Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta)|\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

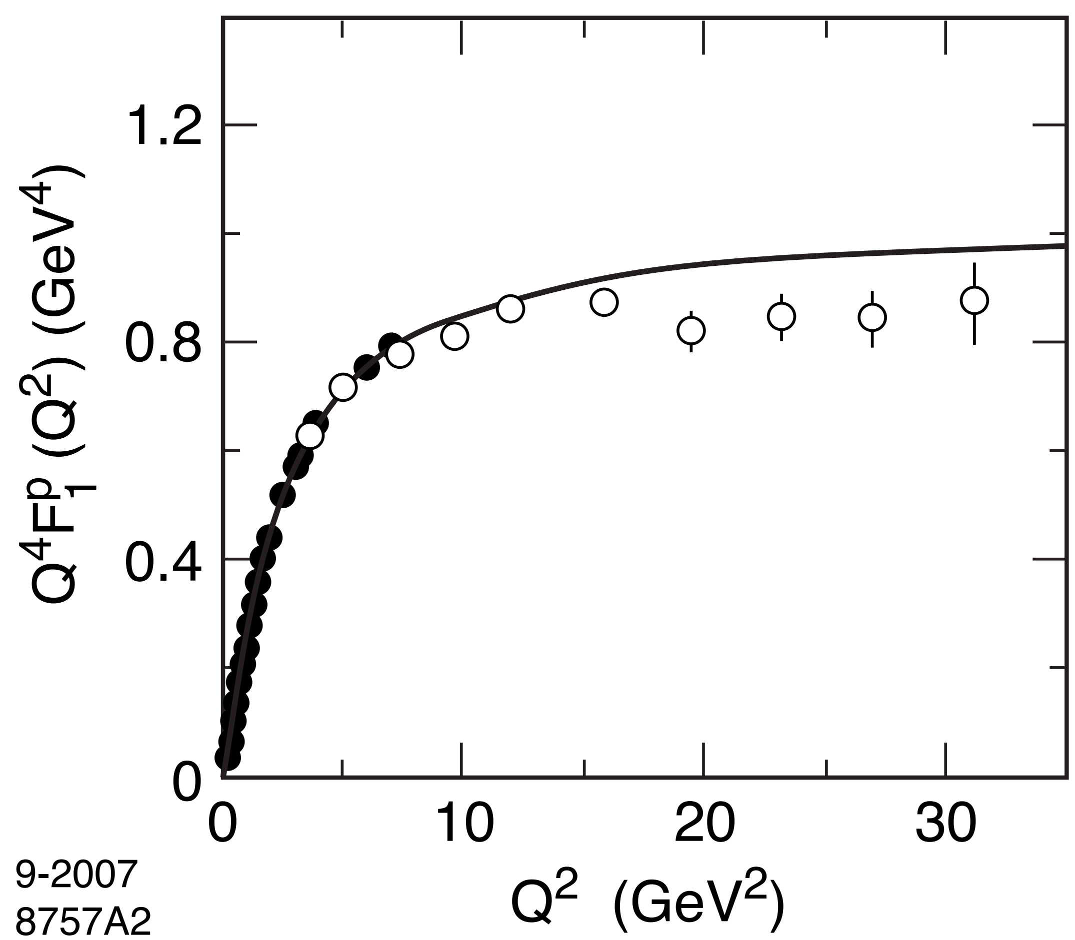
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3}\int d\zeta\, J(Q,\zeta)\left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

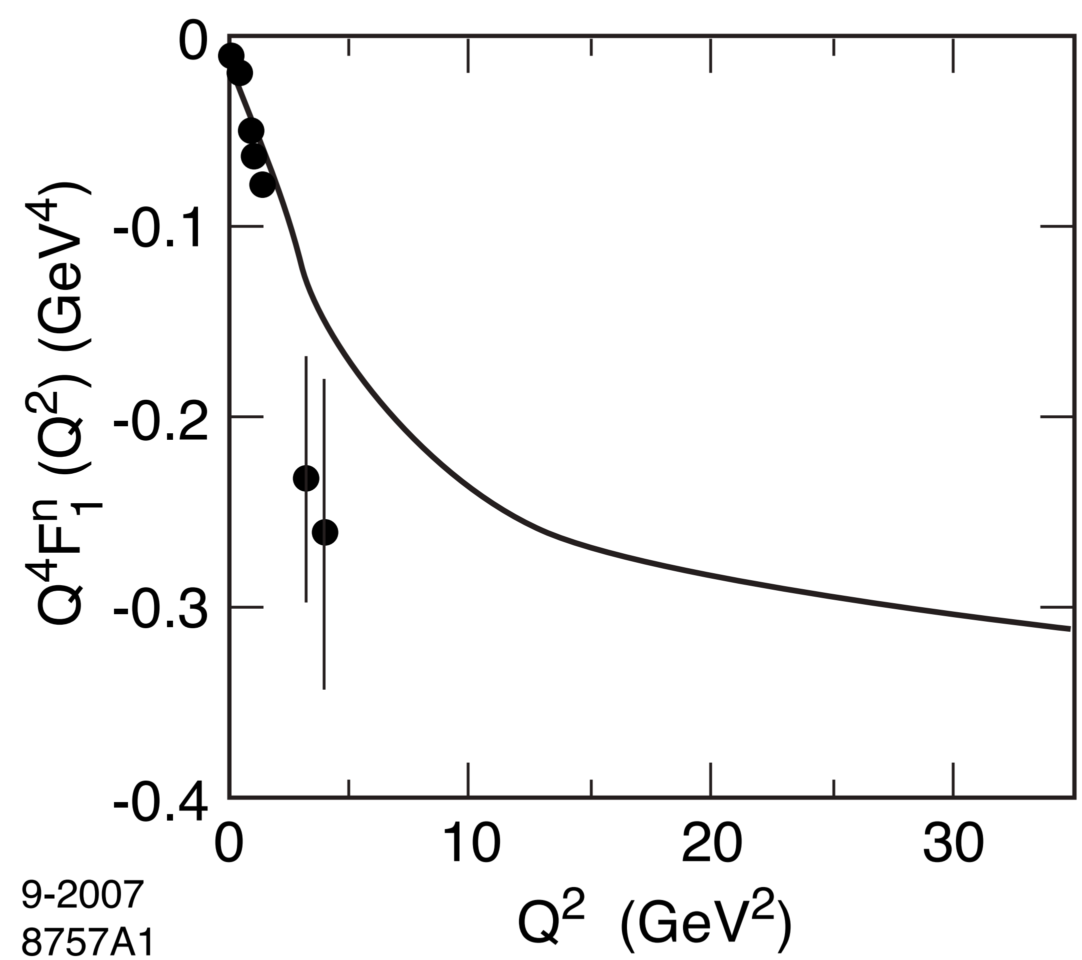
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \mathrm{constant}$ $\boxed{\mathrm{Proton}\ \ \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

