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Jet production rates at LEP and the scale of α_s *

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We discuss the scale dependence of α_s in connection with jet multiplicities on the Z pole in the framework of perturbative QCD. Several scale defining procedures are applied to jet fractions and compared to recent measurements at LEP.

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Cross sections $\sigma_{n-\text{jet}}$ for the production of a fixed number n of jets in $e^+ e^-$ annihilation constitute a good testing ground for perturbative QCD [1–6]. Theoretically the jet multiplicities $f_n = \sigma_{n-\text{jet}}/\sigma_{\text{tot}}$ $(n=2, 3, 4)$ are in perturbative theory up to $O(\alpha_s^2)$ given by

$$\begin{aligned} f_2(s,y) &= 1 + C_{21}(y)\,\alpha_s(s) + C_{22}(y)\,\alpha_s^2(s) \\ f_3(s,y) &= C_{31}(y)\,\alpha_s(s) + C_{32}(y)\,\alpha_s^2(s) \\ f_4(s,y) &= C_{42}(y)\,\alpha_s^2(s) \end{aligned} \tag{1}$$

In a series of papers [7, 8] we have calculated the higher order corrections $C_{22}(y)$ and $C_{32}(y)$ to 2- and 3-jet production in the $\overline{\text{MS}}$ renormalization scheme and with renormalization scale $\mu=\sqrt{s}$, where $\sqrt{s}$ is the *cms* energy of the $e^+ e^-$ beams. y is the parameter for the resolution of jets. Two particles (partons) of momenta p_1 and p_2 are defined to form a jet if their invariant mass $(p_1+p_2)^2$ is smaller than ys. With our choice of renormalization scale μ the coefficients depend only on the dimenionsless resolution parameter y. The dependence on the *cms* energy $\sqrt{s}$ appears only through $\alpha_s(s)$. In addition the theoretical results depend on the QCD parameter $\Lambda_{\overline{\text{MS}}}$ (through α_s) and the number of flavours n_f (which appear in α_s and the second order coefficients $C_{i2}(y)$ $(i=2, 3, 4)$). We shall take $n_f=5$ throughout this paper.

As is well known the QCD coupling $a_s(\mu^2)$ depends on the renormalization point μ^2 used to define it. Although the complete jet rates f_n must be independent of μ^2, the dependence of α_s is compensated by the dependence of the higher order coefficients C_{nj}, any finite order result depends on the renormalization scale μ. Unfortunately it is not known what is the best choice of μ for any particular order of perturbative QCD. This ignorance about μ corresponds to an intrinsic theoretical error of the perturbative result and must be taken into account in any determination of the QCD scale $\Lambda_{\overline{\text{MS}}}$. In our calculations [7, 8] we had chosen $\mu^2=s$, since this is the customary choice of scale for the total hadronic cross section σ_{tot}. Otherwise the coefficient functions would not only depend on the resolution parameter y but also on the other dimensionless parameter $\mu^2/\sqrt{s}$. Physically the jet rates f_n depend on two scales, $\sqrt{s}$ and $\sqrt{ys}$. If we rescale back from $\mu=\sqrt{s}$ to the scale $\mu=\sqrt{ys}$ we introduce terms $\frac{\beta_0}{4\pi}\log y\ C_{21}(y) \sim \log^3 y$ $(\beta_0 = 11 - 2n_f/3)$ into $C_{22}(y)$ and a similar term into $C_{32}(y)$, which could make these coefficients large for small y. The analytic calculations show, however, that such terms with the opposite sign are present in $C_{22}(y)$ and $C_{32}(y)$ which are eliminated by a rescaling from $\mu=\sqrt{s}$ to $\mu=\sqrt{ys}$. From such considerations many authors [7, 8, 9] suggested to choose $\mu=\sqrt{ys}$ as the "natural" scale for jet cross sections. Various other procedures have been suggested in the literature [10–12], i.e. criteria for selecting for each variable the "best" value of μ. For example, it has been suggested by Stevenson [10] to take the scale μ_{PMS} which makes $d\sigma/d\mu=0$, where σ is the considered physical quantity (PMS = principle of minimal sensitivity). The idea behind this proposal is that the exact σ at all orders does not depend on μ. In a healthy case the result should not depend too much on which value of μ was actually chosen. With the Stevenson procedure one chooses the μ value where the change of σ with varying μ is minimal [11]. A second proposal is Grunberg's fastest apparent convergence principle (FAC) [12]. Here the scale μ is fixed by the requirement that the

