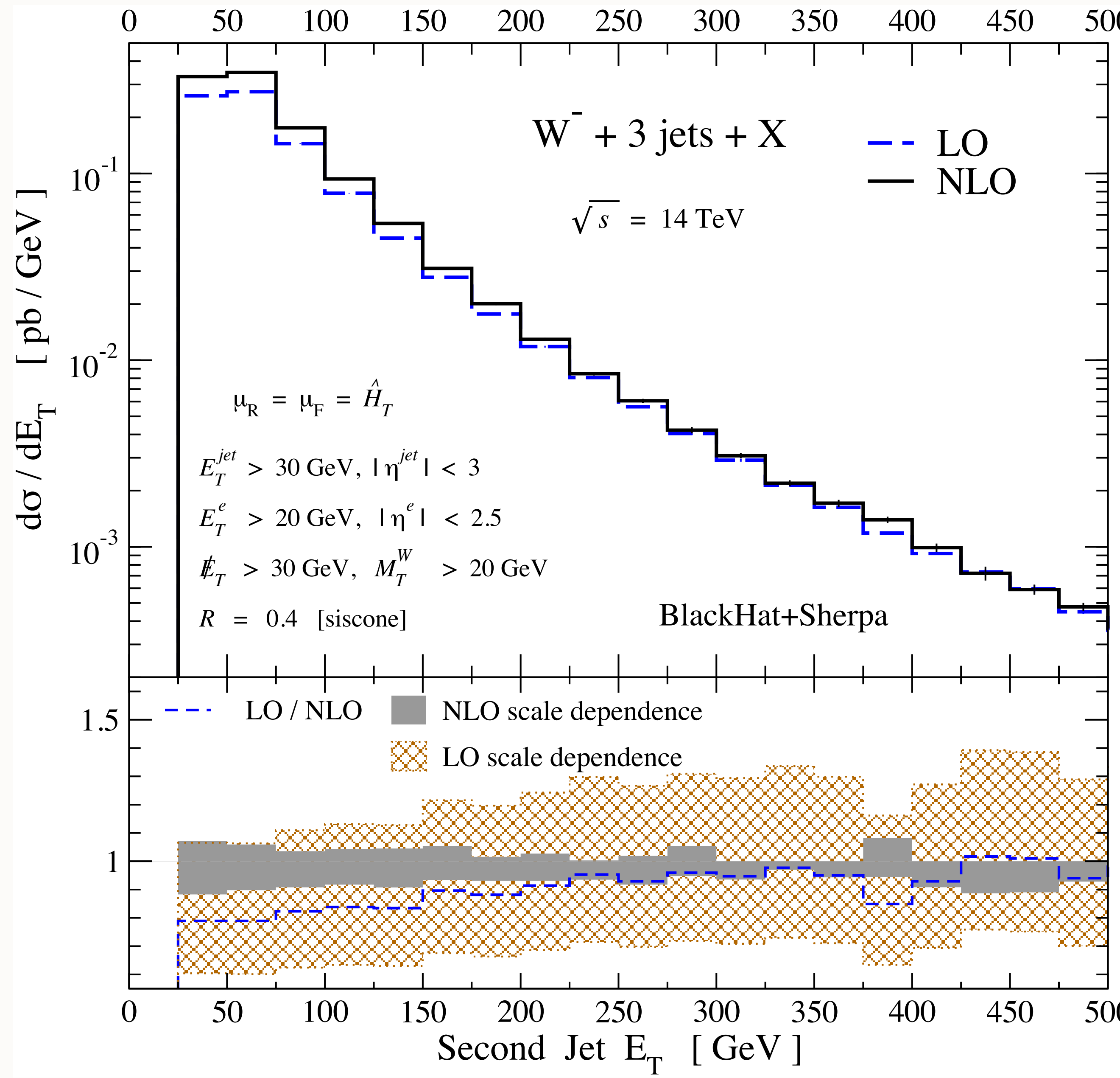
Next-to-Leading Order QCD Predictions for W + 3-Jet Distributions at Hadron Colliders