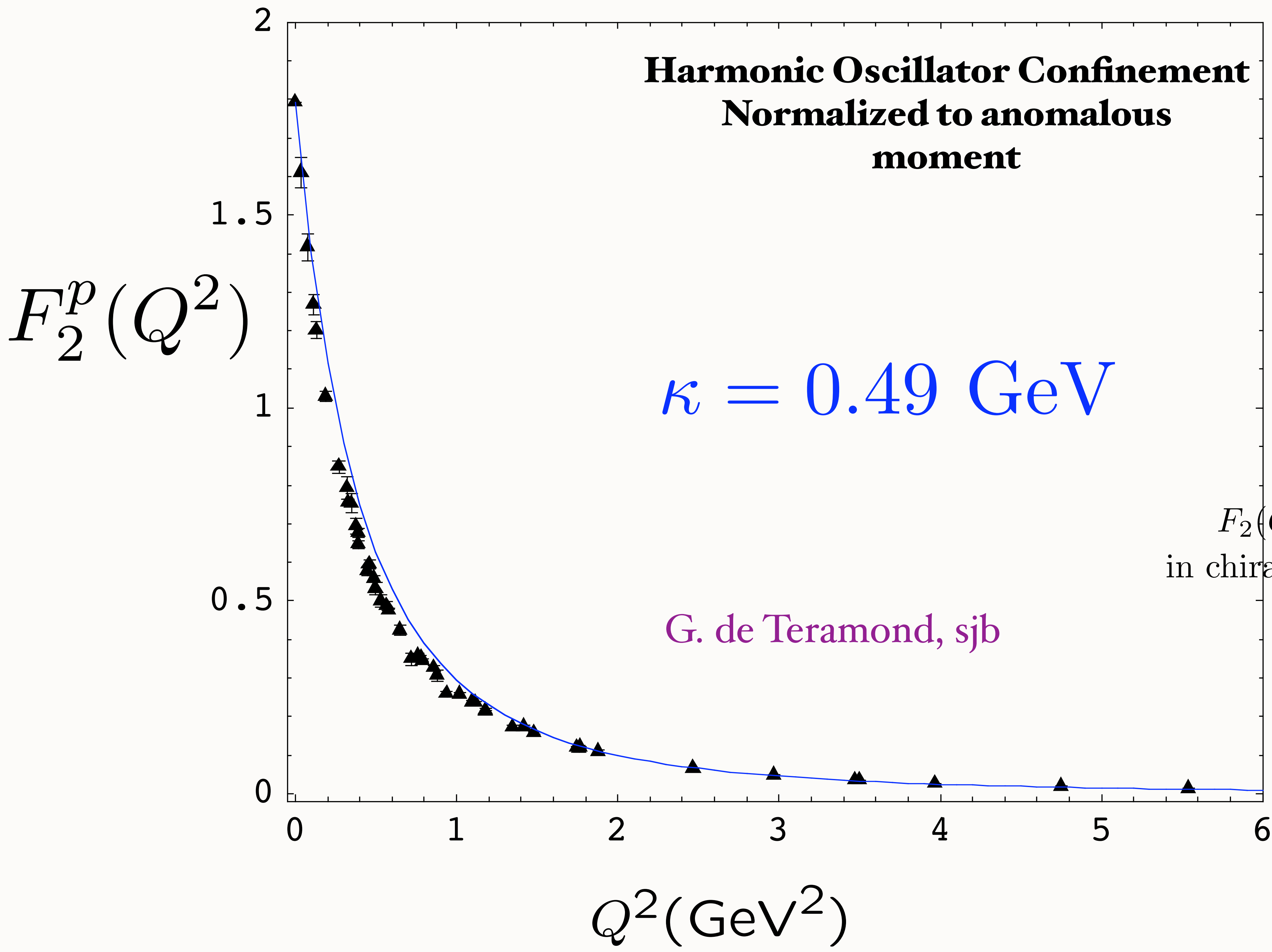


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

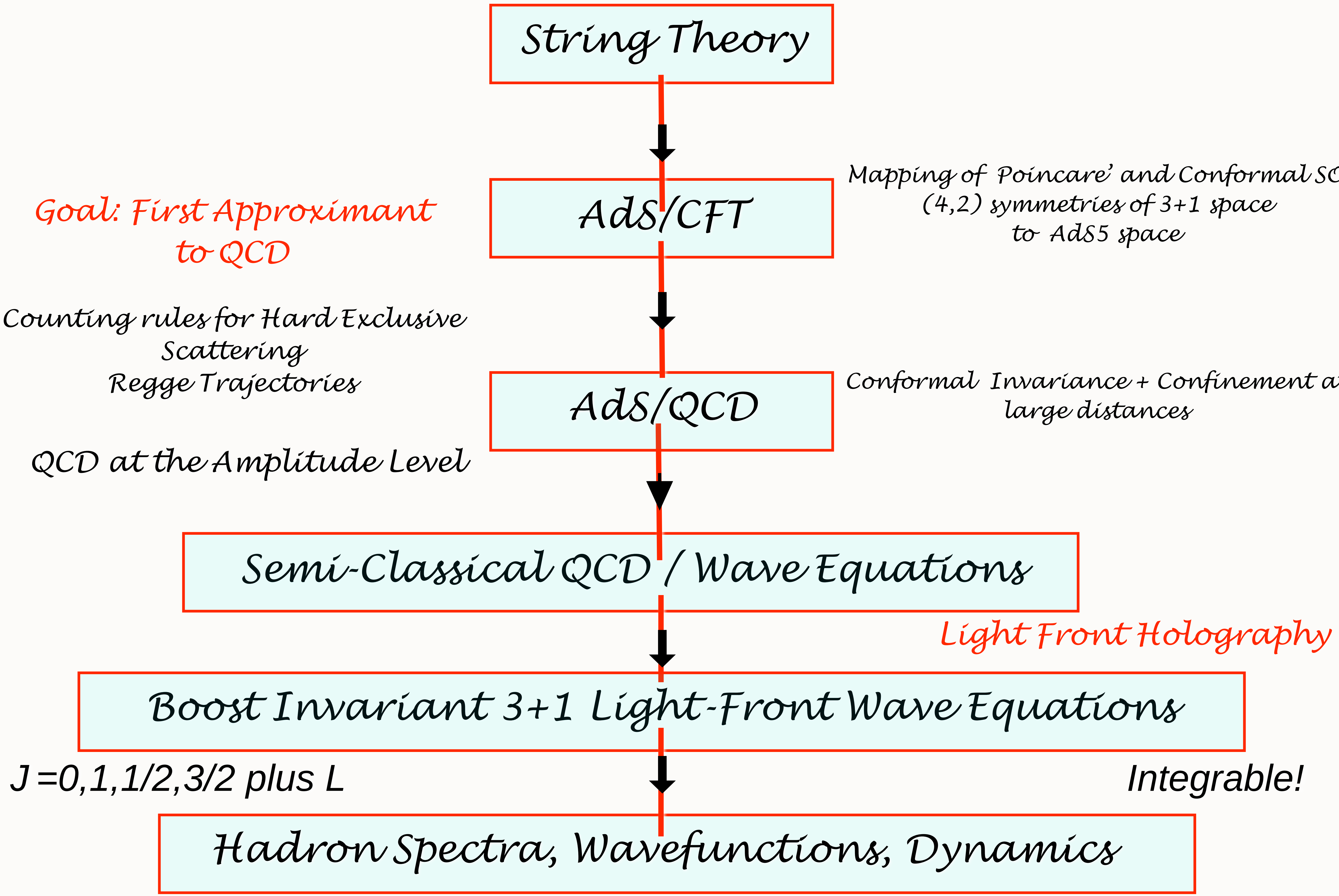
Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



AdS/QCD No chiral divergence!

$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$
in chiral perturbation theory


String Theory
AdS/CFT
Goal: First Approximant to QCD
Mapping of Poincare' and Conformal SO (4,2) symmetries of 3+1 space to AdS5 space
Counting rules for Hard Exclusive Scattering
Regge Trajectories
AdS/QCD
Conformal Invariance + Confinement at large distances
QCD at the Amplitude Level
Semi-Classical QCD / Wave Equations
Light Front Holography
Boost Invariant 3+1 Light-Front Wave Equations
J =0,1,1/2,3/2 plus L
Integrable!
Hadron Spectra, Wavefunctions, Dynamics

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g}\; e^{\phi(z)} \frac{1}{g_5^2} G^2 \qquad \text{where } \sqrt{g} = (\tfrac{R}{z})^5 \text{ and } \phi(z) = +\kappa^2 z^2$$

Define an effective coupling $g_5(z)$

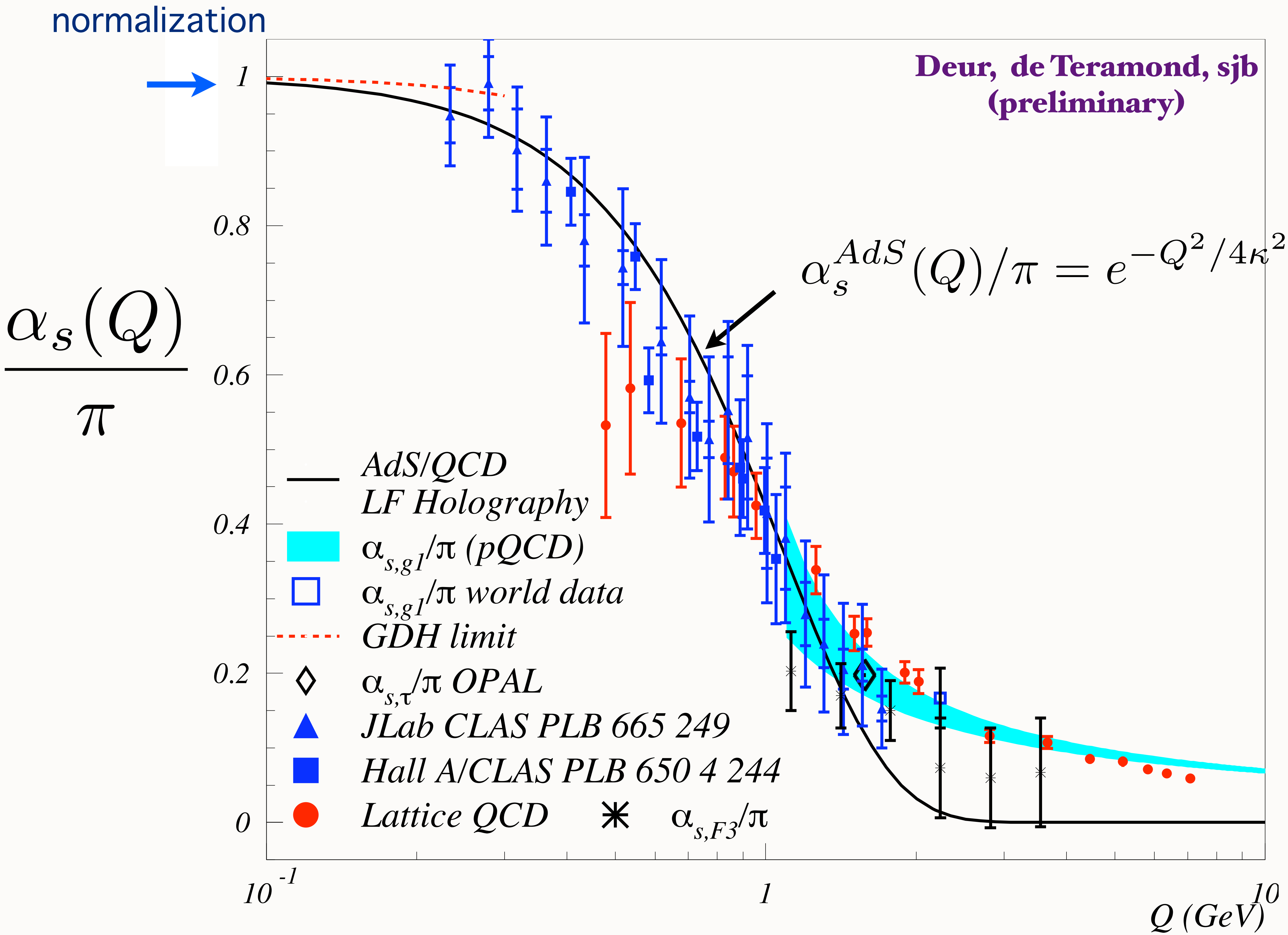
$$S = -\frac{1}{4}\int d^4x dz \sqrt{g} \frac{1}{g_5^2(z)} G^2$$

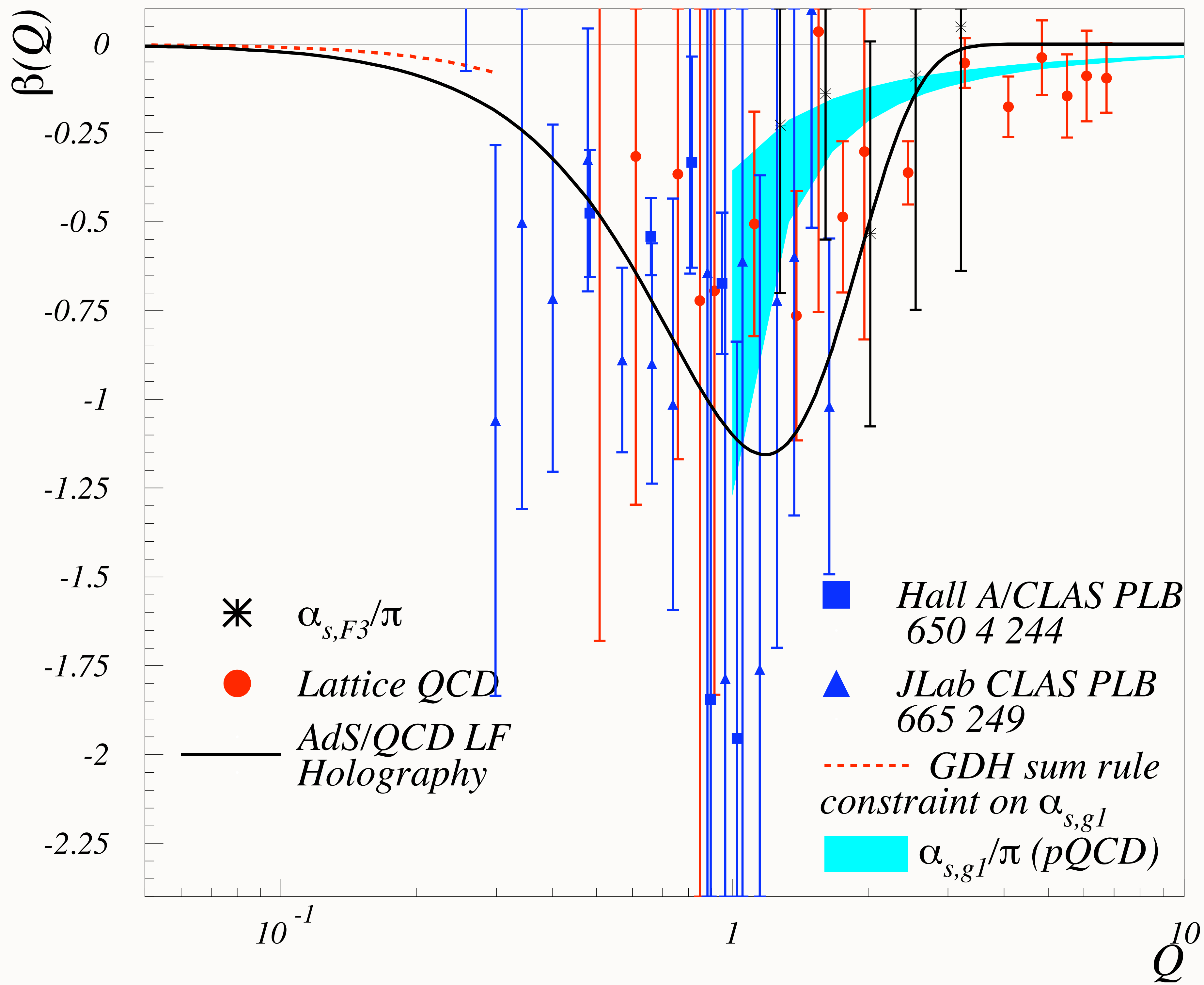
$$\text{Thus } \frac{1}{g_5^2(z)} = e^{\phi(z)} \frac{1}{g_5^2(0)} \text{ or } g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