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second order coefficient in the perturbative expansions of an observable, in our case f_2 and f_3, vanishes. This yields the FAC scale $\mu=\mu_{\rm FAC}$. Some time ago Brodsky, Lepage and Mackensie [13] suggested that whatever the renormaliation scheme used for calculating the higher order coefficients the scale should be chosen so that there is no explicit dependence on the number of flavours n_f. i.e. one chooses $\mu=\lambda\sqrt{s}$ such that for the observable O, given in perturbative theory up to $O(\alpha_s^2)$,

$$O=c_0\,\alpha_s(\lambda^2 s)\left[1+c_1^\lambda\frac{\alpha_s(\lambda^2 s)}{\pi}+\dots\right] \tag{2}$$

the higher order coefficient c_1^λ is n_f independent. The idea is that this leads to the most natural behaviour for the expansion as the *cms* energy is varied across a quark threshold, since the quark vacuum polarization is absorbed into α_s. We shall study this scheme, which yields $\mu_{\rm BLM}=\lambda\sqrt{s}$, starting from our coefficients $C_{i2}(y)$ calculated in the $\overline{\rm MS}$ renormalization scheme [7, 8]. Of course, all these PMS, FAC and BLM rules are somewhat ad-hoc and have no strong justification. Nevertheless, we think that these "principles" are plausible and give us at least a range of scales μ which should be considered.

In this note we shall study the μ dependence of the jet fractions f_2 and f_3 and determine the three scales $\mu_{\rm PMS}$, $\mu_{\rm FAC}$ and $\mu_{\rm BLM}$ as a function of $\Lambda_{\overline{\rm MS}}$. By comparing with recent very accurate measurements of the f_n ($n=2$, 3, 4, 5) by the OPAL collaboration at LEP [6] we can investigate whether the experimental data can be fitted by a universal $\Lambda_{\overline{\rm MS}}$ for all y values in the range $0.01\leqq y\leqq 0.14$ which, of course, may depend on the PMS, FAC or BLM procedure respectively. In addition we shall adopt also the original scale $\mu=\sqrt{s}$ and the scale $\mu=\sqrt{ys}$.

Results obtained at tree level always show a rather steep monotonous behaviour as a function of μ. This is the case for the 4-jet rate which has been calculated only in order α_s^2. If loop corrections are included, which are known for 2- and 3-jets [7, 8], the variation of the jet rates with μ is much smaller. This can be seen in Fig. 1 where we have plotted the μ^2 dependence of f_2 and f_3 for a specific set of parameters $y=0.03$ and $\Lambda=0.1$ GeV for $\sqrt{s}=91$ GeV. We skip the label $\overline{\rm MS}$ in the following. We find that the result is quite stable against scale variations so even if we do not trust the particular numbers, we obtained, for example, for the scales $\mu_{\rm PMS}$, $\mu_{\rm FAC}$, $\mu_{\rm BLM}$ or $\mu=\sqrt{ys}$, we get a prediction for the jet rates with an estimated error of less than 5%, if we limit the range of accepted scales μ to the vicinity where f_2 or f_3 is maximal (neighbourhood of $\mu_{\rm PMS}$). For comparison we have also included the corresponding curves for PETRA energies ($\sqrt{s}=34$ GeV) and see that the scale dependence at LEP ($=91$ GeV) is much weaker ($\Delta\mu^2=30$ GeV^2 versus $\Delta\mu^2=500$ GeV^2 in f_3), since α_s is smaller at LEP for Λ kept fixed. We observe, though, that the ratio of $\Delta\mu^2$ over μ^2 at the extremum does not change if we go from 34 to 91 GeV. In Fig. 1 we have marked the points where the special scales $\mu_{\rm PMS}$, $\mu_{\rm FAC}$, $\mu_{\rm BLM}$, $\sqrt{ys}$ and $\sqrt{s}$ lie for the f_3 at $\sqrt{s}=91$ GeV. The scales $\sqrt{y}$, $\mu_{\rm PMS}/\sqrt{s}$, $\mu_{\rm FAC}/\sqrt{s}$ and $\mu_{\rm BLM}/\sqrt{s}$ for the other y's between 0.01 and 0.14 are shown in Fig. 2 and Fig. 3 for f_2 and f_3 respectively. They are obtained from the higher order coefficients of the 2- and 3-jet rates [7, 8]. We see that these scales are different for f_2 and f_3 (except $\sqrt{y}$), but not very much. For PMS and FAC the scales are almost constant for $y\geqq 0.05$ and increase for y below 0.05. This increase is stronger for the scale in f_3 than in f_2. For $y\geqq 0.05$ these scales are rather small $\mu/\sqrt{s}\simeq 0.08$ which comes from large C_{i2}/C_{i1} ($i=2$, 3) in (1). For the small y's we expect the behaviour as for $\mu/\sqrt{s}=\sqrt{y}$. This is not the case. The reason is that C_{i2}/C_{i1} ($i=1$, 2) decreases for decreasing $y\leqq 0.03$. (They stay almost constant in the range $0.03\leqq y\leqq 0.14$). For