Black Hat

$$\mu_R = \mu_F = E_T^W$$



$$\mu_R = \mu_F = \hat{H}_T$$

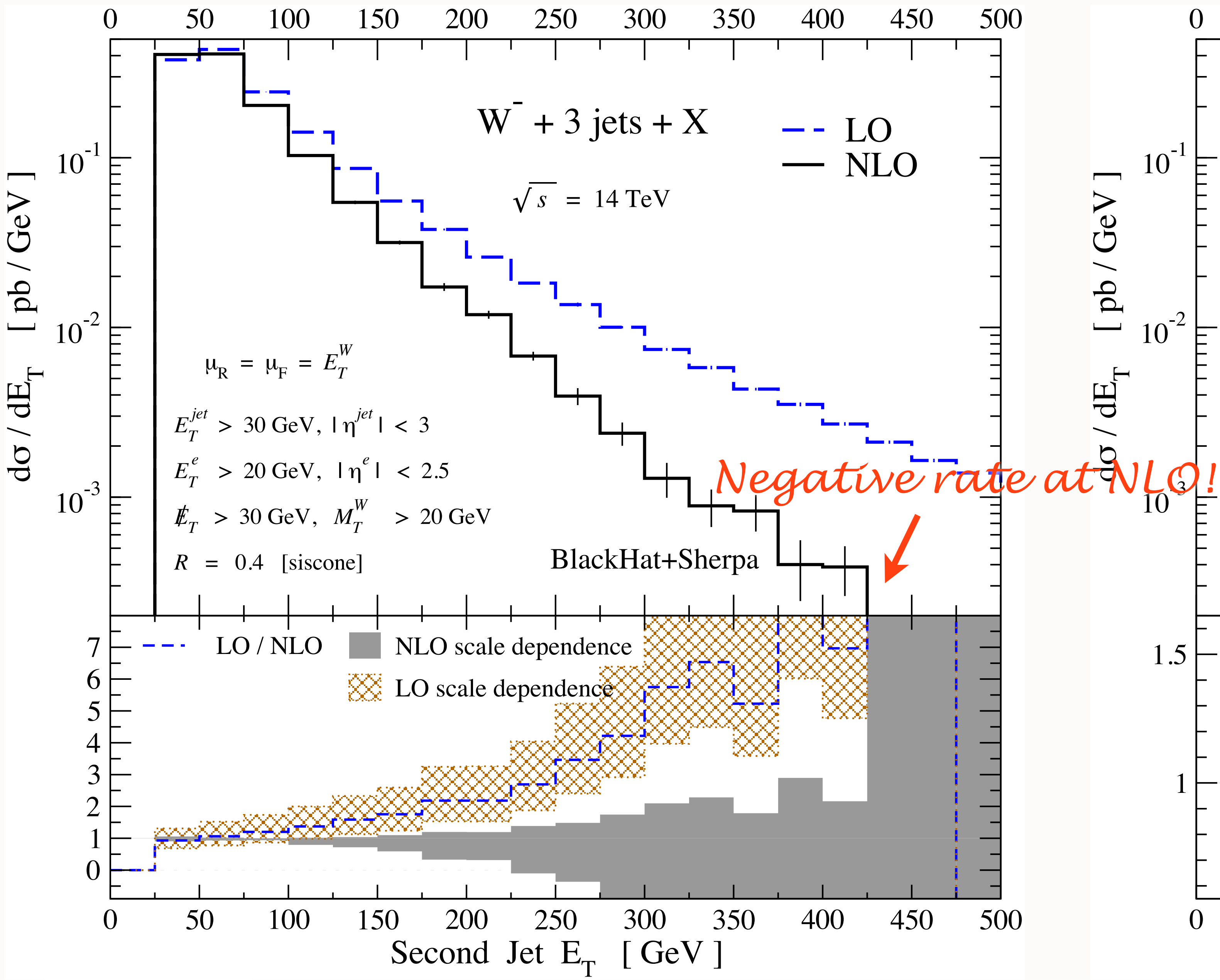


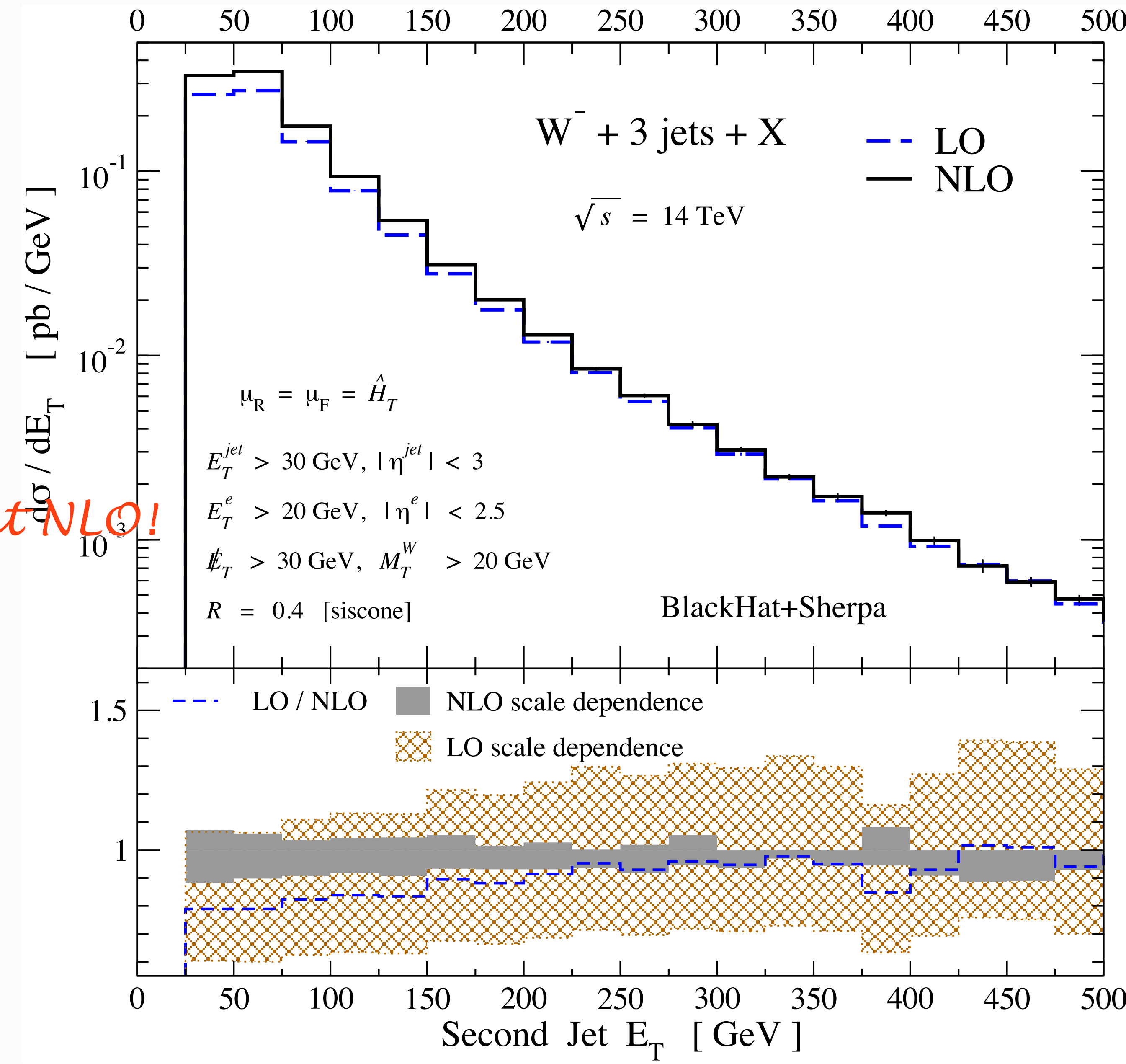
. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maıtre

Next-to-Leading Order QCD Predictions for W + 3-Jet Distributions at Hadron Colliders

Black Hat

$$\mu_R = \mu_F = E_T^W \qquad\qquad \mu_R = \mu_F = \hat{H}_T$$





. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maître

Myths concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale

- Can ignore possibility of multiple physical scales

- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$

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Clearly heuristic. Wrong in QED, Scheme dependent!

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These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED, Scheme dependent!

Principle of Maximal Conformality

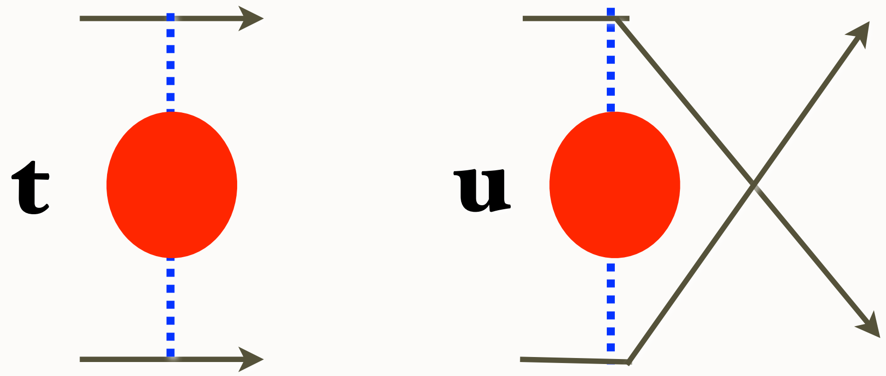
- BLM: Set scale in each skeleton graph to absorb all nonzero beta terms.
- In practice easier to set a single global scale
- Consider general hard subprocess: $a + b \to c + d + e + \cdots$

  e.g., $pp \to W + 3 \text{ jets} + X$
- Set of invariants q_i^2
- Global scale $\hat{\mu}^2 = \Pi_i (q_i^2)^{w_i}$
- Weights $\{w_i\}$ $\sum_i w_i = 1$

Identify w_i from $\frac{dM}{d \log q_i^2}$

Di Giustino, sjb

Example: Spinless electron-electron scattering



$$M = \frac{s-t}{t}\alpha(t) + \frac{s-u}{u}\alpha(u)$$

Scales sum VP to all orders

$$M \simeq [\tfrac{s-t}{t} + \tfrac{s-u}{u}]\alpha(\mu_0^2)$$
$$+[\tfrac{s-t}{t}]\frac{\alpha^2(\mu_0^2)}{3\pi} n_\ell \log(\tfrac{t}{\mu_0^2}) + [\tfrac{s-u}{u}]\frac{\alpha^2(\mu_0^2)}{3\pi} n_\ell \log(\tfrac{u}{\mu_0^2})$$

$$M = [\frac{s-t}{t} + \frac{s-u}{u}]\alpha(\hat{\mu}^2)$$

$$\hat{\mu}^2 = t^{w_t} \times u^{w_u}$$

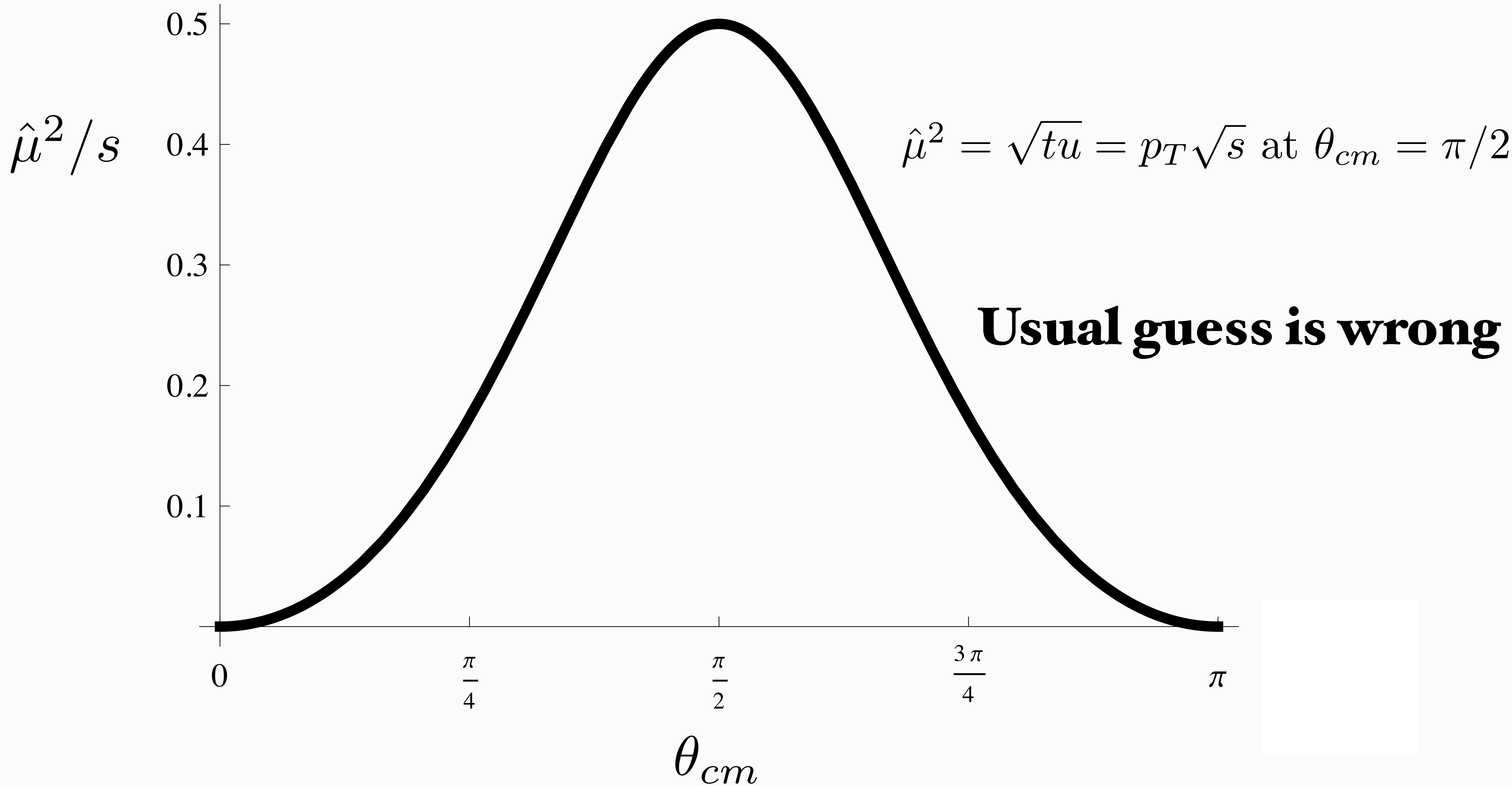
$$w_t = \frac{\frac{s-t}{t}}{\frac{s-t}{t} + \frac{s-u}{u}} \qquad w_u = \frac{\frac{s-u}{u}}{\frac{s-t}{t} + \frac{s-u}{u}}$$