Light-Front Holography: $z \to \zeta = b_\perp \sqrt{x(1-x)}$

$$\alpha_s(q^2) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \qquad \text{where } \alpha_s(z) = e^{-\kappa^2 z^2} \alpha_s(0)$$

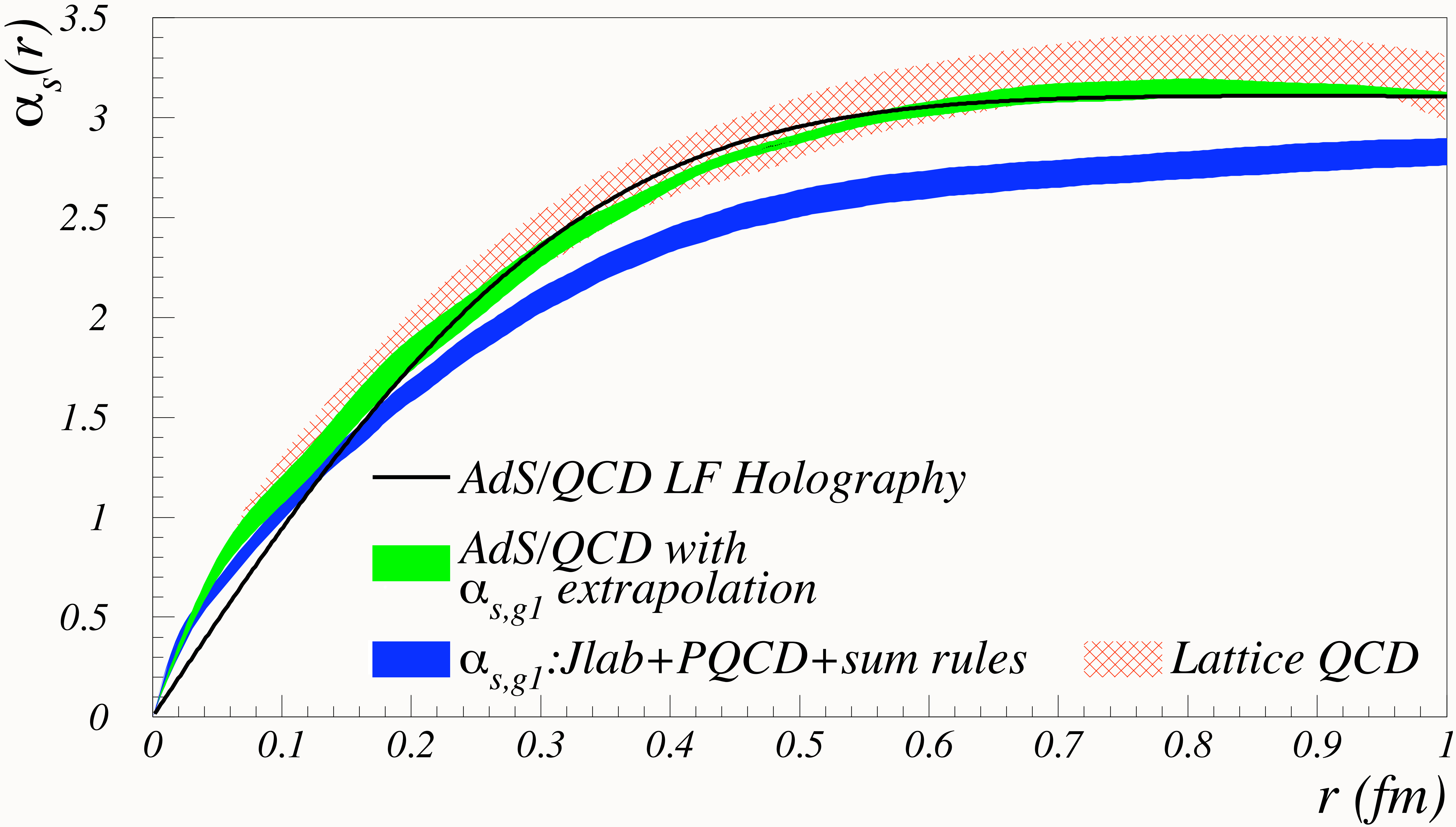
Running Coupling from AdS/QCD





Deur, de Teramond, sjb, (preliminary)

Deur, de Teramond, sjb,
(preliminary)



Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

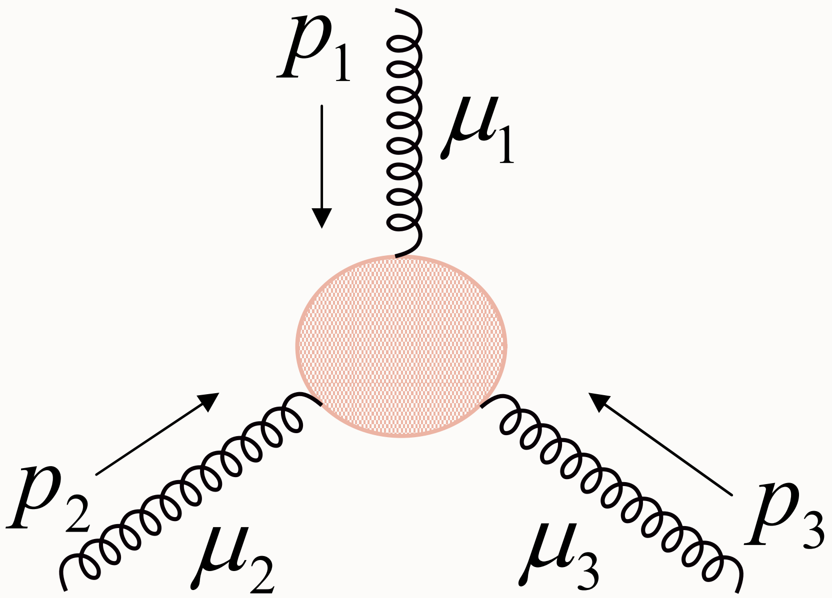
The Renormalization Scale Problem

$$\rho(Q^2) = C_0 + C_1\alpha_s(\mu_R) + C_2\alpha_s^2(\mu_R) + \cdots$$

$$\mu_R^2 = CQ^2$$

Is there a way to set the renormalization scale μ_R?

What happens if there are multiple physical scales ?



PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

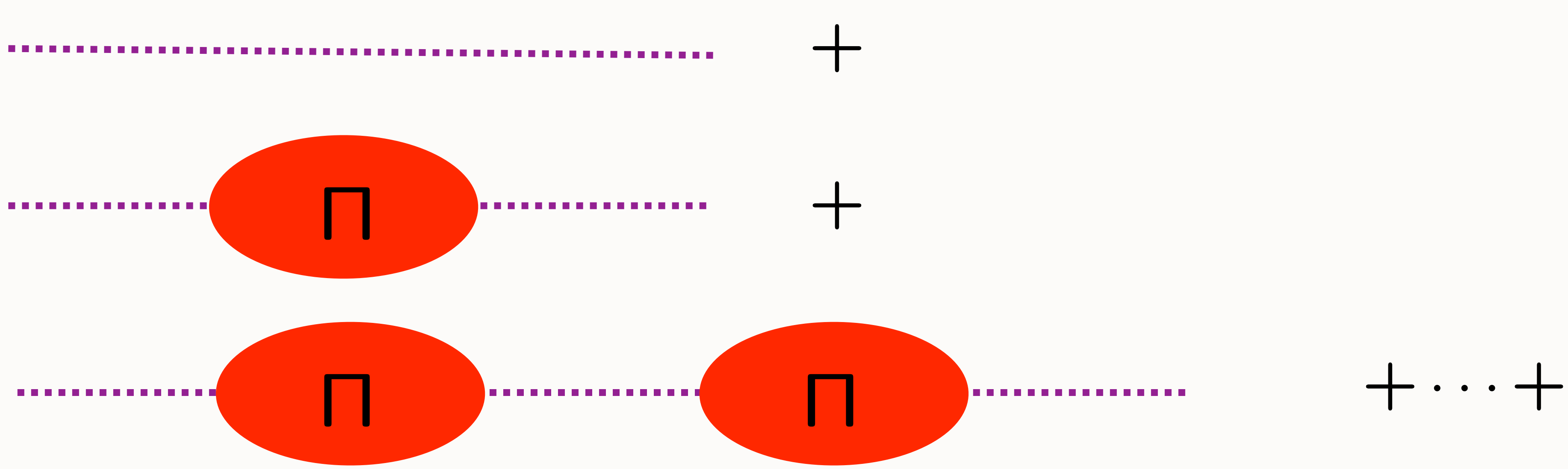
$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

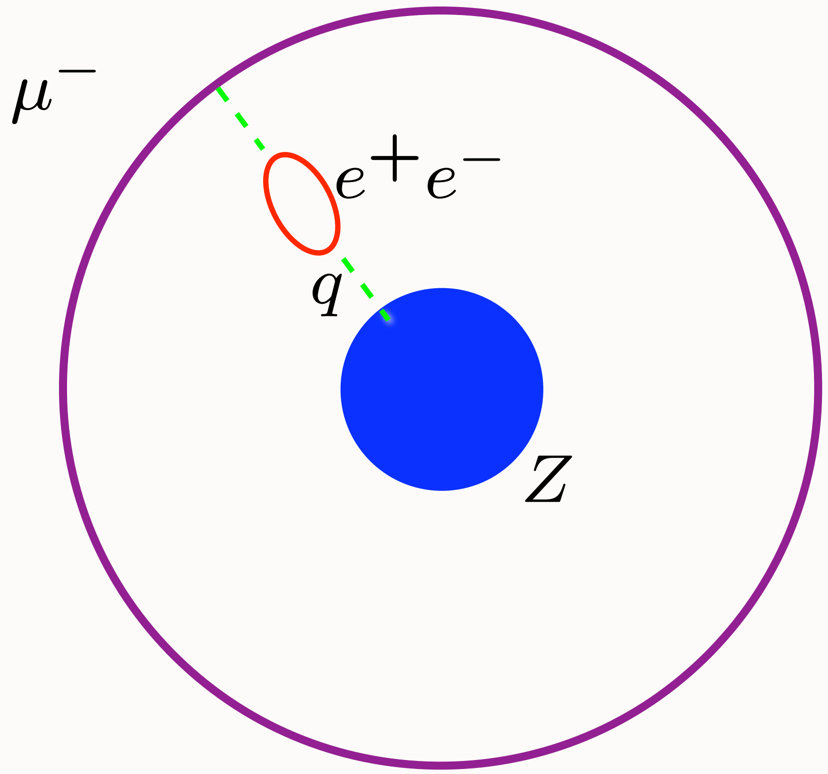
All-orders lepton loop corrections to dressed photon propagator



$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_0)}{1-\Pi(t_0)}$$

Initial scale t_0 is arbitrary -- Variation gives RGE Equations
Physical renormalization scale t not arbitrary

Another Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

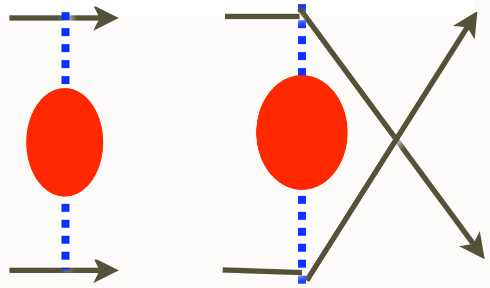
Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Electron-Electron Scattering in QED

- No renormalization scale ambiguity!