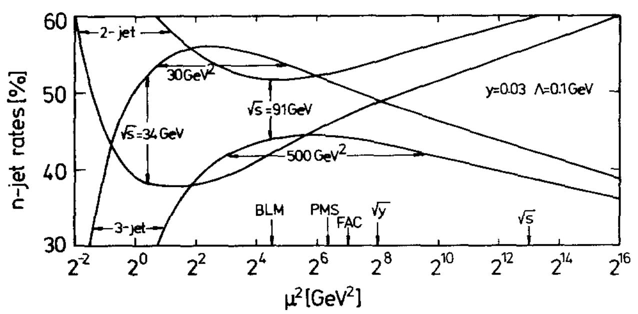

Fig. 1. n-jet production rates in % of the total hadronic cross section at PETRA (34 GeV) and LEP (91 GeV) energies as a function of the renormalization scale μ for $y=0.03$ and $\Lambda=0.1$ GeV. We have marked the BLM, PMS and FAC scale (for 3 jets) and the scales $\sqrt{ys}$ and $\sqrt{s}$ for the LEP energy

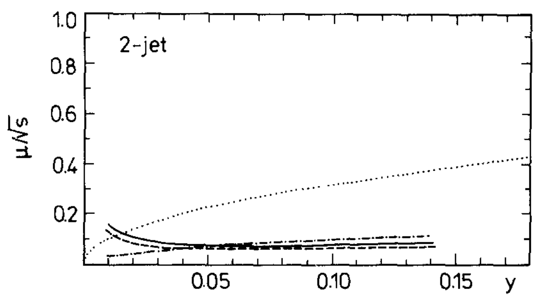

Fig. 2. The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full) and $\sqrt{y}$ (dotted) procedure for 2-jet rate f_2 as a function of y for $\sqrt{s}=91$ GeV and $\Lambda=0.1$ GeV