Identify w_t from $\frac{dM}{d\log t}$

Remaining $\mathcal{O}(\alpha^2)$ correction is conformal

Spinless electron-electron scattering

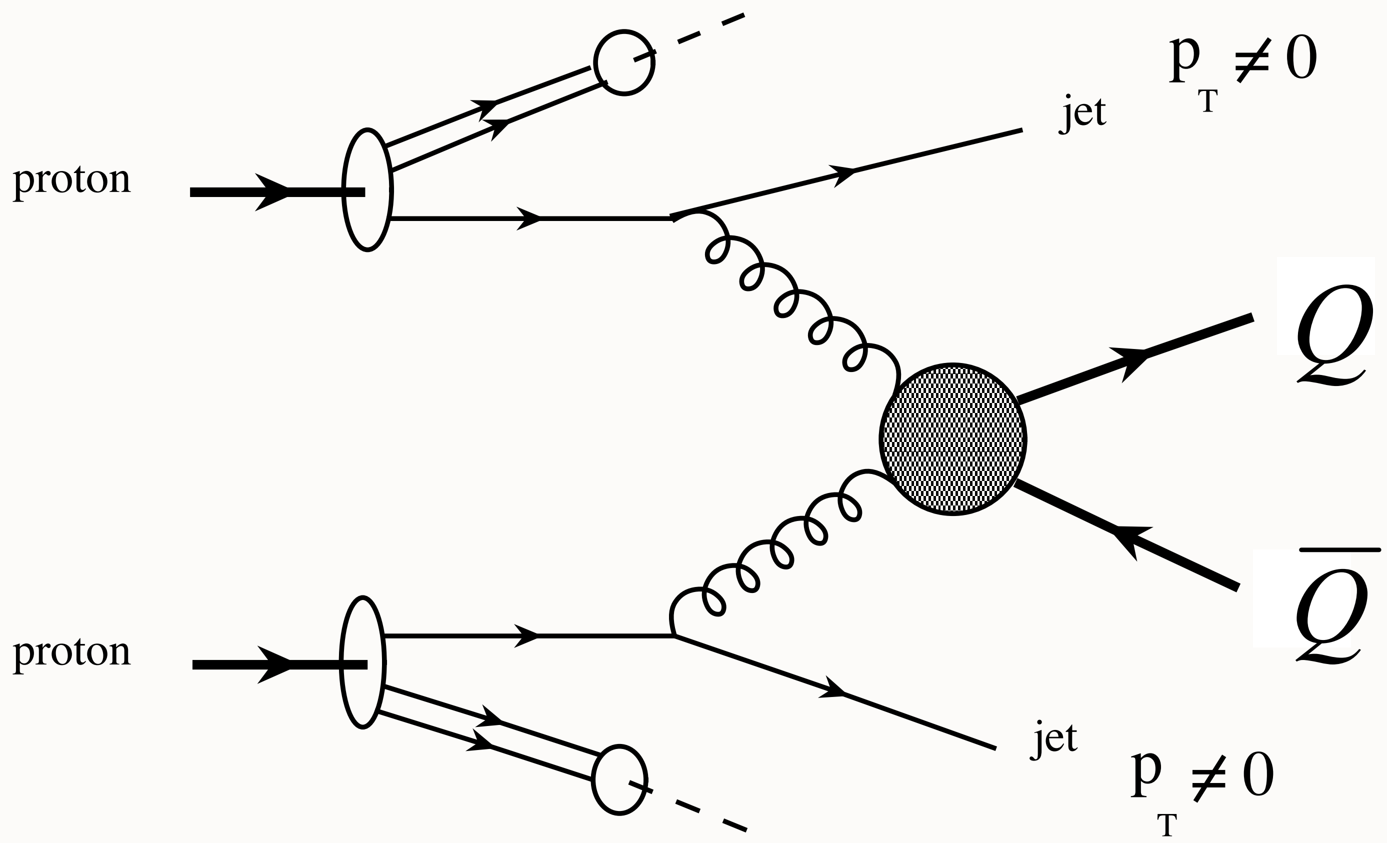
$$\hat{\mu}^2 = t^{w_t} \times u^{w_u}$$

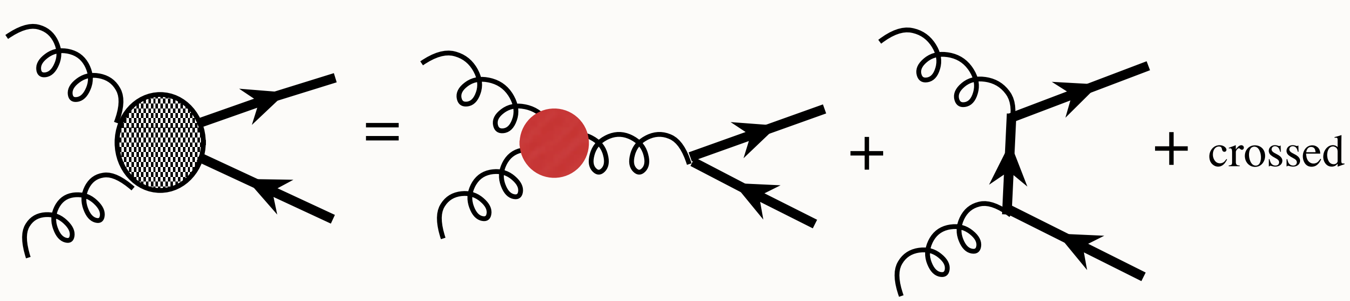


PMC

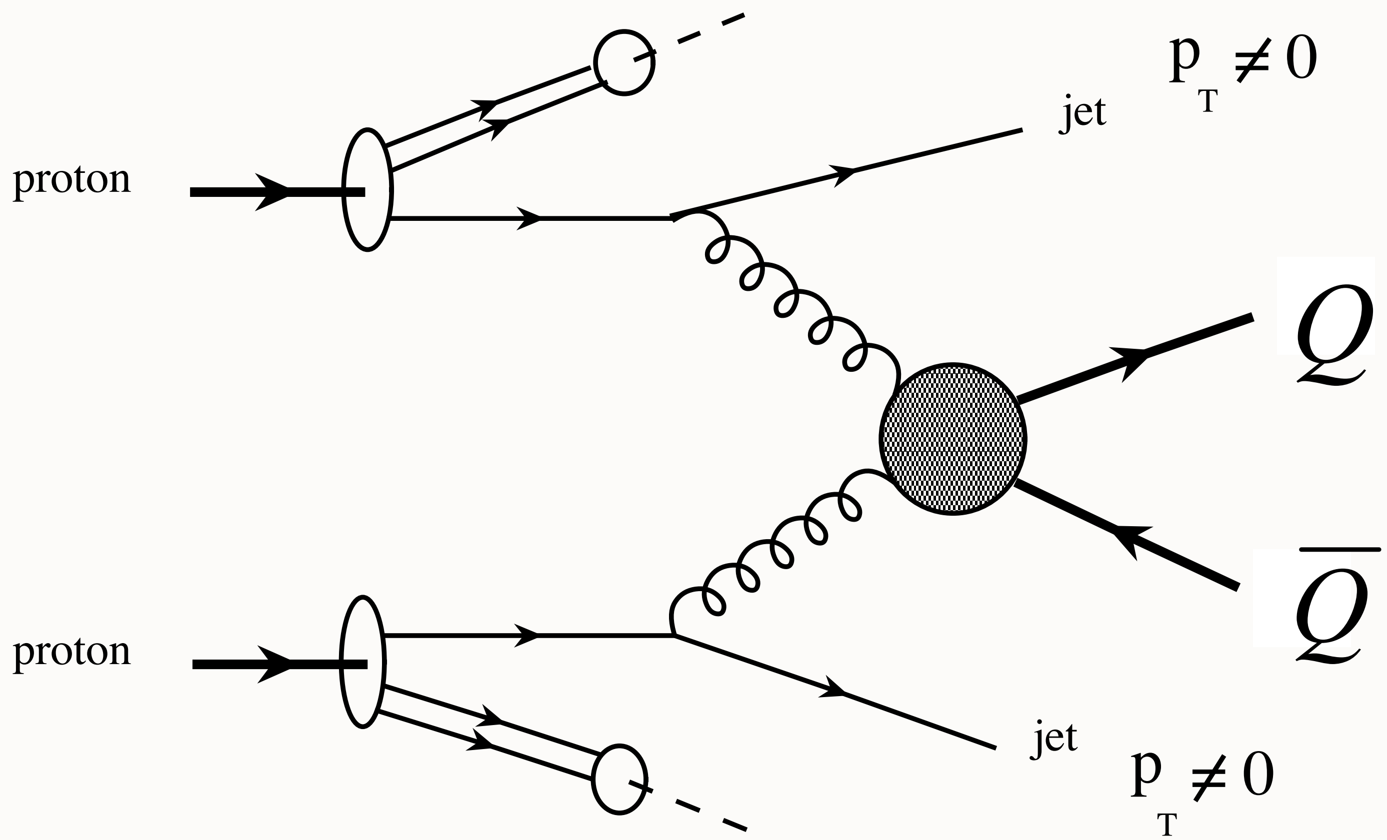
- PMS/FAC - incorrectly sums conformal terms -- even minimizes physical asymmetries!
- PMC/BLM: exposes conformal series - no renormalons
- Conformal series has new physics -- not associated with renormalization
- PMC: No need to analyze diagrams or codes -- simply identify non-conformal logarithms -- then shift scale
- PMC: Applies to subprocesses with multiple final particles- recursive procedure
- PMC/BLM: Agrees with QED in Abelian limit
- PMC/BLM: Result is independent of scheme and initial scale choice