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!   Scheme independent.
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!
- Two separate physical scales.
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.

Conventional wisdom concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale

- Can ignore possibility of multiple physical scales

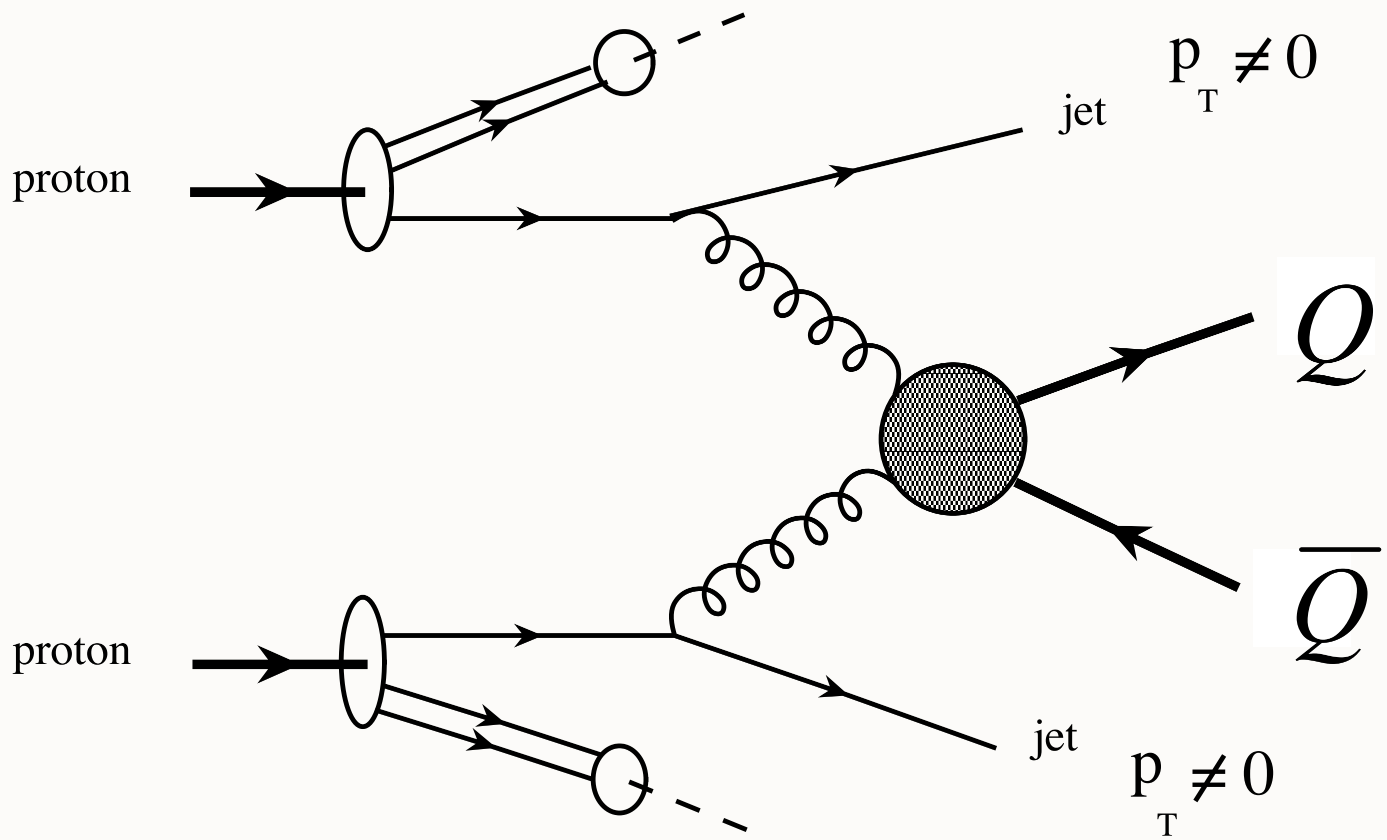
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$

- with an arbitrary range $Q/2 < \mu_R < 2Q$

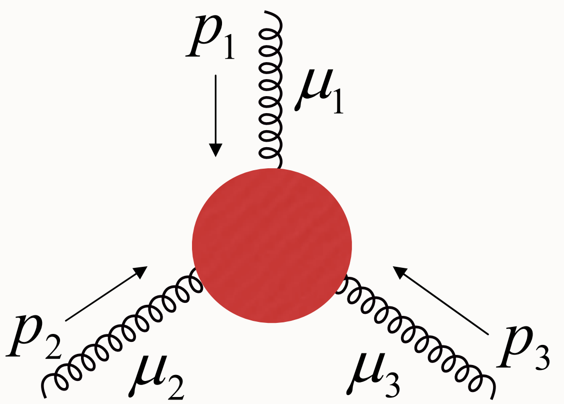
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

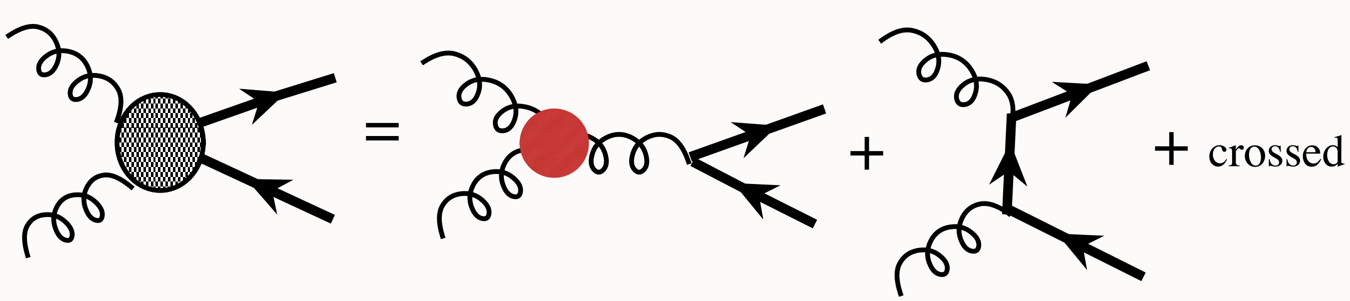
These assumptions are untrue in QED and thus they cannot be true for QCD!

Heavy Quark Hadroproduction



3-gluon coupling depends on 3 physical scales





Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q* sets the number of active flavors
- Only n_f dependence required to determine renormalization scale at NLO
- Result is scheme independent: Q* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- **Resulting series identical to conformal series!**
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

BLM Scale Setting

$\beta_0 = 11 - \frac{2}{3} n_f$

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2}\beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right] ,$$

where

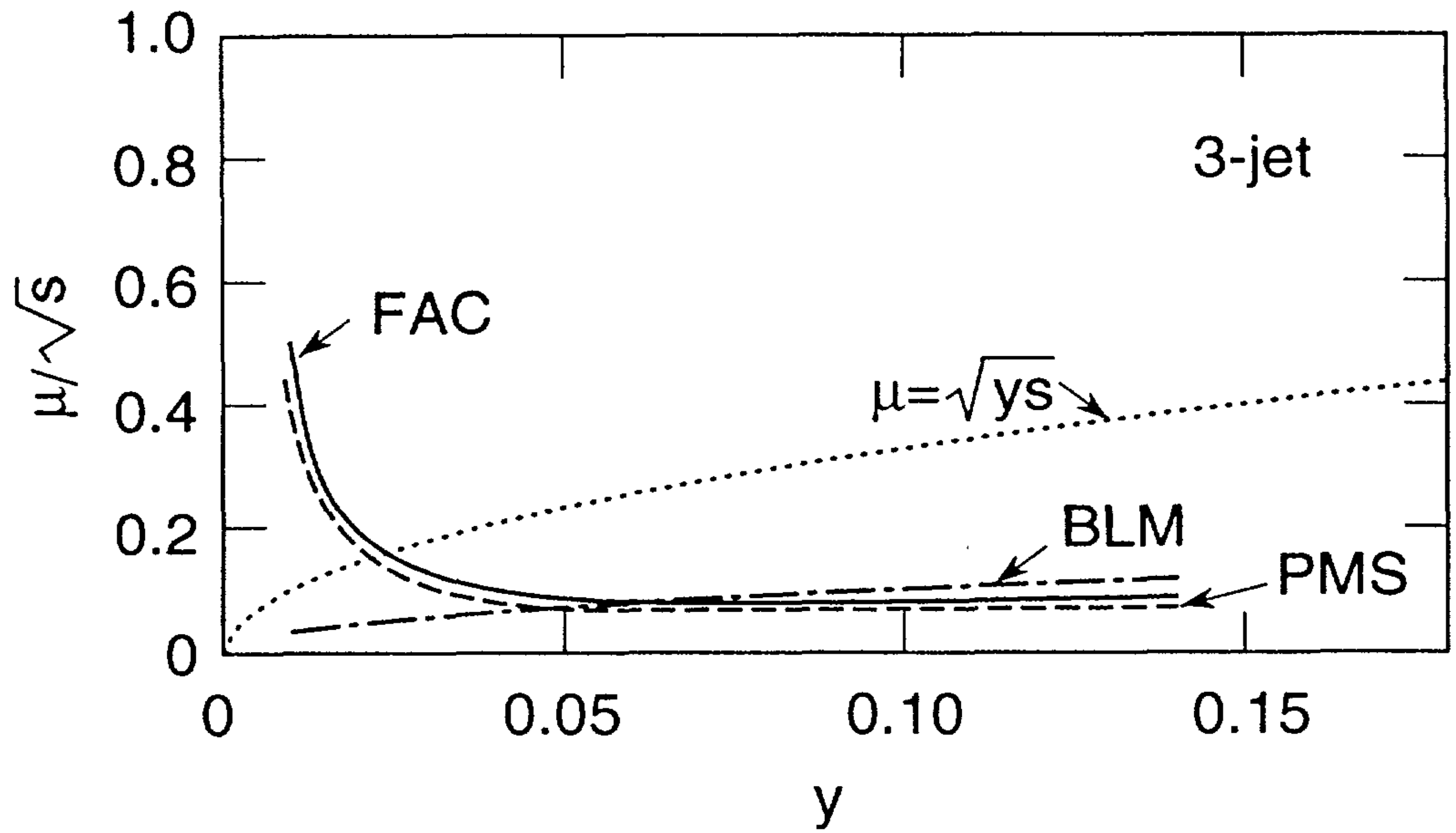
Conformal coefficient - independent of β

$$Q^* = Q \exp(3A_{VP}) ,$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B .$$