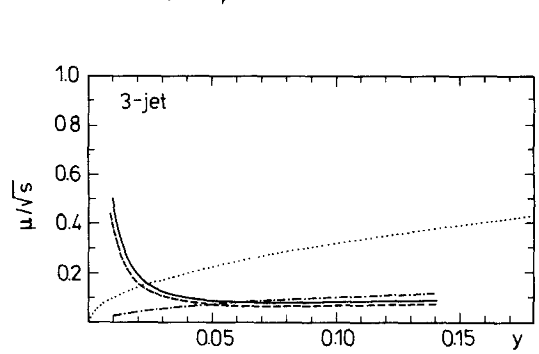

Fig. 3. Same as Fig. 2 for 3-jet rate f_3

small $y \leqq 0.02$ the whole approach might be questionable since not all higher-order terms are likely to be summed by changing the scale in α_s. In ref. [7] we have shown that for $y \leqq 0.02$ perturbation theory breaks down and that one has to include the effect of the radiation of an arbitrary number of gluons. This can be accommodated by "exponentiating" the $O(\alpha_s^2)$ result. It was shown that for $y \leqq 0.02$ the results from (1) and from the exponentiated version differ quite strongly. Therefore we should not take the theoretical predictions for $y \leqq 0.02$ too seriously. The BLM procedure, on the other hand, has the nice feature that the scale increases with increasing y as roughly $\mu = \sqrt{ys}$ does. This scale is not very different from the PMS and FAC scale for $y \geqq 0.05$. Only below $y = 0.05$ the PMS and FAC scales and the BLM scale differ appreciably. We also notice that for all three cases the scales from f_2 and f_3 are almost equal for $y \geqq 0.05$. In this range of y's the contribution of 4 jets is very small so that the C_{i2}/C_{i1} $(i = 2, 3)$ are almost equal for f_2 and f_3. The scales for f_3 as a function of y have been calculated earlier for lower energies [8, 14]. Only the PMS scale can depend on $\sqrt{s}$. But it is found that $\mu_{\mathrm{PMS}}/\sqrt{s}$ has only little energy dependence, so that the results in Fig. 2 and 3 are independent of $\sqrt{s}$. With the scales plotted in Fig. 2 and Fig. 3 we have calculated f_2 and f_3 and $f_4 = 1 - f_2 - f_3$ as a function of y between $y = 0.01$ and $y = 0.14$, where higher order coefficients are available [7, 8]. The results are shown in Figs. 4, 5, 6 and 7 for the PMS, FAC, BLM and $\sqrt{ys}$ scale, respectively. We fixed $\Lambda = 0.10$ GeV for PMS, FAC and BLM and $\Lambda = 0.11$ GeV in the case of the $\sqrt{ys}$ scale. For α_s we use the second order formula as in our earlier paper [7]. Our results are compared with OPAL data at the Z-resonance [6]. These data were corrected for final acceptance and resolution of the detector but not for hadronization effects. The hadronization effects, however, are very small, at least for the cluster algorithm E_0, used by the OPAL group [1, 6, 15]. From this comparison we observe that the experimental data for f_2 and f_3 are very well fitted for all y values above 0.05. The 4-jet rate is also correctly described although in this range there are only three measured data points. To get agreement with f_2(or f_3) we had to adjust $\Lambda = 0.1$ GeV (for the $\sqrt{ys}$ scale we have increased $\Lambda = 0.11$ GeV to obtain a better fit). Using our coefficients C_{ij} [7, 8] the OPAL

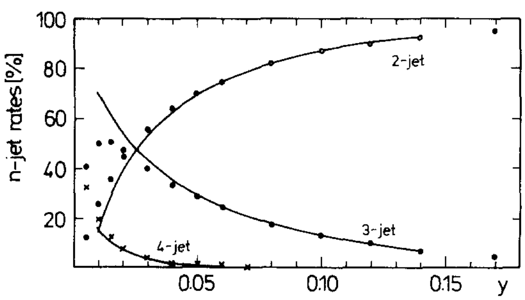

Fig. 4. n-jet production rates in % of total hadronic cross section at $\sqrt{s} = 91$ GeV with PMS scale and $\Lambda = 0.1$ GeV compared to OPAL data [6]