Heavy Quark Hadroproduction

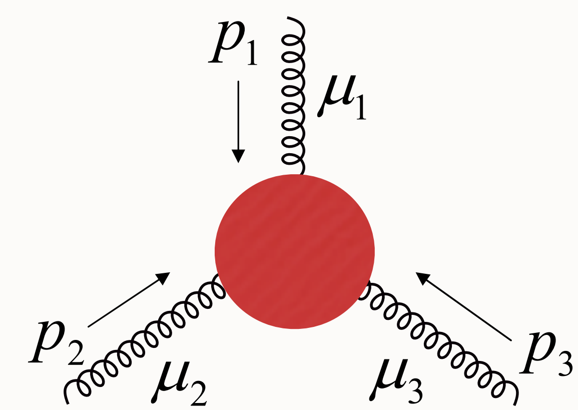


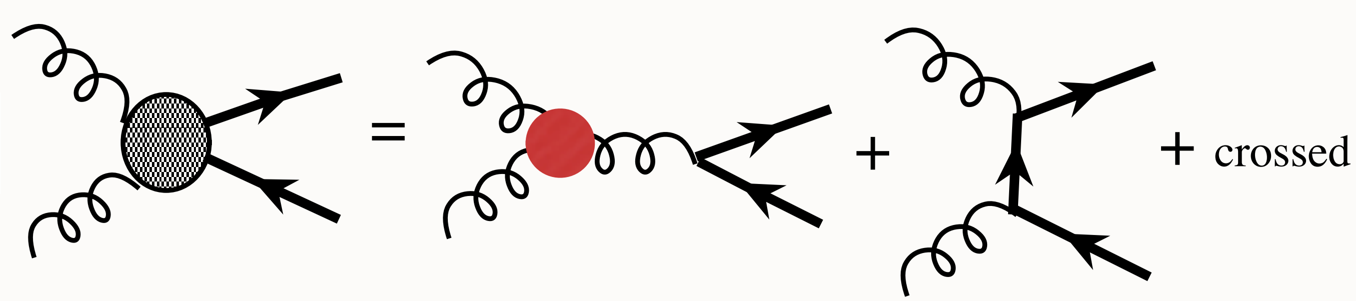


Heavy Quark Hadroproduction



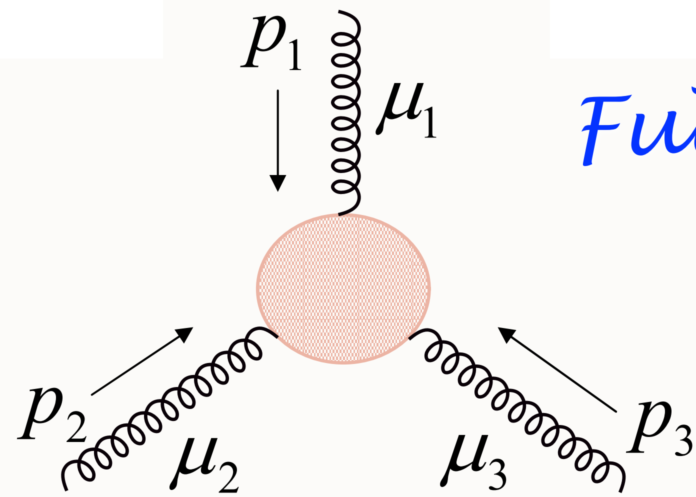
3-gluon coupling depends on 3 physical scales





General Structure of the Three-Gluon Vertex

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



Full analytic calculation, general masses, spin
Pinch Scheme

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3
with $p_1 + p_2 + p_3 = 0$

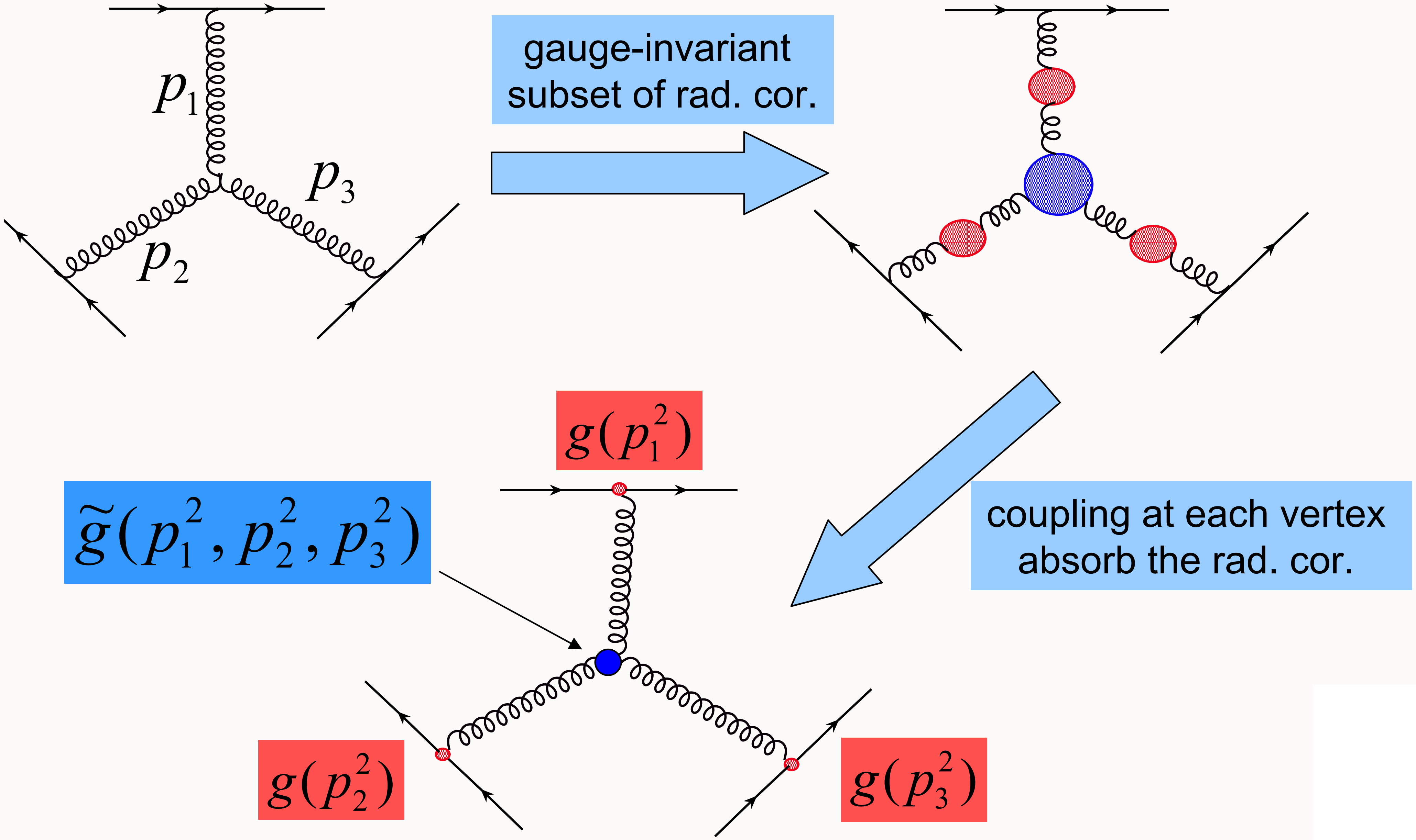
⇒ 14 basis tensors and form factors

PHYSICAL REVIEW D 74, 054016 (2006)

Form factors of the gauge-invariant three-gluon vertex

Michael Binger* and Stanley J. Brodsky†

Multi-scale Renormalization of the Three-Gluon Vertex



3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi}\beta_0\left(L(a,b,c) - \frac{1}{\varepsilon} + \cdots\right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