The term $33A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

Use skeleton expansion: Gardi, Grunberg, Rathsman, sjb

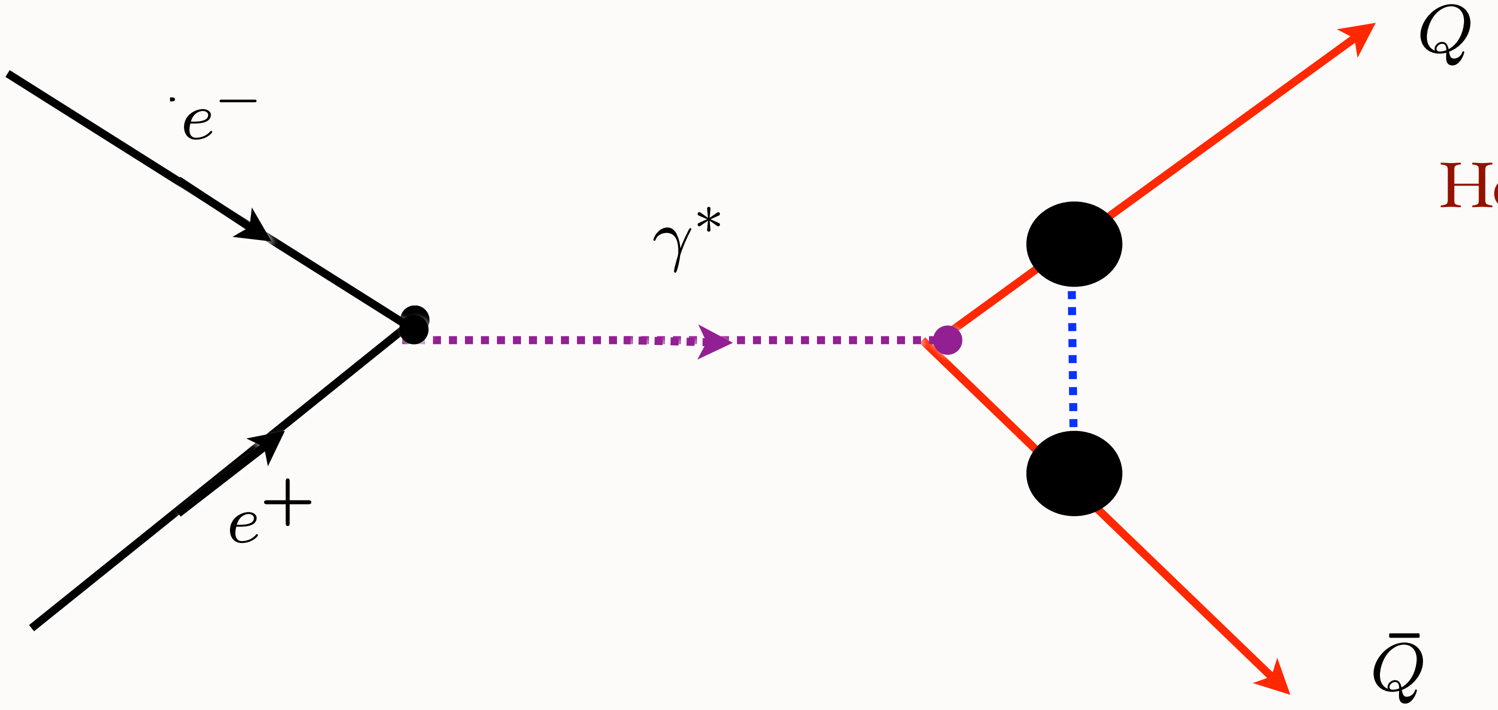


Kramer & Lampe

Three-Jet rate in electron-positron annihilation

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

Other Jet Observables: Rathsman



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = 1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta} - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}$$

$$\cong \left(1 - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}\right)\left(1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta}\right)$$

Example of Multiple BLM Scales

Angular distributions of massive quarks and leptons close to threshold.

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

Define QCD Coupling from Observable

Grunberg

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2\ [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu)\times[1+\frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations:
Relate observable to observable at commensurate scales

Effective Charges: analytic at quark mass thresholds, finite at small momenta

H.Lu, Rathsman, sjb

Pinch scheme: Cornwall, et al

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648} f^2\right\}.$$

Eliminate MSbar, Find Amazing Simplification

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1+\frac{\alpha_R(s^*)}{\pi}][1-\frac{\alpha_{g_1}(q^2)}{\pi}]=1$$

$$\sqrt{s^*}\simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

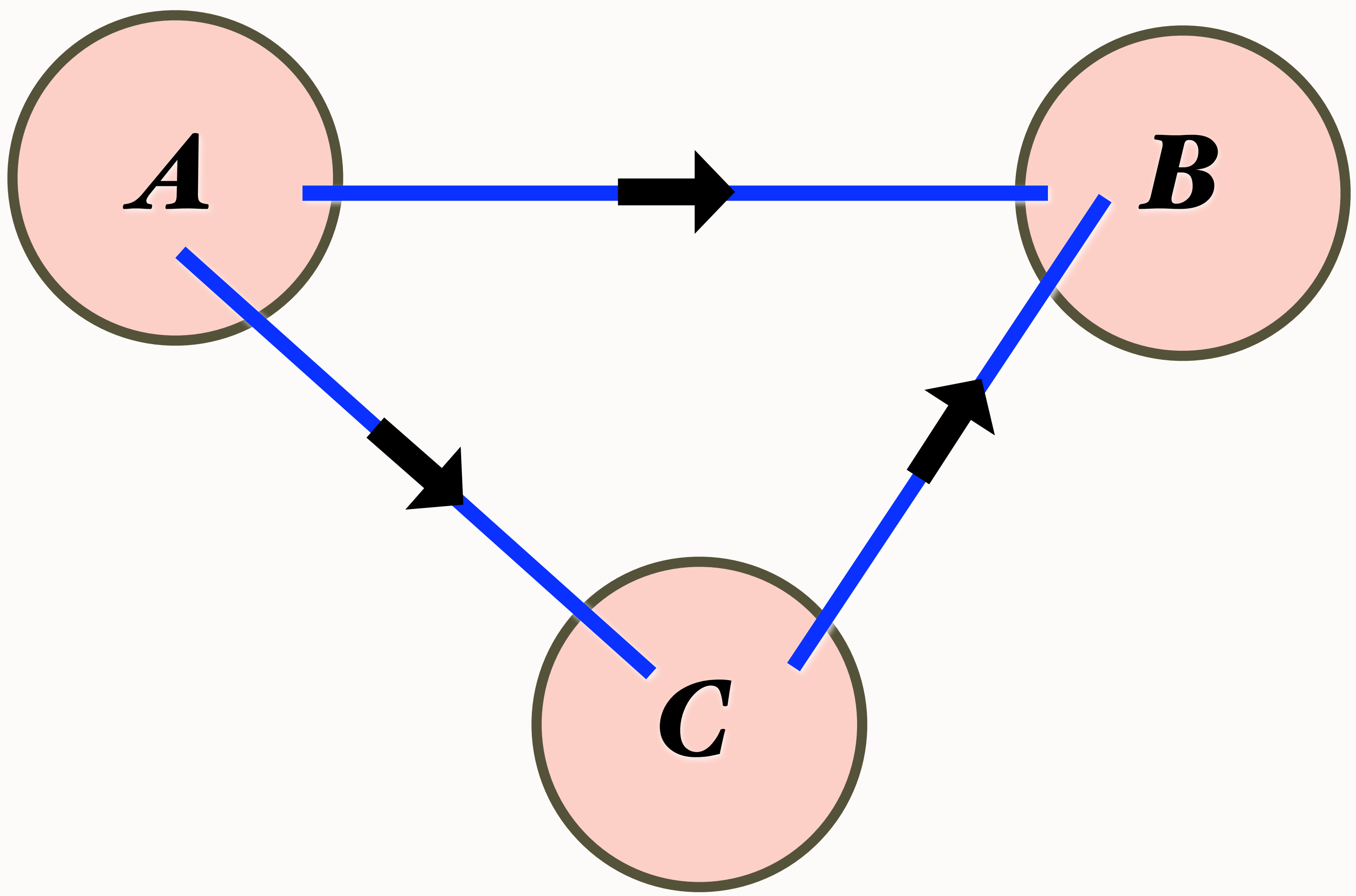
No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

Analytic matching at quark thresholds

No renormalization scale ambiguity!

Transitivity Property of Renormalization Group

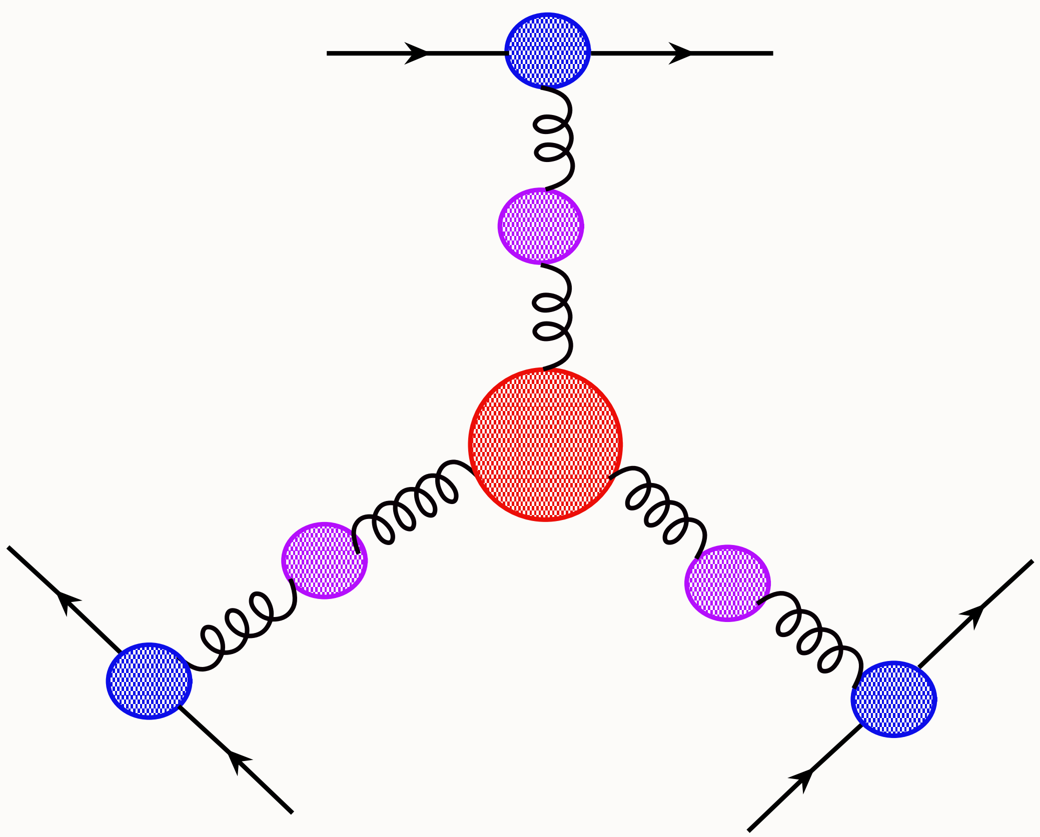


A ➡ ***C*** ***C*** ➡ ***B*** ***identical to*** ***A*** ➡ ***B***

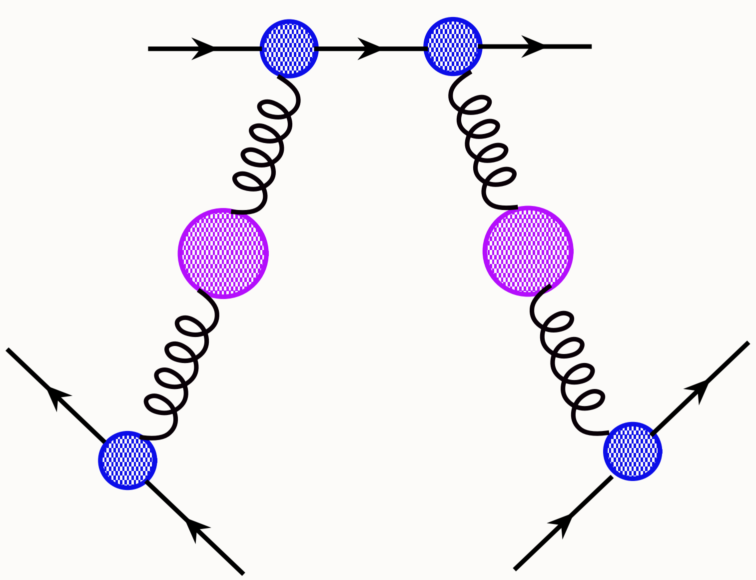
Relation of observables independent of intermediate scheme C