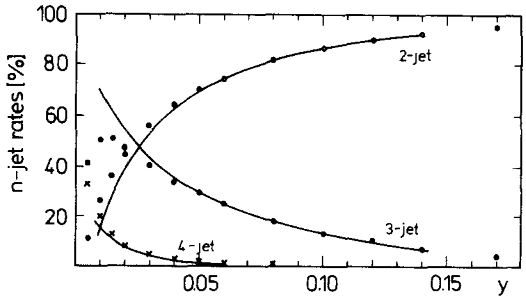

Fig. 5. Same as Fig. 4 with FAC scale

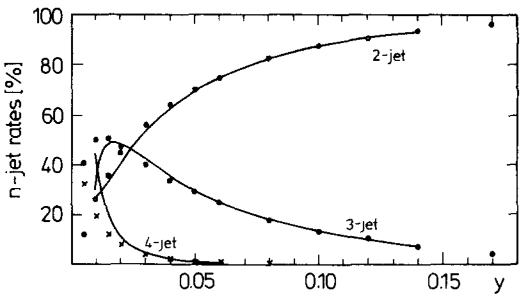

Fig. 6. Same as Fig. 4 with BLM scale

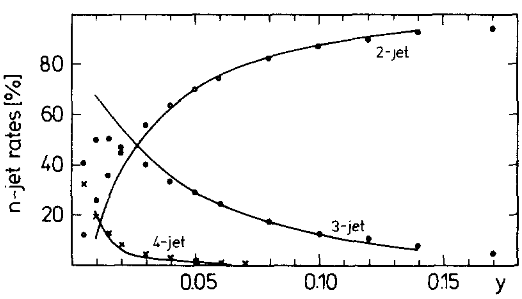

Fig. 7. Same as Fig. 4 with $\sqrt{y}$ scale

collaboration has compared their data to the theory with the scale $\mu = \sqrt{s}$ and obtained $\Lambda = 0.20$ GeV. (They quote $\Lambda = 0.23$ GeV. This corresponds to $\Lambda = 0.20$ GeV if for α_s the second order formula as in [7] is used and not the linearized formula as in [16]). Their fit was equally good as the fit with our small scales following from PMS, FAC, BLM or $\sqrt{ys}$ shown in Figs. 4, 5, 6 and 7. For $y < 0.05$ the theoretical predictions deviate from the experimental data appreciably for PMS, FAC, $\sqrt{ys}$ and $\sqrt{s}$. For example f_3 at $y = 0.015$ is experimentally (50.5 ± 0.9)%, whereas we get: 60.5%, 60.5%, 61.5% and 65.3% for the PMS, FAC, $\sqrt{ys}$ and $\sqrt{s}$ scale respectively. In the case of the BLM scale the theoretical curves follow approximately the data points also for $y < 0.05$. For example $f_3 = 48.5\%$ at $y = 0.015$. This means that, if one wants to describe f_2 and f_3 correctly also for $y < 0.05$, the scale μ/Q must remain small also in this region and should not increase when y is lowered as it is the case in particular for the PMS and FAC scales obtained from

f_3 (see Fig. 3). Therefore the authors of [6] could fit their data, and also those at lower energies [17], with a universal, i.e., s and y independent, optimal scale $\mu_{opt}/\sqrt{s}=0.041$. Of course, their Λ comes out similarly to ours, i.e. $\Lambda=0.1$ GeV (they quote $\Lambda=0.11$ GeV on the basis of the linearized formula for α_s [16]). The quality of their fit with this universal scale is slightly better than with the BLM scale.

What can we conclude from this analysis? It is clear that if we restrict the comparison with the experimental data of f_2 and f_3 to the large y range ($y \geqq 0.05$) the scale is uncertain and by changing the scale we obtain different values of Λ. Thus varying the scale ratio $\mu/\sqrt{s}$ from 0.05 to 1.0 results in Λ values between 0.1 and 0.2 GeV (with some additional small error originating from the experimental error of f_2 and f_3). Only if we require also to fit the 4-jet rate to the two experimental points in this y range we must choose the small scale as we get from the PMS, FAC, BLM and $\sqrt{y}$ procedure and then obtain $\Lambda=0.1$ GeV. This procedure to fit f_4 and so to determine Λ is questionable since tree level results always show a strong monotonous dependence on the scale μ. Only when the scale is known from other sources are tree level results useful for an estimate of Λ. Of course, the PMS, FAC and BLM procedures can be applied only to observables for which higher order corrections are known, i.e., to f_2 and f_3 and not to f_4. Although these rules are ad-hoc and have no strong justification we think they are plausible and this gives us, together with the $\sqrt{ys}$ choice, a range of scales μ which should be preferred. Since all these procedures lead to equal scales (except $\sqrt{ys}$) we are inclined to prefer these small scales over the scale $\mu=\sqrt{s}$ and thus get $\Lambda=0.1$ GeV.

We can also look how well the theoretical predictions describe the data in the small y region ($y \leqq 0.05$). Here all predictions with small scale and $\Lambda \simeq 0.1$ GeV describe f_4 equally well similar to the larger y values. As we already mentioned, f_2 and f_3 are reproduced satisfactorily only with the BLM procedure in this small y region. In total we would like to conclude that this comparison suggests a preference for a small scale and thus $\Lambda \simeq 0.1$ GeV.