L(a,b,c) = 3-scale “log-like” function

L(a,a,a) = log(a)

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54 |a|$$

$$Q^2_{eff}(a,a,c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c|$$

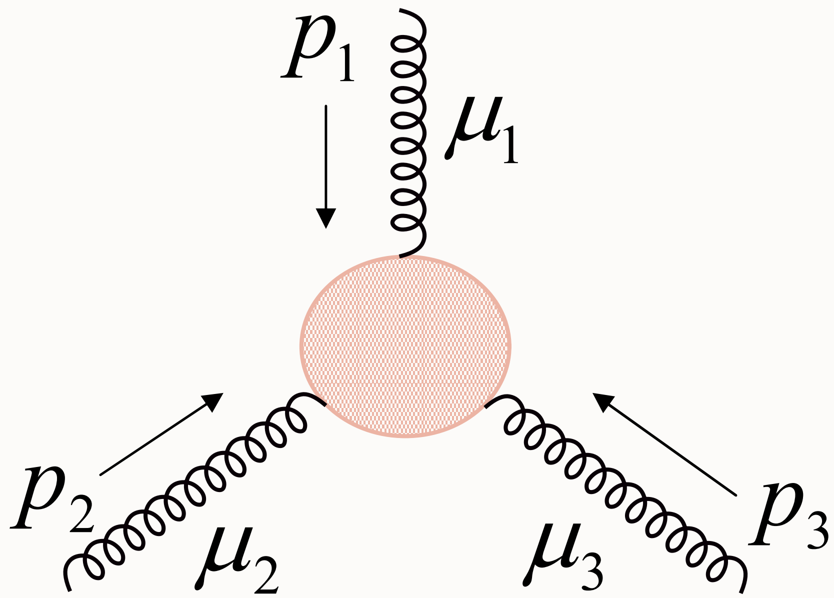
$$Q^2_{eff}(a,-a,c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c|$$

Pinch Scheme

Surprising dependence on Invariants

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$

H. J. Lu

Scale determines effective number of flavors

Define QCD Coupling from Observables

Grunberg

Effective Charges: analytic at quark mass thresholds, finite at small momenta

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu)\times[1+\frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations:
Relate observable to observable at commensurate scales

H.Lu, Rathsman, sjb

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32} C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32} C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648} f^2\right\}.$$

Eliminate MS
Find Amazing Simplification

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

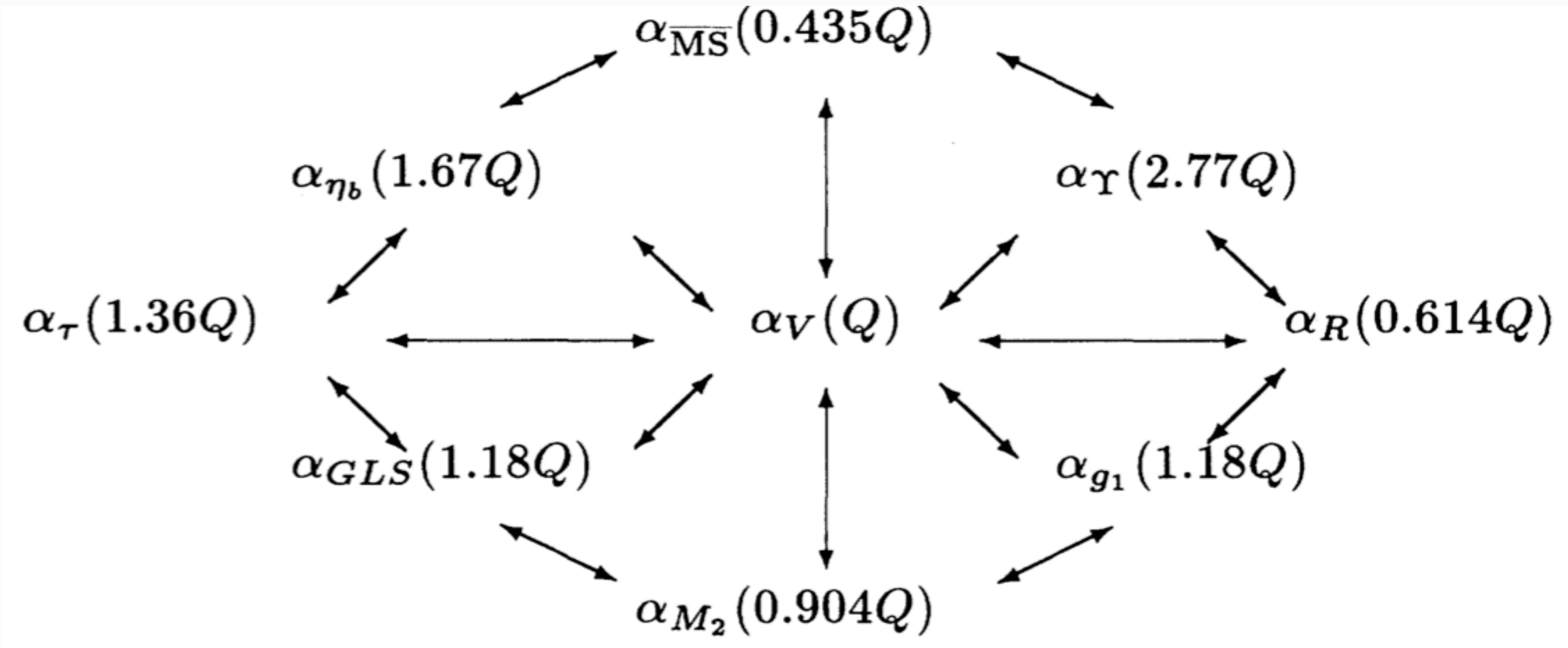
Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

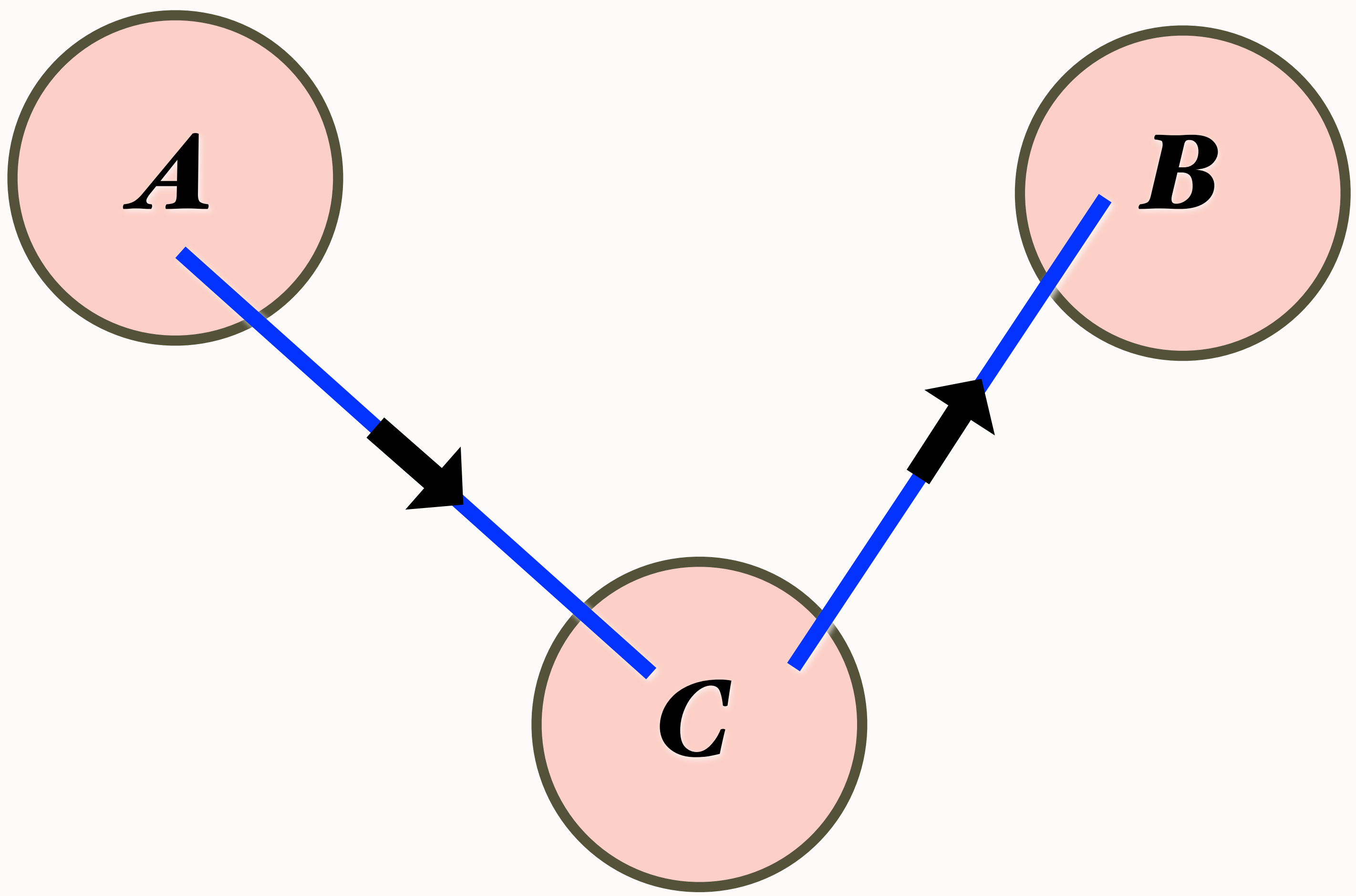

$\alpha_{\overline{MS}}(0.435Q)$
$\alpha_{\eta_b}(1.67Q)$
$\alpha_{\Upsilon}(2.77Q)$
$\alpha_{\tau}(1.36Q)$
$\alpha_V(Q)$
$\alpha_R(0.614Q)$
$\alpha_{GLS}(1.18Q)$
$\alpha_{g_1}(1.18Q)$
$\alpha_{M_2}(0.904Q)$

$$\frac{\alpha_\tau(M_\tau)}{\pi} = \frac{\alpha_R(Q^*)}{\pi},$$

$$Q^* = M_\tau \exp\left[-\frac{19}{24} - \frac{169}{128}\frac{\alpha_R(M_\tau)}{\pi}\right]$$