3 Gluon Vertex In Scattering Amplitudes

Pinch-Technique approach :
fully dress with gauge-invariant Green's functions



(A)

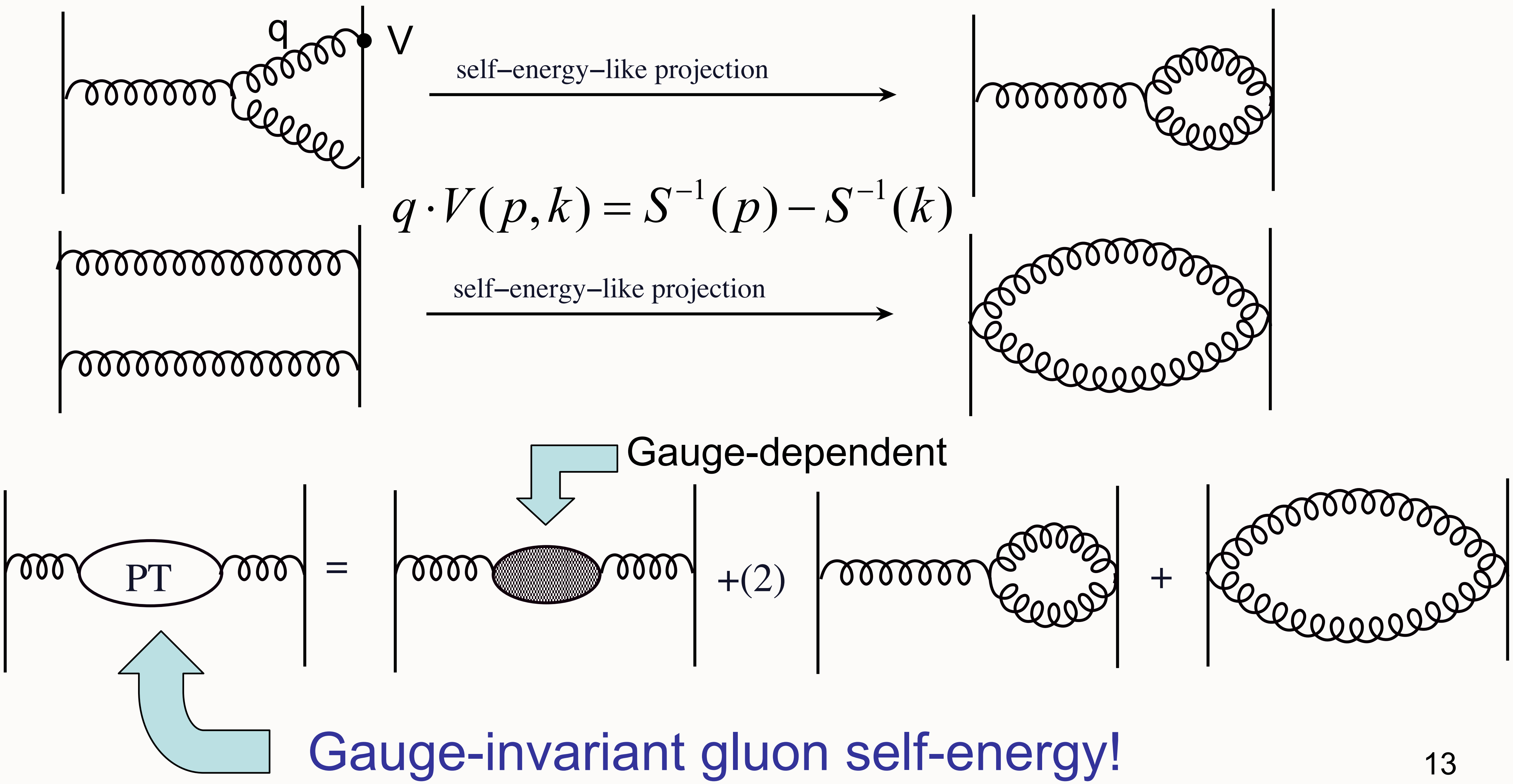


(B)

+ perms

The Pinch Technique

(Cornwall, Papavassiliou)



$$q \cdot V(p,k) = S^{-1}(p) - S^{-1}(k)$$

Gauge-invariant gluon self-energy!

13

natural generalization of QED charge

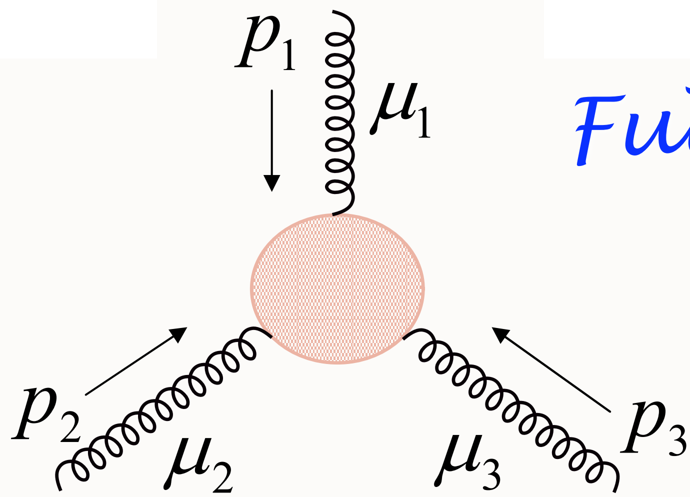
Pinch Scheme (PT)

- J. M. Cornwall, Phys. Rev. D 26, 345 (1982)
- Equivalent to Background Field Method in Feynman gauge
- Effective Lagrangian Scheme of Kennedy & Lynn
- Rearrange Feynman diagrams to satisfy Ward Identities
- Longitudinal momenta from triple-gluon coupling, etc. hit vertices which cancel ("pinch") propagators
- Two-point function: Uniqueness, analyticity, unitarity, optical theorem
- Defines analytic coupling with smooth threshold behavior

Binger, sjb

General Structure of the Three-Gluon Vertex

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



Full analytic calculation, general masses, spin Pinch Scheme

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3

with $p_1 + p_2 + p_3 = 0$

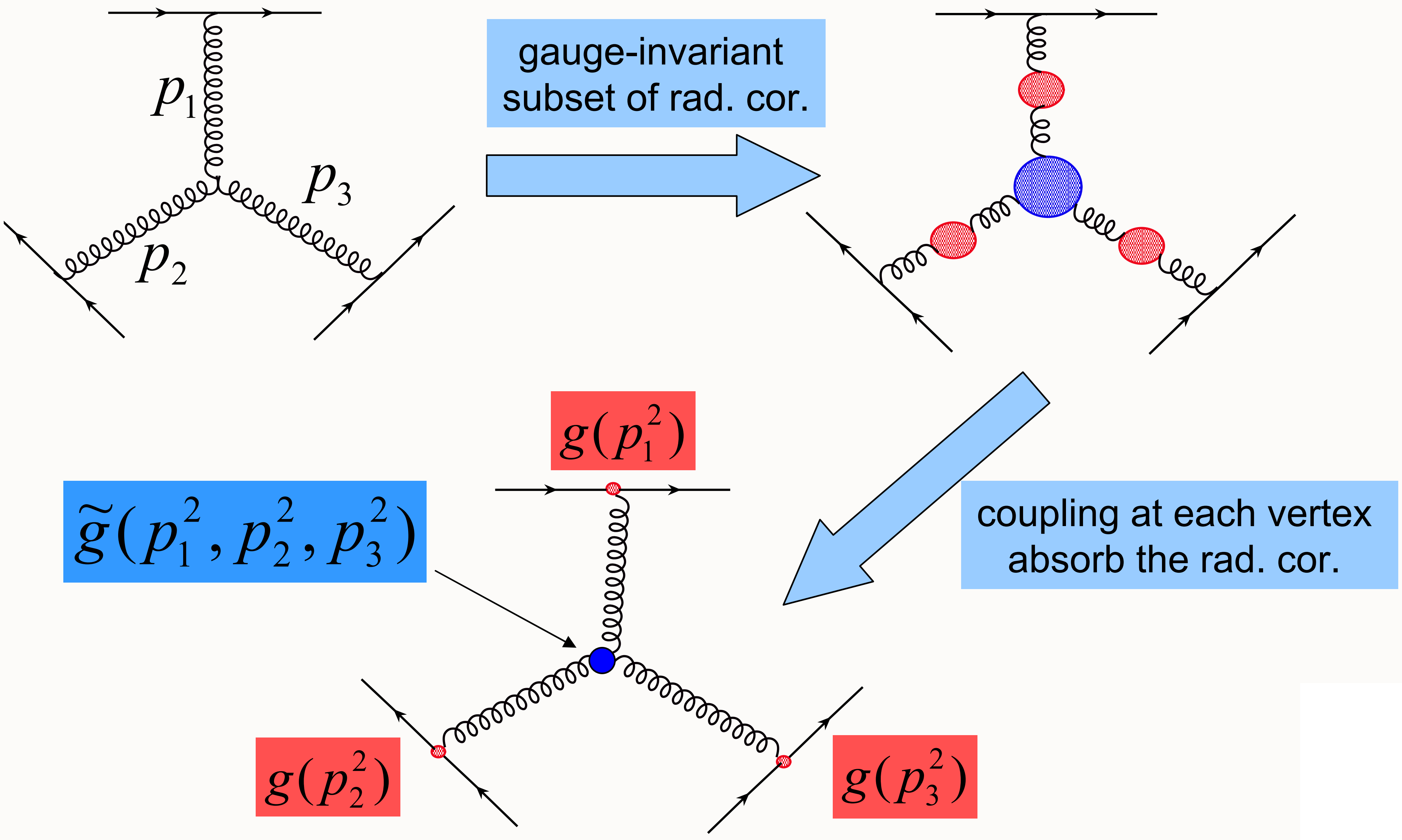
⇨ 14 basis tensors and form factors

PHYSICAL REVIEW D **74**, 054016 (2006)