The experimental error of the OPAL jet rates is very small, smaller than 1%. This leads to an error of Λ not larger than approximately 20 MeV. This is shown for the BLM prediction in Fig. 8a, b, where we have compared the experimental points of f_2 and f_3 in the range $0.05 \leqq y \leqq 0.14$ with the BLM curve using $\Lambda=0.08$ GeV (upper curve in Fig. 8a and lower curve in Fig. 8b) and $\Lambda=0.12$ GeV. We see that the measured data lie well inside the band of the two curves. So we determine $\Lambda=(0.10 \pm 0.02)$ GeV if we restrict ourselves to a scale fixing procedure favouring a small scale like the BLM appraoch. For the two other approaches PMS and FAC the result is quite similar. With this value of Λ we find on the Z pole $\alpha_s=0.107 \pm 0.003$. The error on α_s includes only the experimental error and not the error from possible (although small) hadronization effects and the unknown systematic error from not knowing the scale μ. In addition there is the error from the recombination dependence. In this analysis we used the coefficients C_{ij} based on the higher order corrections called KL' in [7]. It was found that the 3-jet cross section depends on the way the variables describing 3 jets were formed out of the momenta of the 4 partons. In [7] we used two possible ways for defining these 3-jet variables which lead to the two results KL and KL' for the 3-jet cross sections. Further details are presented in [7, 15].

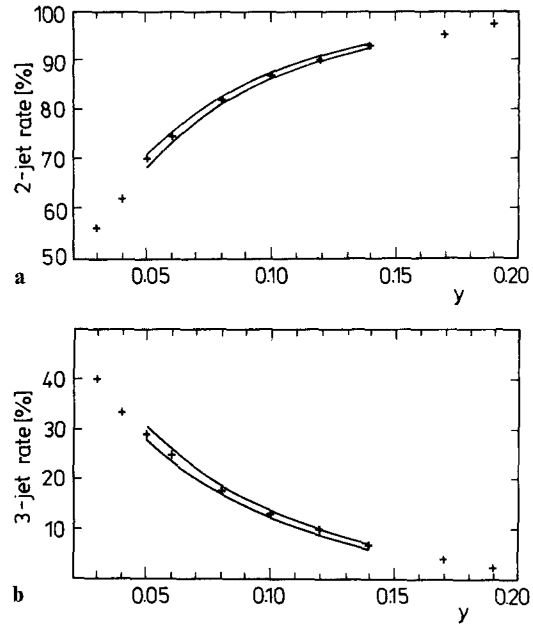

Fig. 8. **a** 2-jet production rate in % of total hadronic cross section as a function of y with BLM scale for $\Lambda=0.08$ GeV (upper curve) and $\Lambda=0.12$ GeV (lower curve) compared to OPAL data [6]. **b** 3-jet production rate in % of total hadronic cross section as a function of y with BLM scale for $\Lambda=0.08$ GeV (lower curve) and $\Lambda=0.12$ GeV (upper curve) compared to OPAL data [6]

In conclusion we state that the scale defining procedures PMS, FAC, BLM and $\sqrt{y}$ applied to 2- and 3-jet fractions lead to good agreement with recent OPAL data. The BLM approach fits the data even at small mass cut $\sqrt{y}$ and is preferred over the other three approaches. All four procedures lead to a smaller Λ parameter, $\Lambda \simeq 0.1$ GeV, than the usual perturbation prediction with scale $\sqrt{s}$. The 4-jet multiplicity comes out much larger with the small scales originating from these procedures than with the scale $\sqrt{s}$ in agreement with the data. This is needed if one wants to develop realistic Monte Carlo models including fragmentation on perturbative QCD matrix elements up to order α_s^2 [17, 15].

It is true that one can explain the data with $\mu=\sqrt{s}$ and $\Lambda=0.2$ GeV. However, the choice $\mu=\sqrt{s}$ contradicts any physical intuition, because the jets are formed at invariant mass $\sqrt{ys} \ll \sqrt{s}$. The best thing to do is to use the small scales which come out of the optimization procedures and to determine the average value of Λ which one gets from that. This is what leads to $\Lambda=(0.10 \pm 0.02)$ GeV which should be considered as the main result of this paper.

The discussion in this paper was devoted to theoretical uncertainties in the determination of Λ originating from the unknown scale. We did not undertake a chi-squared analysis including the errors of the highly correlated experimental data points. In Fig. 4–8 the comparison with the data of ref. 6 results from eyeball fits. Therefore our error estimate of Λ obtained from Fig. 8a, b is not really quantitative. For an improved analysis one must use the differential distribution $D_2(y)$ of the jet rates [6, 18], in which the statistical errors in bins of $D_2(y)$ are independent from each other since each event enters the distribution only once. Such fits of $D_2(y)$ to our theoretical calculations have been performed [6] with the result that for a particular chosen scale the resulting Λ values are the same as from the fits to the integral presentation of the jet production rates.

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