Transitivity Property of Renormalization Group

Relation of observables must be independent of intermediate scheme



A* ➡ *C*   *C* ➡ *B*   identical to   *A* ➡ *B

Violated by PMS!

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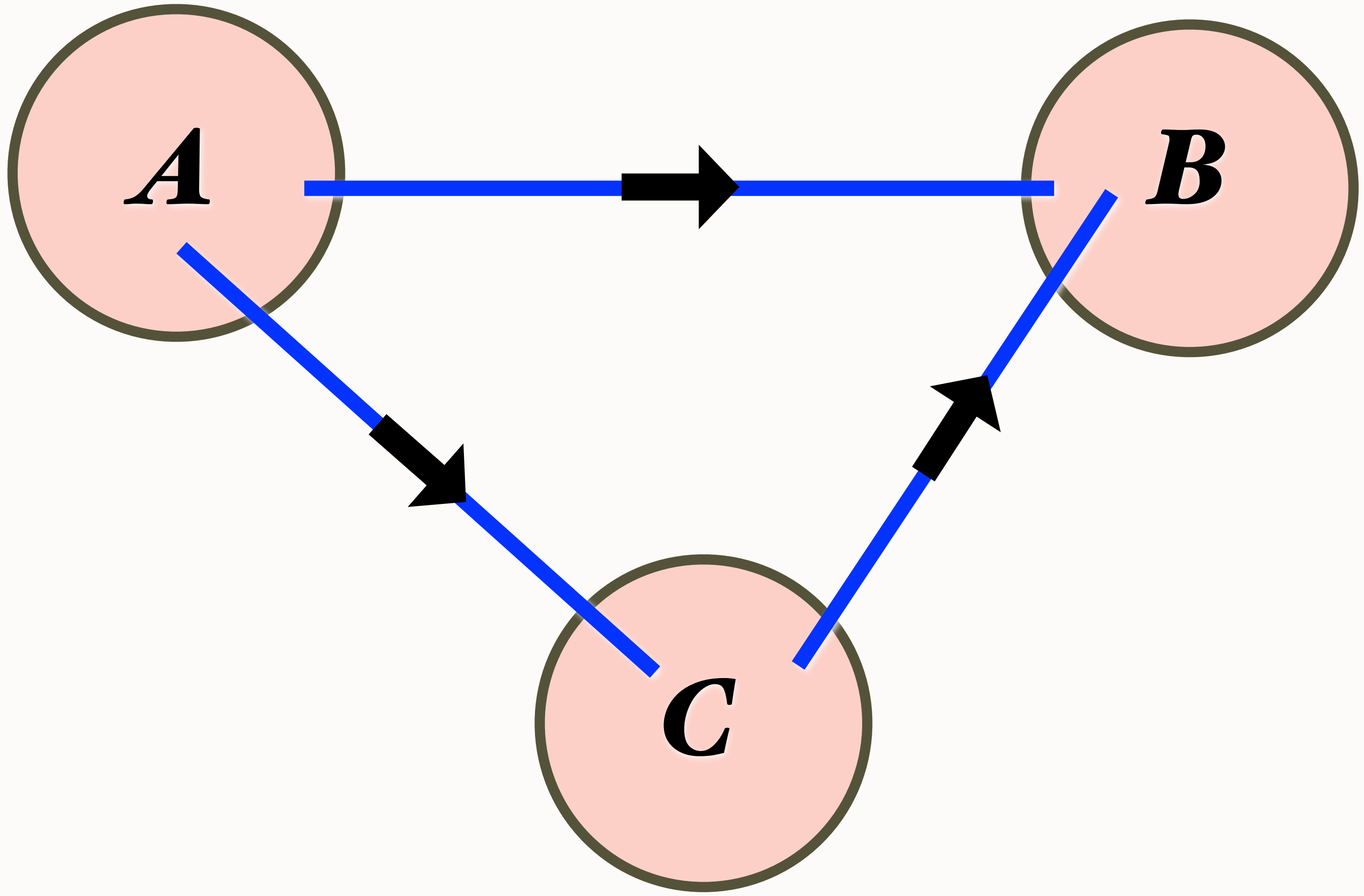
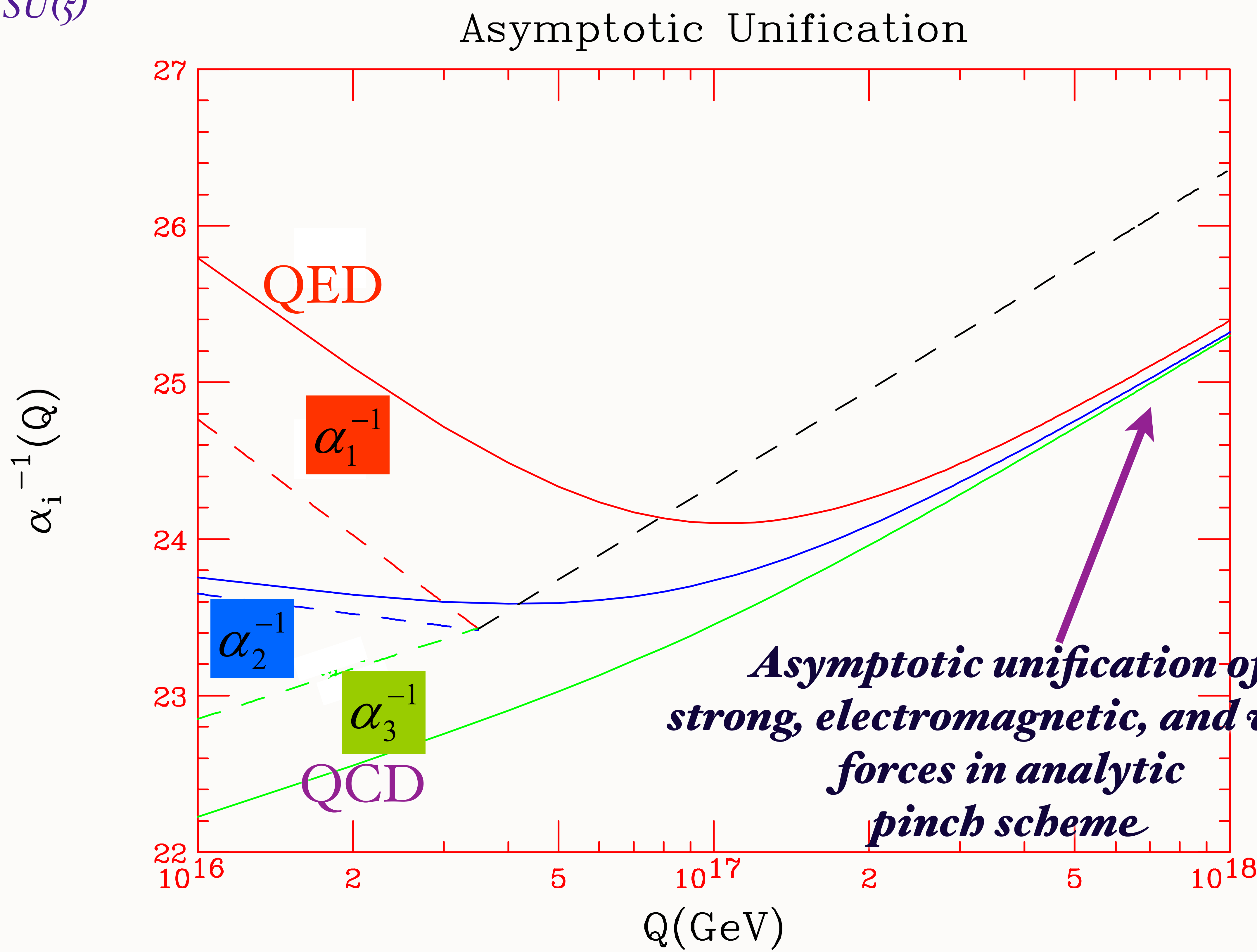


A → C   C → B   identical to A → B

Violated by PMS!