Form factors of the gauge-invariant three-gluon vertex

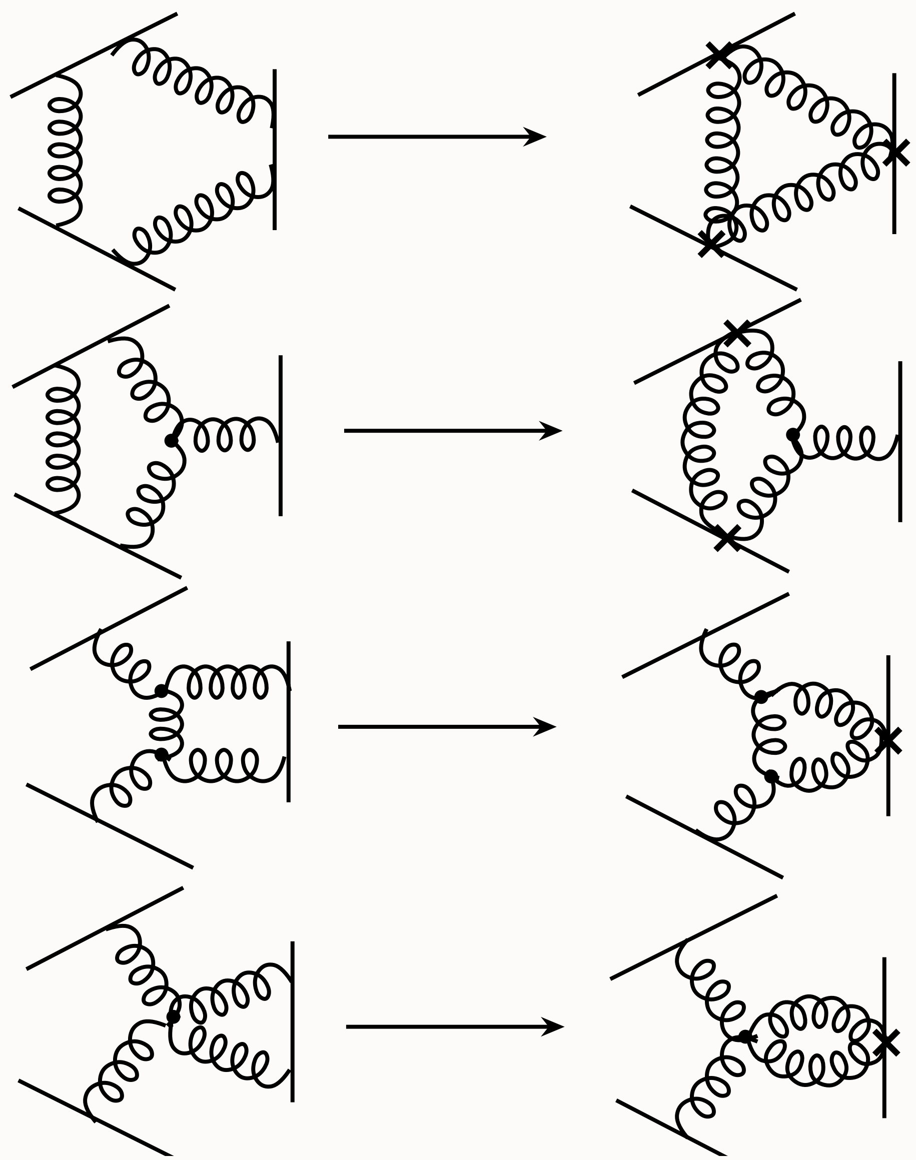
Michael Binger* and Stanley J. Brodsky†

Multi-scale Renormalization of the Three-Gluon Vertex

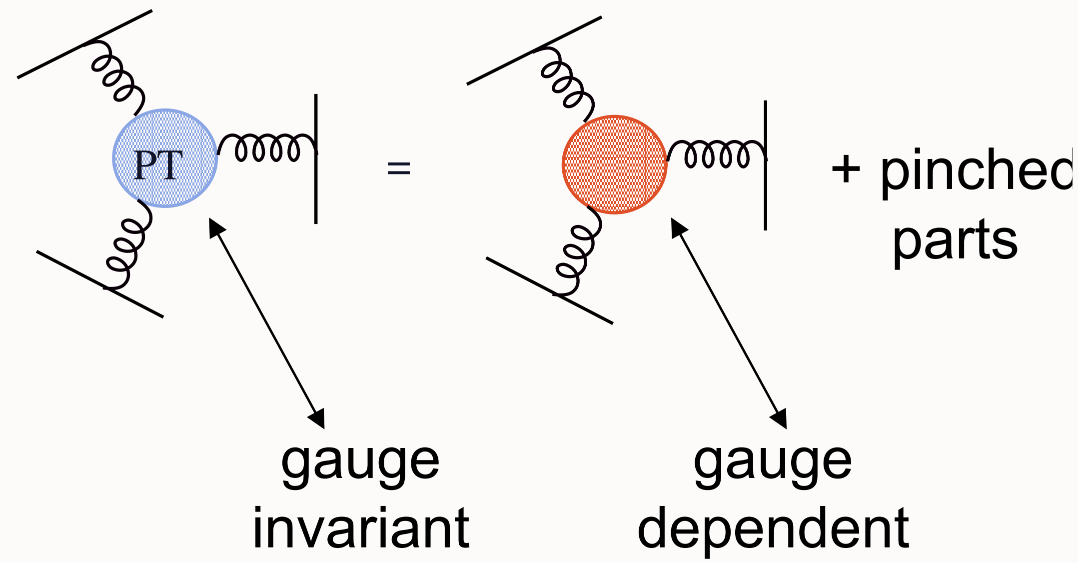


The Gauge Invariant Three Gluon Vertex

Cornwall and Papavassiliou performed the PT construction :



The “pinched” parts are added to the “regular” 3 gluon vertex



Form Factors : Supersymmetric Relations (Massless)

….but certain linear sums are simple :

$$\Sigma_{QG}(F) \equiv \frac{d-2}{2} F_Q + F_G \longrightarrow 0$$

for 7 of the 13 FF's (in physical basis) $\pm$

⇨ Simple N=1 SUSY contribution in d=4

$$F_G + 4F_Q + (10-d)F_S = 0$$

For all FF's !!

⇨ N=4 SUSY in d=4 gives 0

These are off-shell generalizations of relations found in SUSY scattering amplitudes by Z. Bern, L.J. Dixon, D.C. Dunbar, and D.A. Kosower (NPB 425,435)

Vanishing sum contribution of the N=4 supermultiplet in d=4 dimensions

Form Factors : Supersymmetric Relations (Massive)

Equal masses for massive gauge bosons (MG), quarks (MQ), and scalars (MS)

$$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$$

1 d.o.f. “eaten” by MG

Massive gauge boson (MG) inside of loop might be the X and Y gauge bosons of SU(5), for example

External gluons remain unbroken and massless

$$\Sigma_{MQG}(F) \equiv \frac{d-1}{2} F_{MQ} + F_{MG}$$ is simple

3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi}\beta_0\left(L(a,b,c) - \frac{1}{\varepsilon} + \cdots\right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

L(a,b,c) = 3-scale “log-like” function

L(a,a,a) = log(a)

3 Scale Effective Scale

$$L(a,b,c) \equiv \log\left(Q^2_{eff}(a,b,c)\right) + i\,\mathrm{Im}\,L(a,b,c)$$

Governs strength of the three-gluon vertex

$$\frac{1}{\widetilde{\alpha}(a,b,c)} = \frac{1}{\widetilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\widetilde{\alpha}(a,b,c)}$$

Generalization of BLM Scale to 3-Gluon Vertex

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54 |a|$$

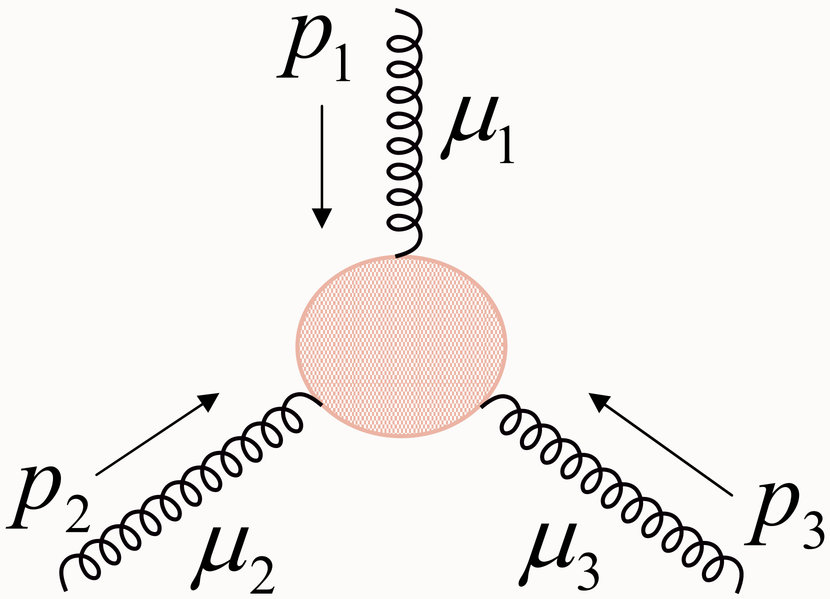
$$Q^2_{eff}(a,a,c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,-a,c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c|$$

Surprising dependence on Invariants

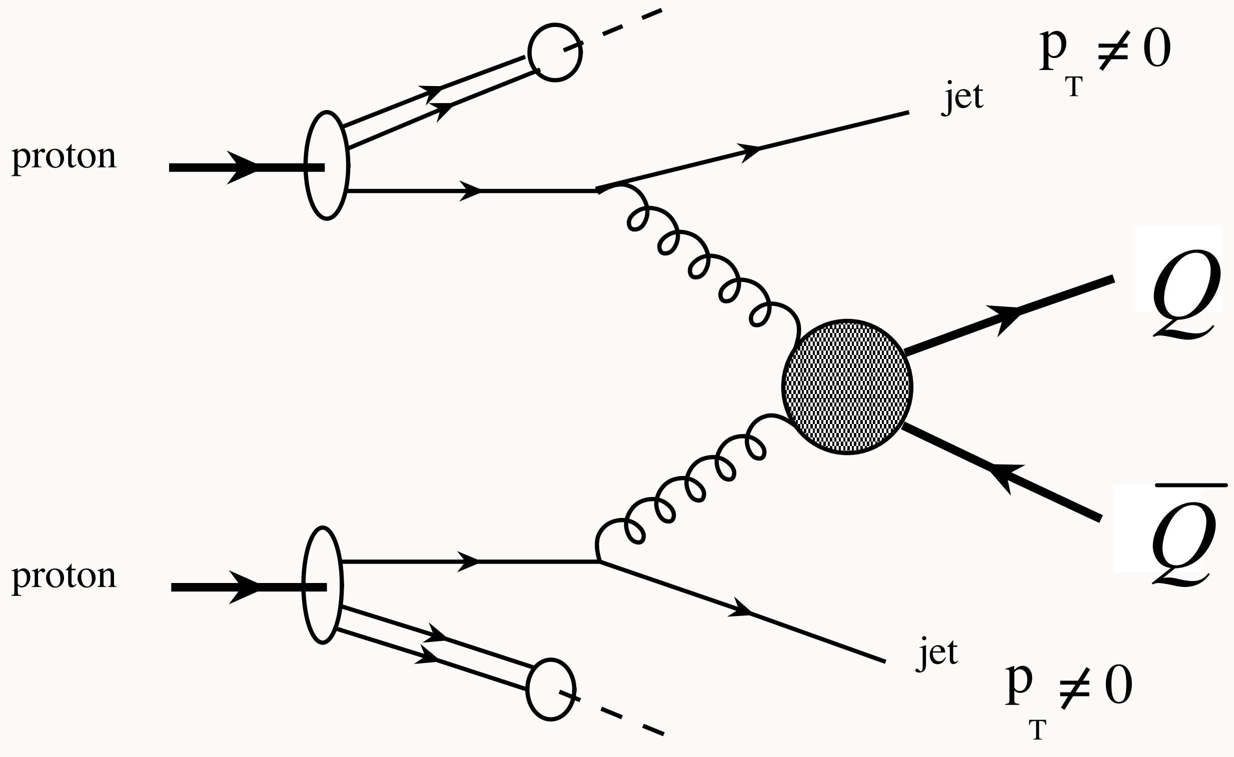
$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$