Supersymmetric SU(5)

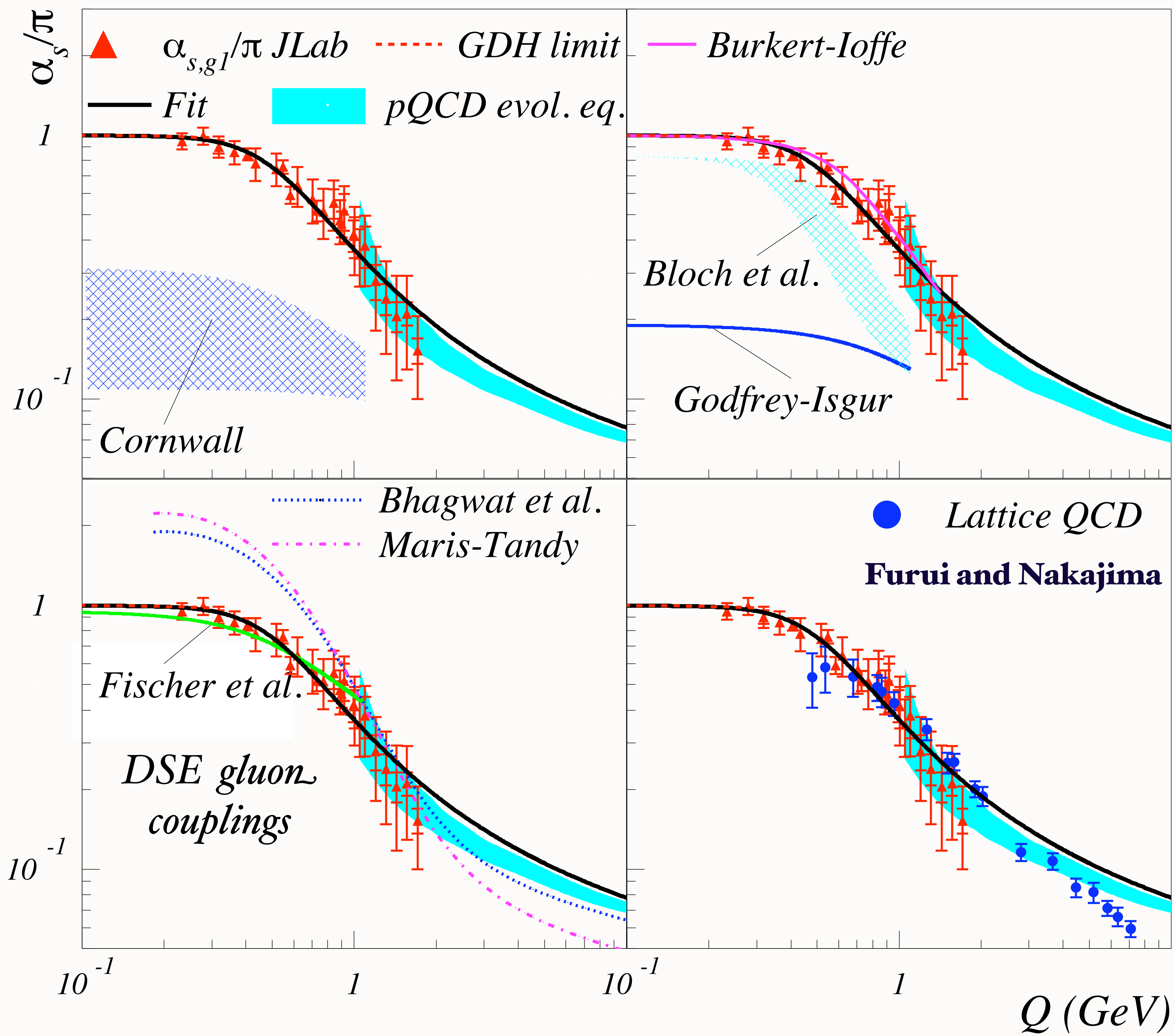
Binger, sjb



Asymptotic unification of strong, electromagnetic, and weak forces in analytic pinch scheme

Deur, Korsch, et al.

Evidence for IR Fixed Point



IR Fixed Point for QCD?

- Effective Gluon Mass Cornwall
- Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point! Alkofer, Fischer, von Smekal et al.
- Lattice Gauge Theory **Furui and Nakajima**
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small Q^2
- Justifies application of AdS/CFT in strong-coupling conformal window

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

Deur, de Teramond, sjb,

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g}\; e^{\phi(z)} \frac{1}{g_5^2} G^2 \quad \text{where } \sqrt{g} = \left(\frac{R}{z}\right)^5 \text{ and } \phi(z) = +\kappa^2 z^2$$

Define an effective coupling $g_5(z)$

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g} \frac{1}{g_5^2(z)} G^2$$

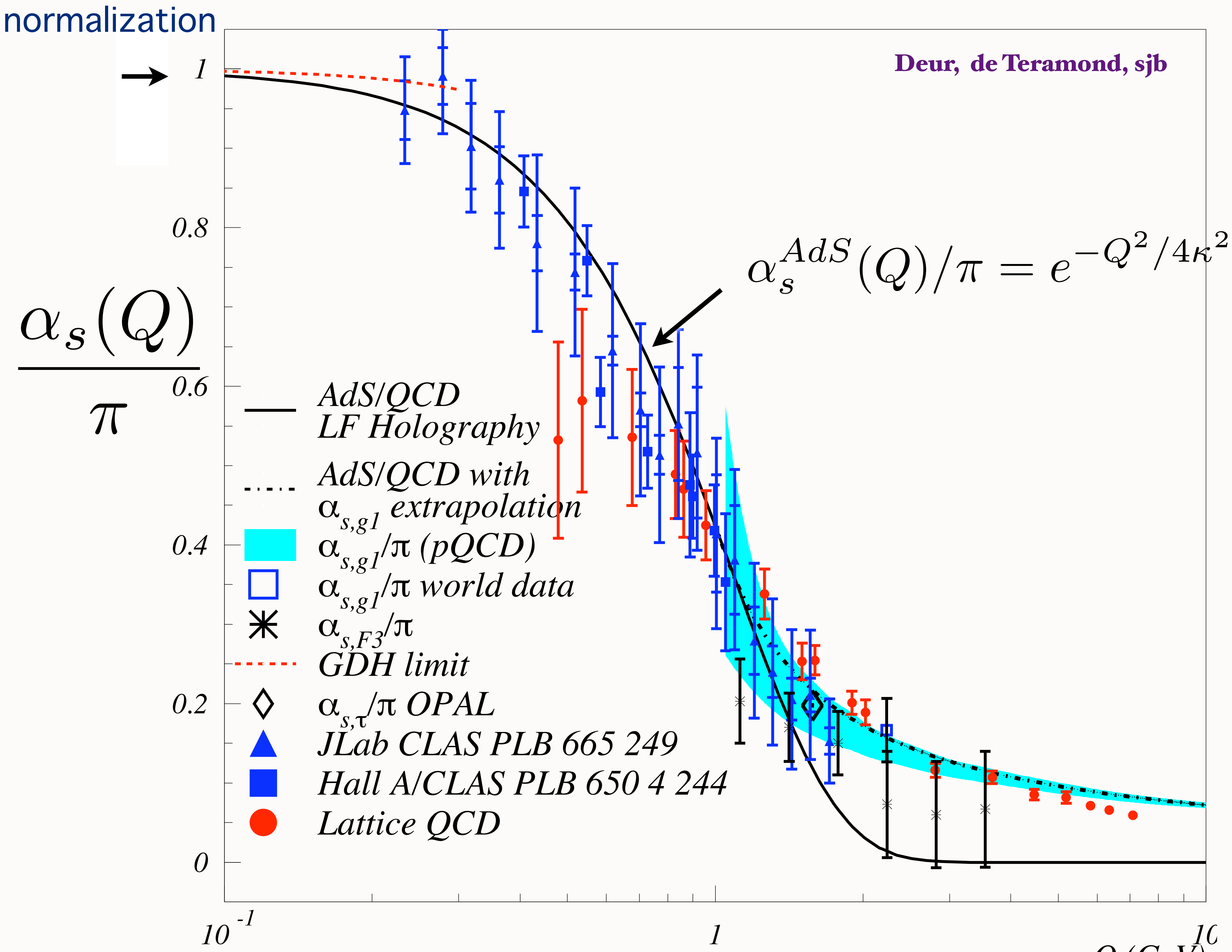
$$\text{Thus } \frac{1}{g_5^2(z)} = e^{\phi(z)} \frac{1}{g_5^2(0)} \text{ or } g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