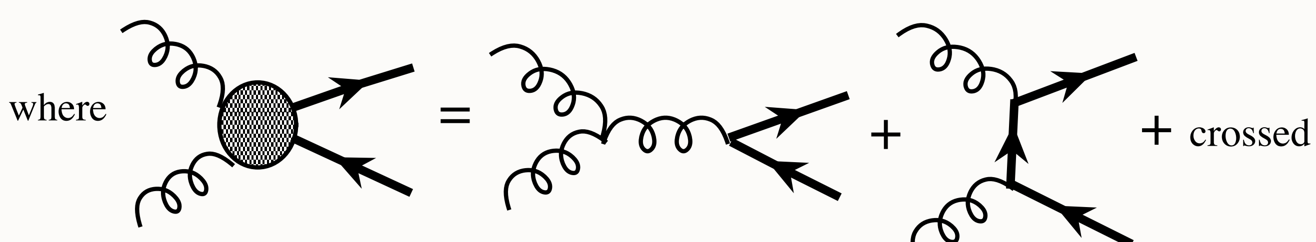


H. J. Lu

$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$

Heavy Quark Hadro-production



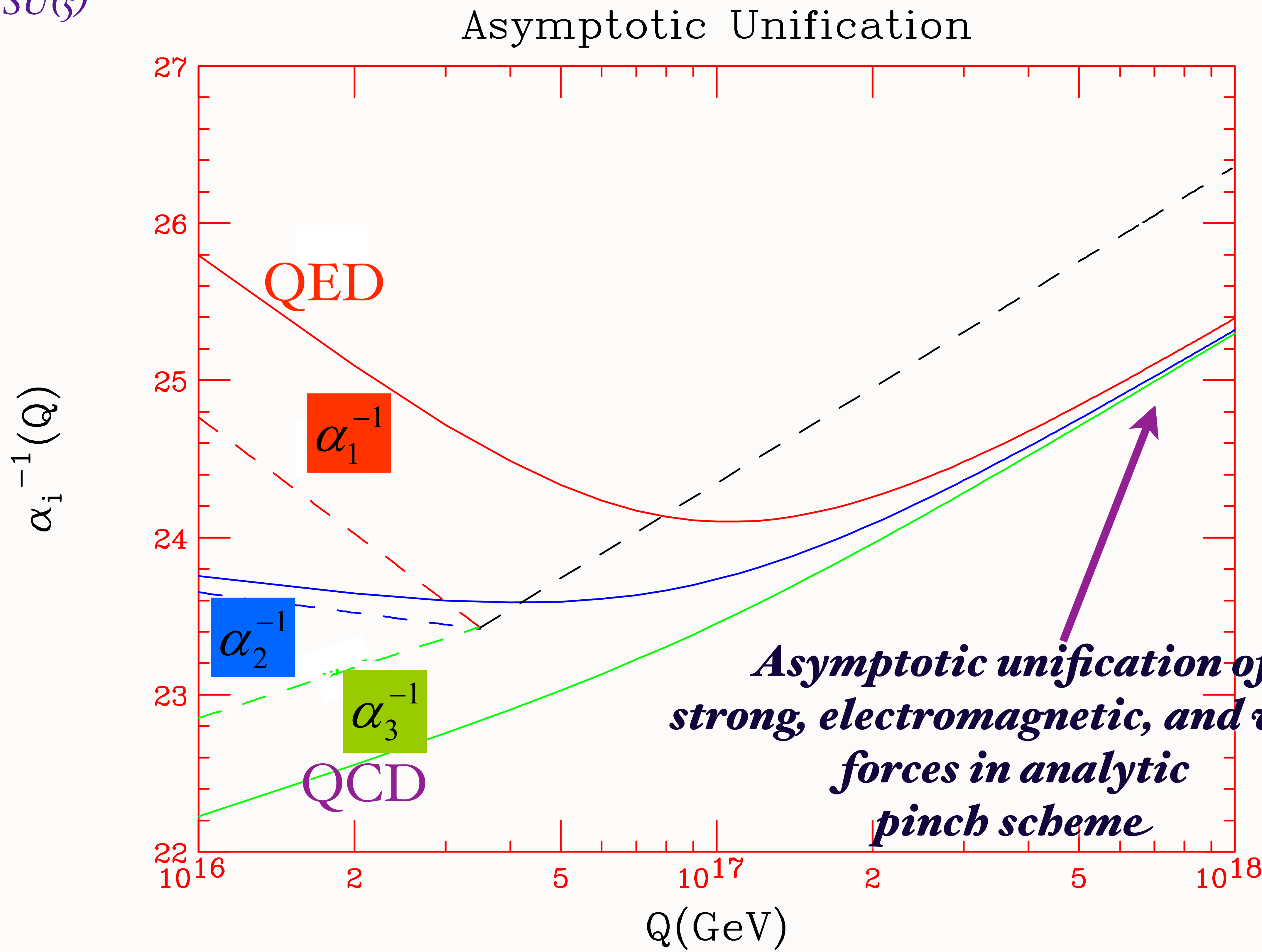


- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale → much larger cross section than $\overline{MS}$ with scale $\mu_R = M_{Q\overline{Q}}$ or M_Q
- Future : repeat analysis using the full mass-dependent results and include all form factors

Expect that this approach accounts for most of the one-loop corrections

Unification in Physical Schemes

- Smooth analytic threshold behavior with automatic decoupling
- More directly reflects the unification of the forces
- Higher "unification" scale than usual



Asymptotic unification of strong, electromagnetic, and weak forces in analytic pinch scheme

Conformal Template

- BLM scale-setting: Retain conformal series; nonzero β-terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.

- ***Commensurate Scale Relations*** based on conformal template

- Pinch Scheme -- provides analytic, gauge invariant, 3-g form factors

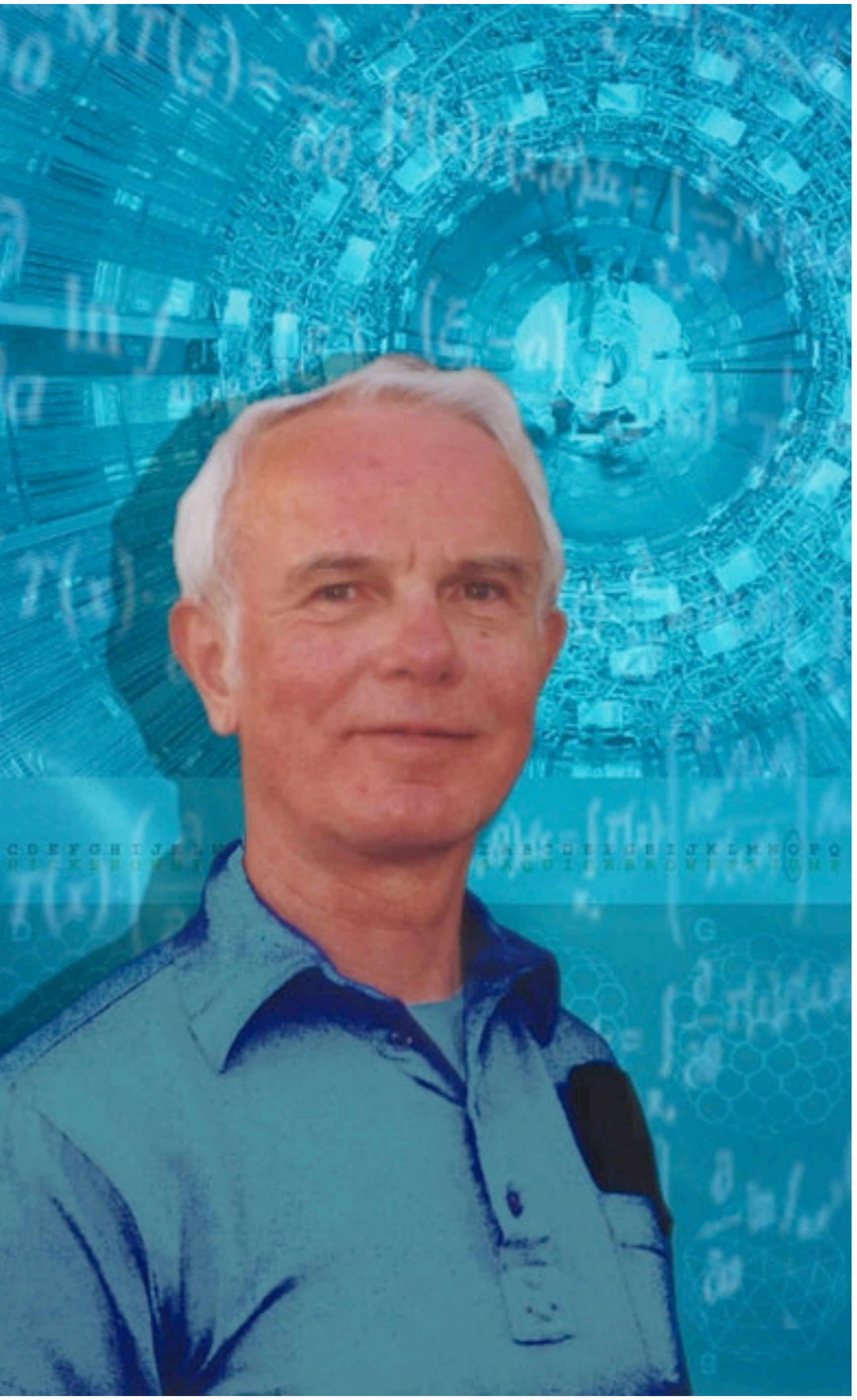
- Analytic scheme for coupling unification

- IR Fixed point -- conformal symmetry motivation for AdS/CFT

- Light-Front Schrödinger Equation: analytic first approximation to QCD

- Dilaton-modified AdS_5: Predict Hadron Spectrum, Form Factors, α_s, β

- Light-Front Wave Functions from Holography



Congratulations, Mike!

For pioneering so many Important Directions in QCD

Quantum Field Theory and Beyond: Celebration of Mike Cornwall's 75th Birthday