Light-Front Holography: $z \to \zeta = b_\perp \sqrt{x(1-x)}$

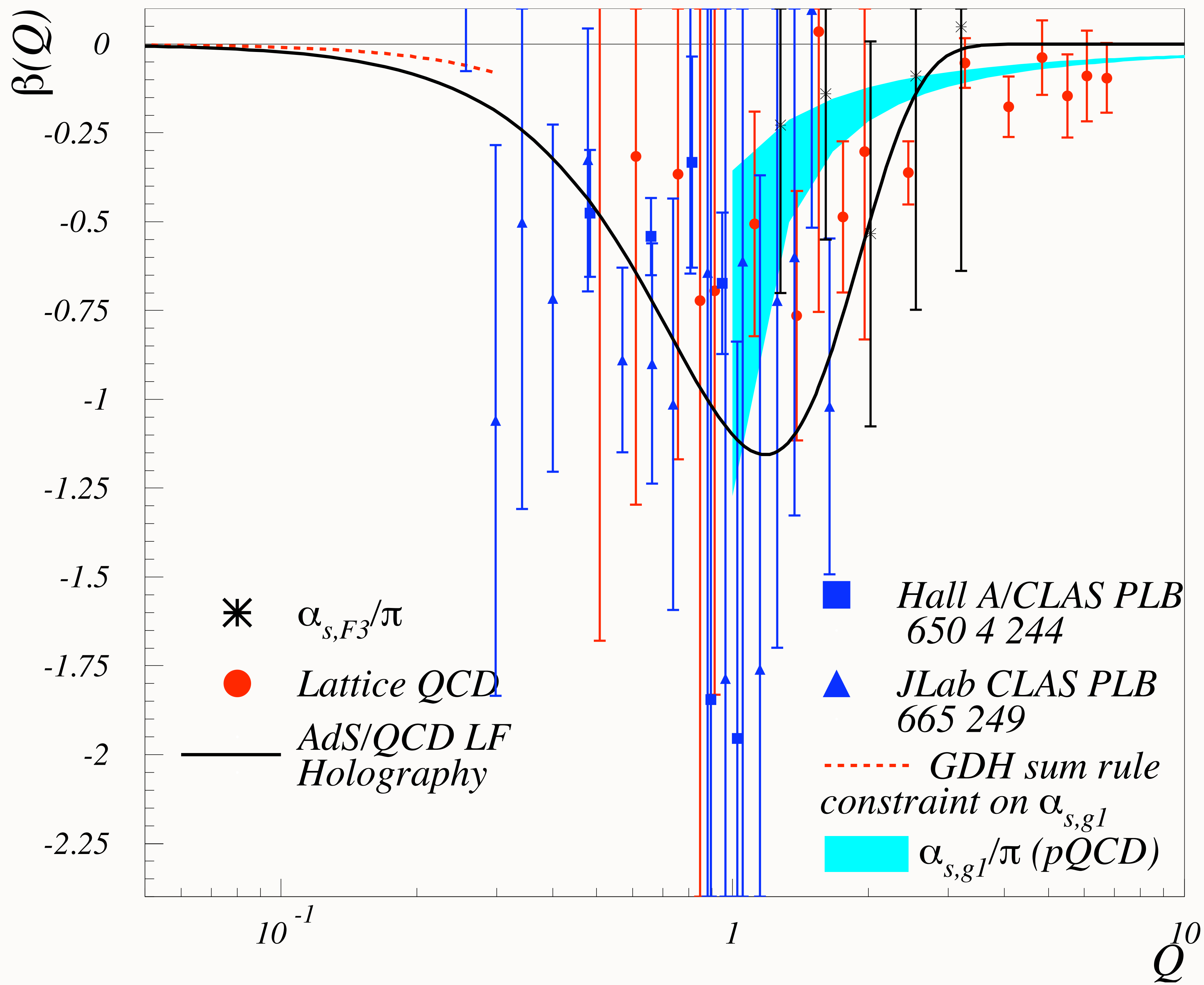
$$\alpha_s(Q^2) \propto \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \propto e^{-Q^2/4\kappa^2}$$

Principle of Maximal Conformality

Running Coupling from AdS/QCD

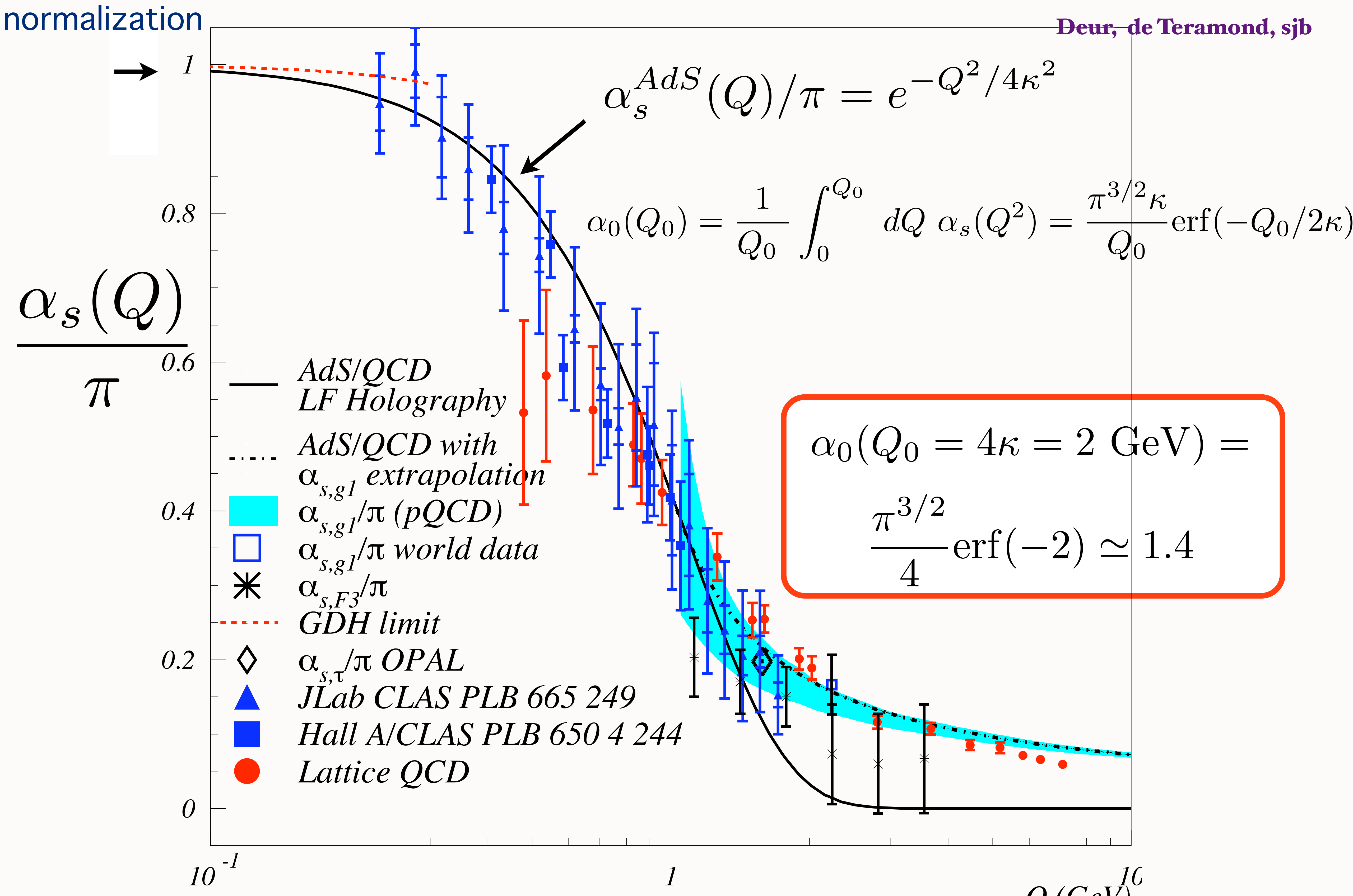


β - Function of AdS/QCD Coupling



Deur, de Teramond, sjb

Running Coupling from AdS/QCD



Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain**
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**
- **Similar in spirit to Cornwall's Effective Gluon mass**

$$(x-y)^2 < \Lambda^{-2}_{QCD}$$

Quark and Gluon vacuum polarization insertions decouple: IR Fixed-Point

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map AdS_5 X S_5 to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window (IR Fixed-Point)**

$$\alpha_s(Q^2) \simeq \text{const at small } Q^2$$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

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Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb — **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling; scheme independent
- Standard procedure in QED
- Resulting series identical to conformal series
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scales depend on all invariants

Conformal Template

- BLM scale-setting: Retain conformal series; nonzero β-terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.
- Principle of Maximal Conformality: Single Effective Scale
- ***Commensurate Scale Relations*** based on conformal template; scheme-independent
- Pinch Scheme -- provides analytic, gauge invariant, 3-g form factors
- Analytic scheme for coupling unification
- IR Fixed point -- conformal symmetry motivation for AdS/CFT
- Light-Front Schrödinger Equation: analytic first approximation to QCD
- Dilaton-modified AdS5: Predict Hadron Spectrum, Form Factors, α_s, β
- Light-Front Wave Functions from Holography: Hadronization at the amplitude level

Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders