



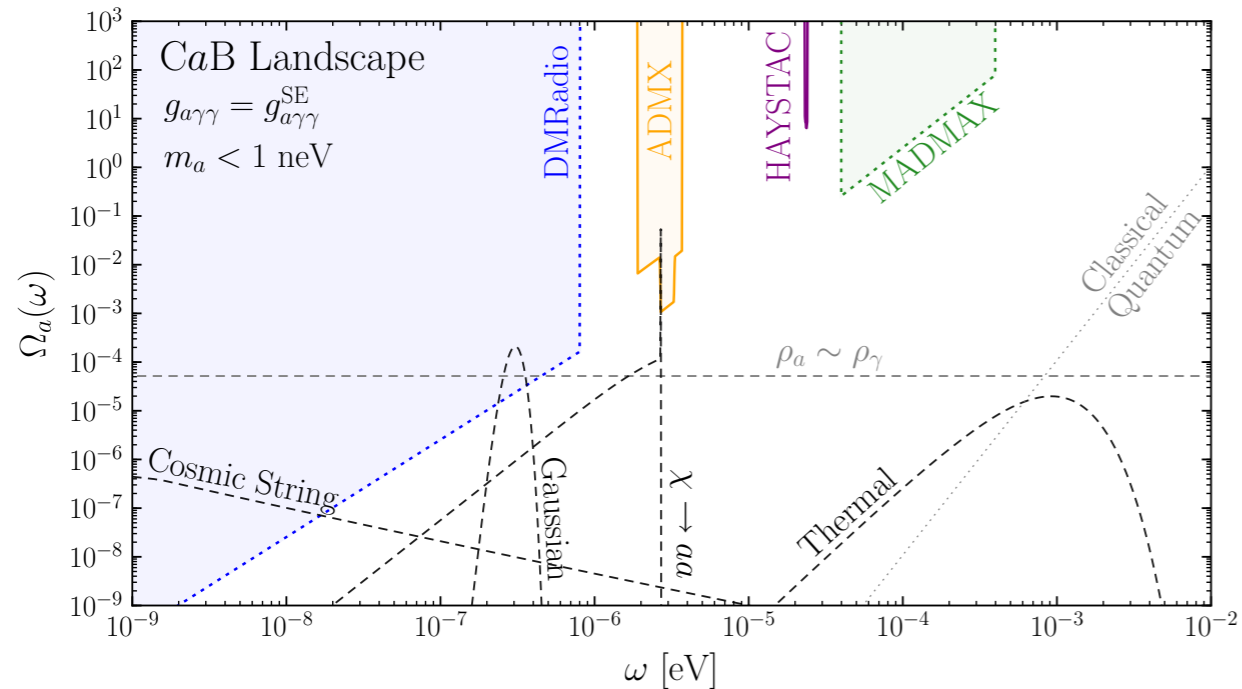
Echos of the Early Universe in Axion Haloscopes



The Cosmic Axion Background

PRD 2021

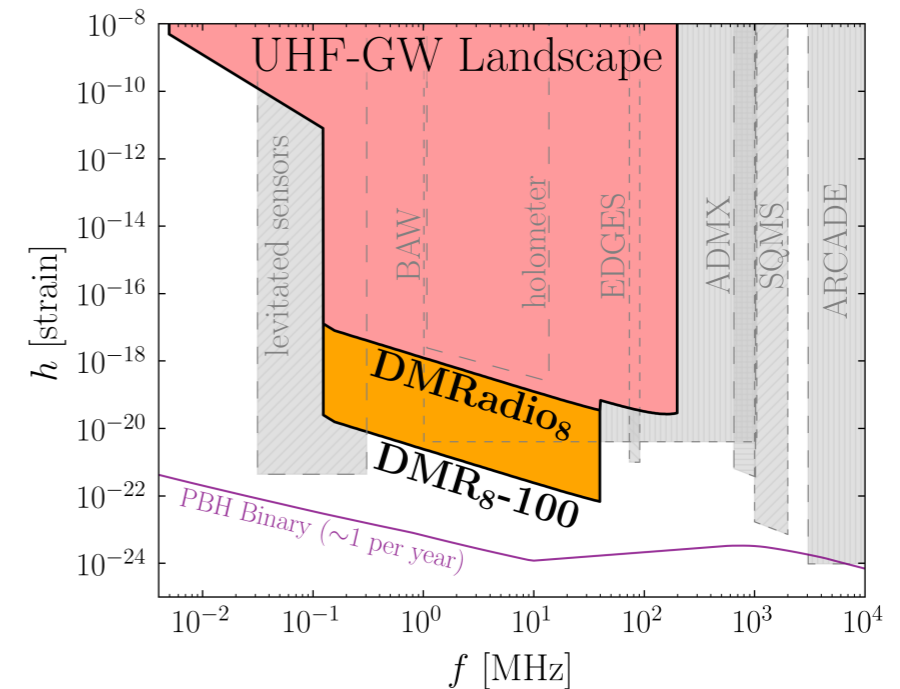
w/ Jeff Dror, Hitoshi Murayama



High-Frequency Gravitational Waves

PRL 2022

w/ Valerie Domcke, Camilo Garcia-Cely



Echos of the Early Universe in Axion Haloscopes



Motivation

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a (F \tilde{F})$$

The axion

Motivation

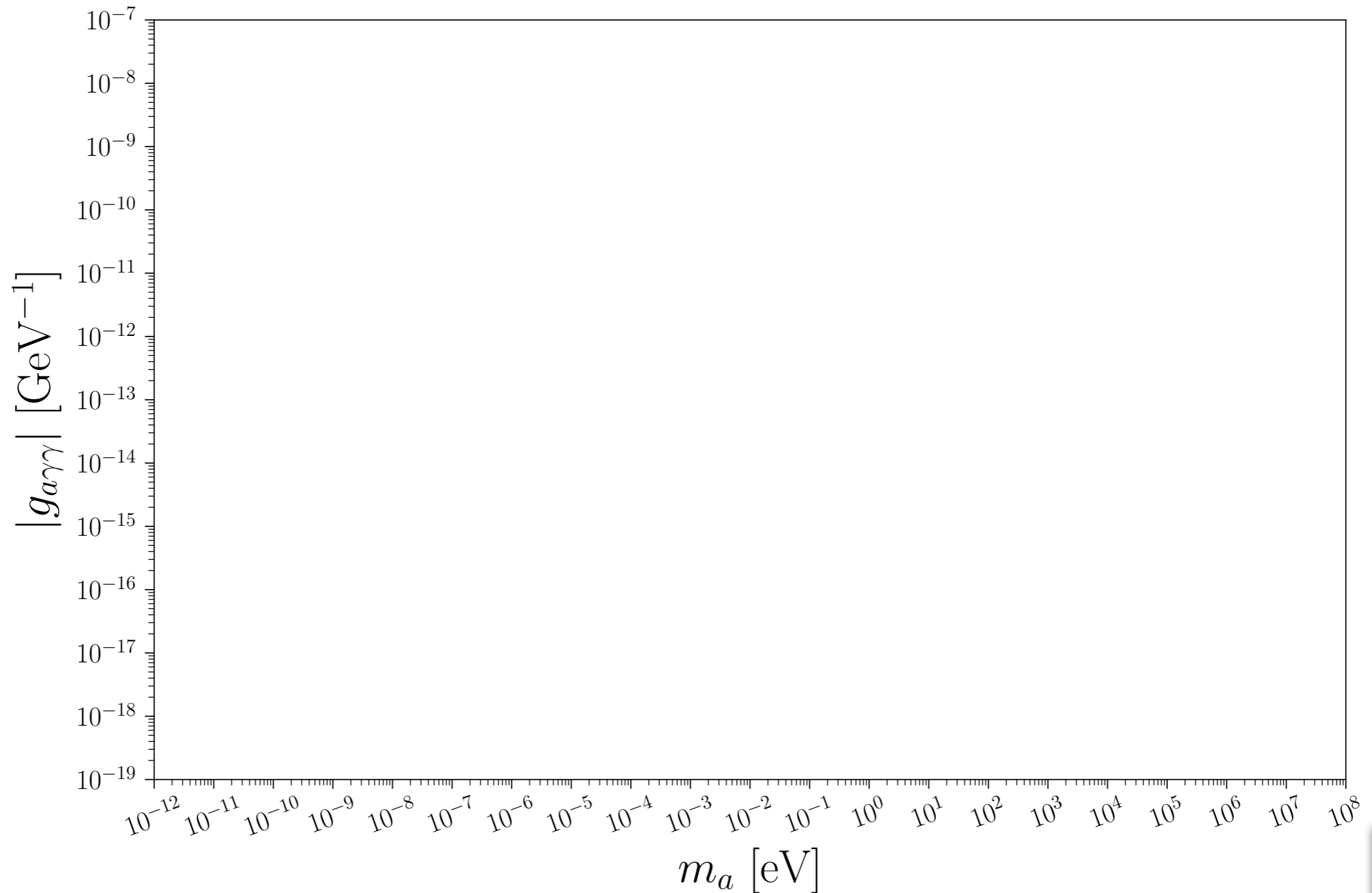
$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma\gamma}a(F\tilde{F})$$

Introduces a new source for E&M fields

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J} + g_{a\gamma\gamma} \mathbf{B} \partial_t a$$

Motivation

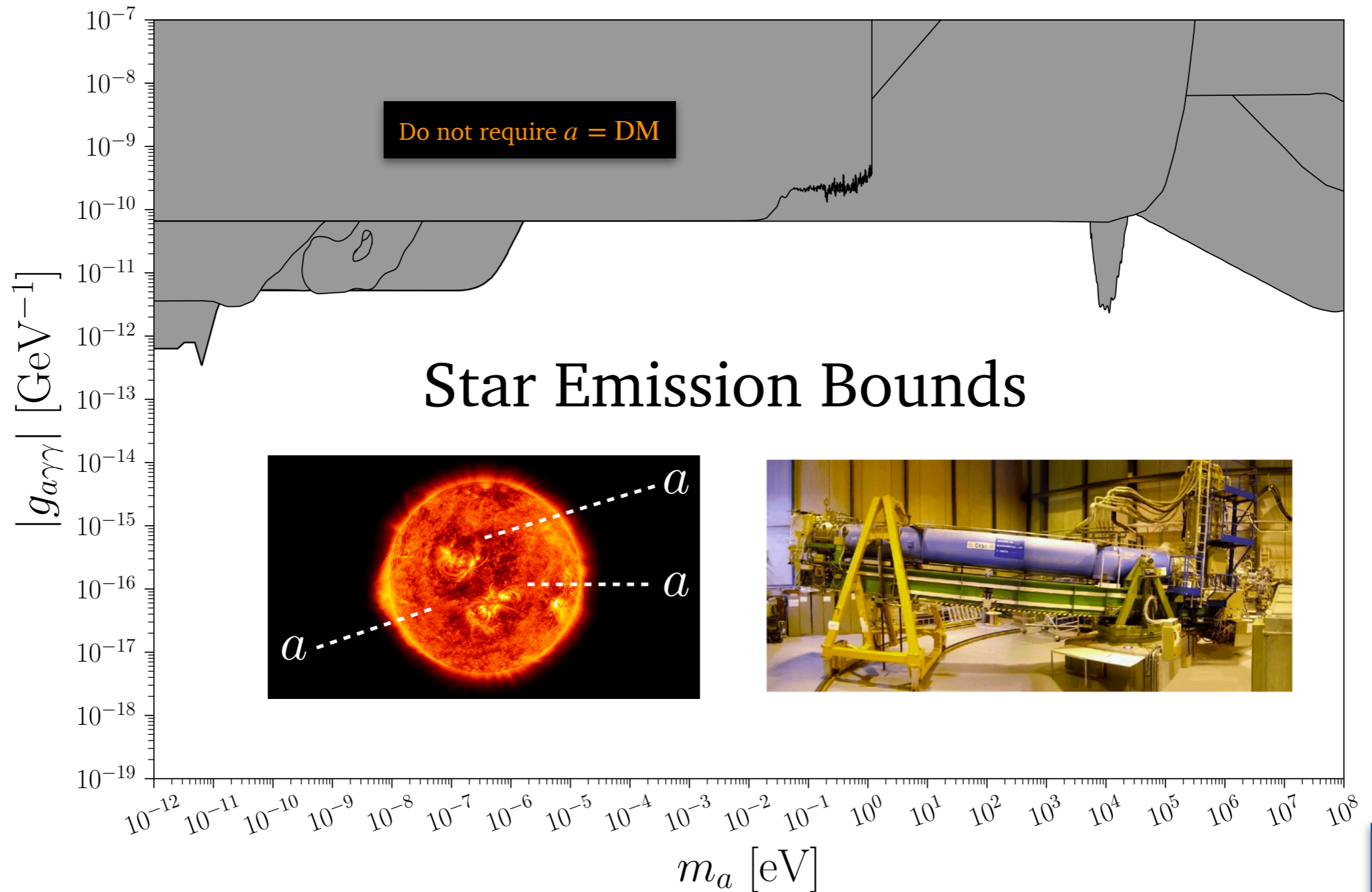
$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma\gamma}a(F\tilde{F})$$



Partial summary
[O'Hare github]

Motivation

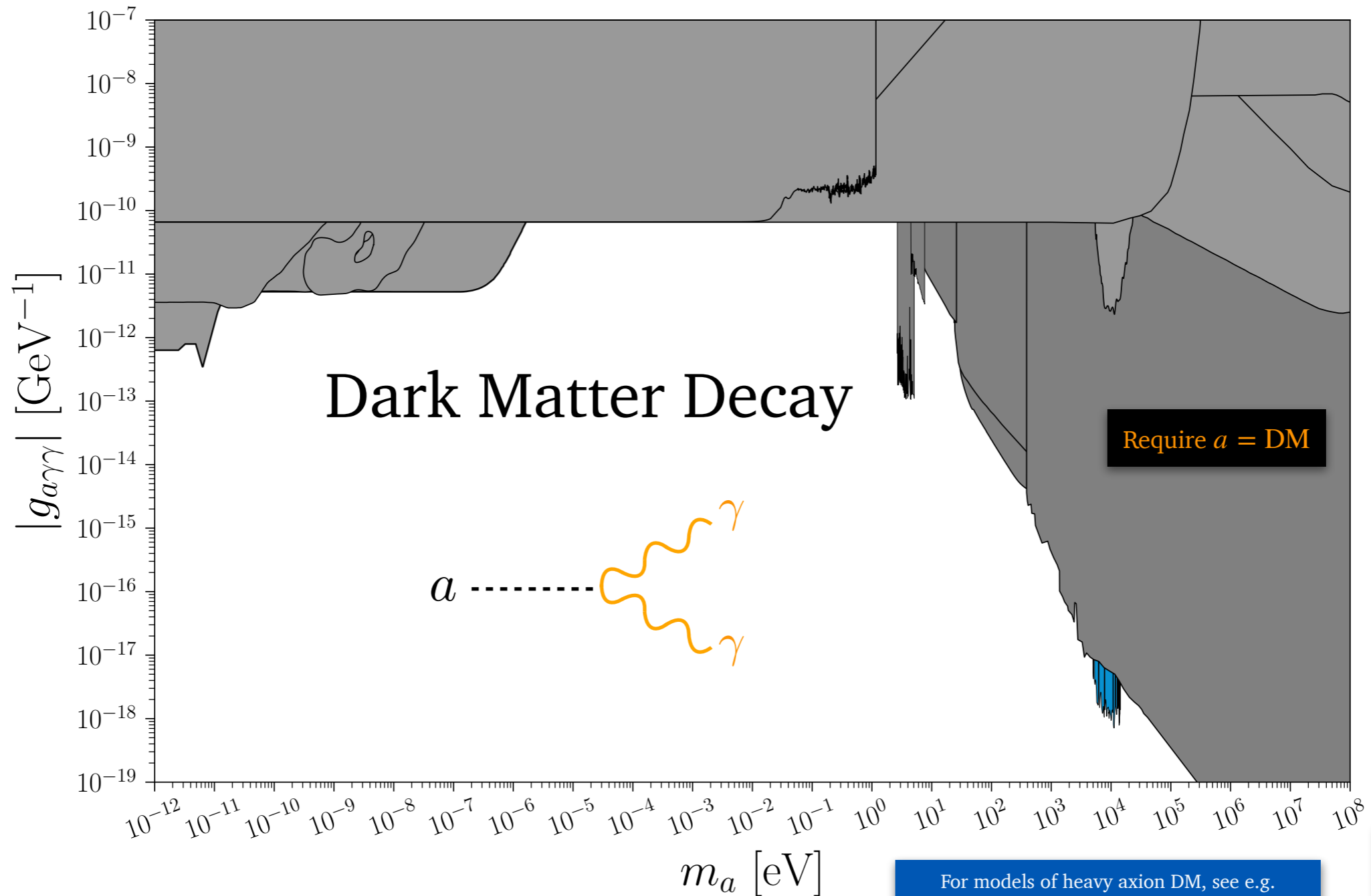
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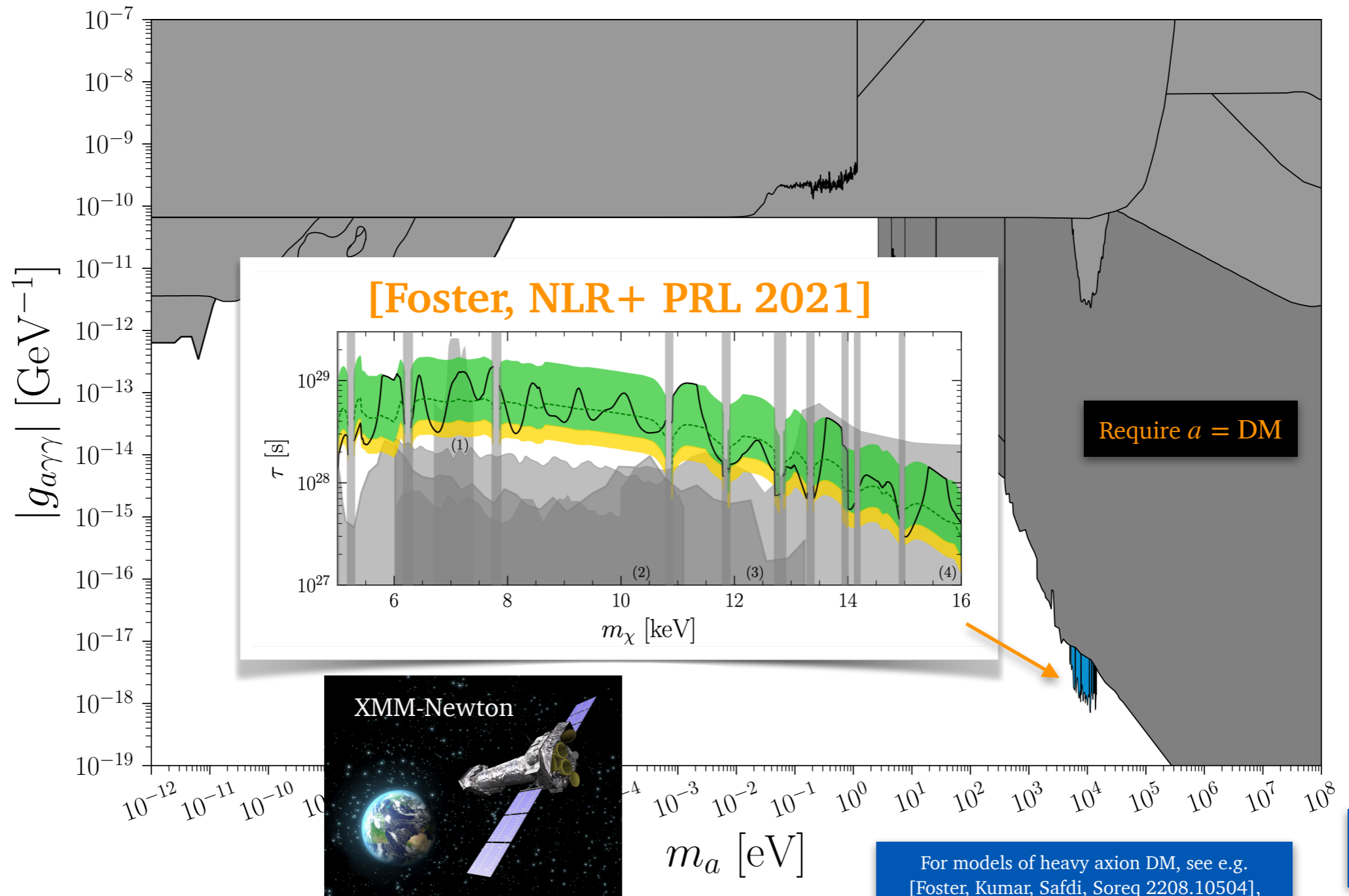
For models of heavy axion DM, see e.g.
 [Foster, Kumar, Safdi, Soreq 2208.10504],
 [Panci, Redigolo, Schwetz, Ziegler 2209.03371]

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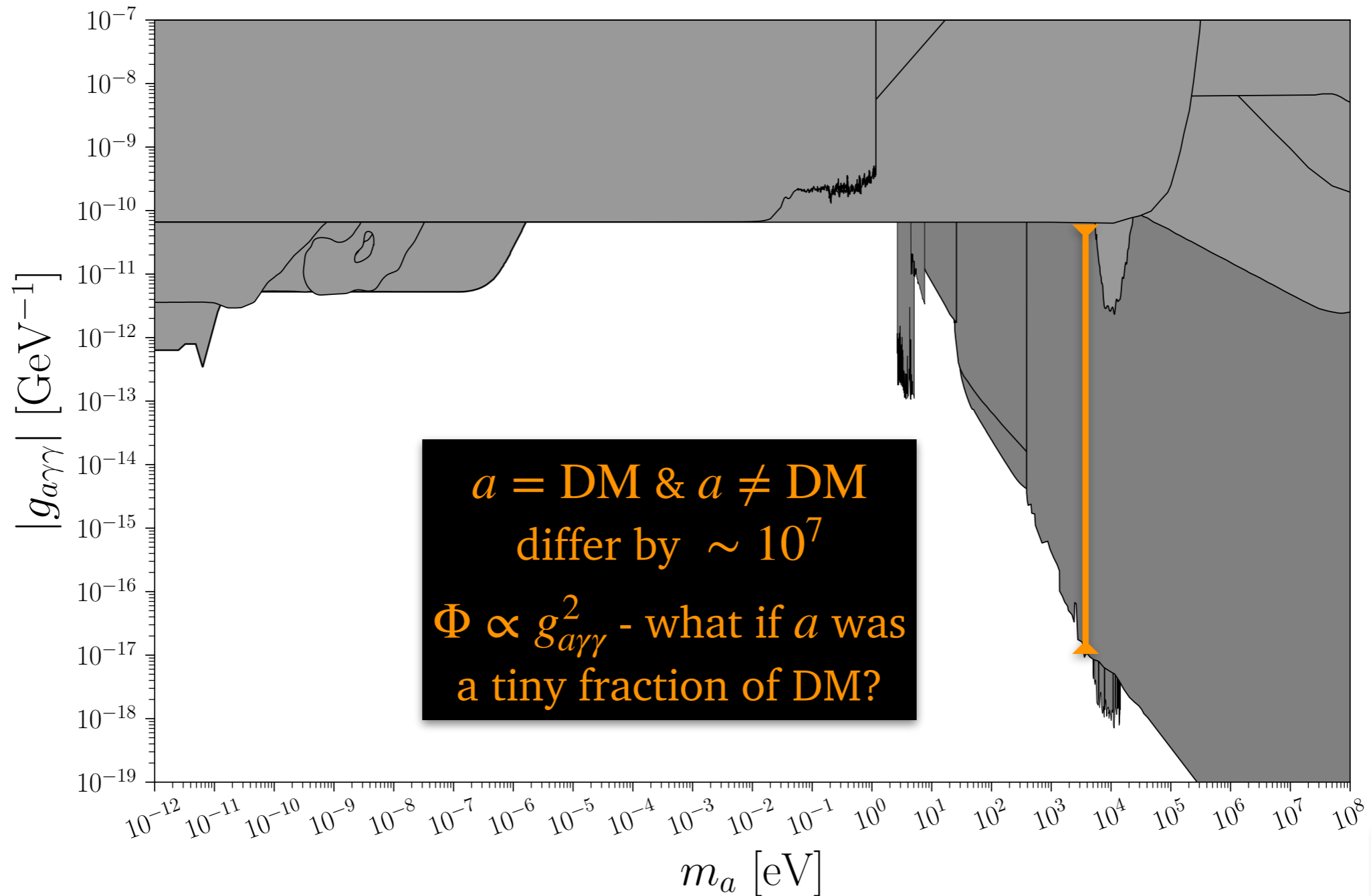
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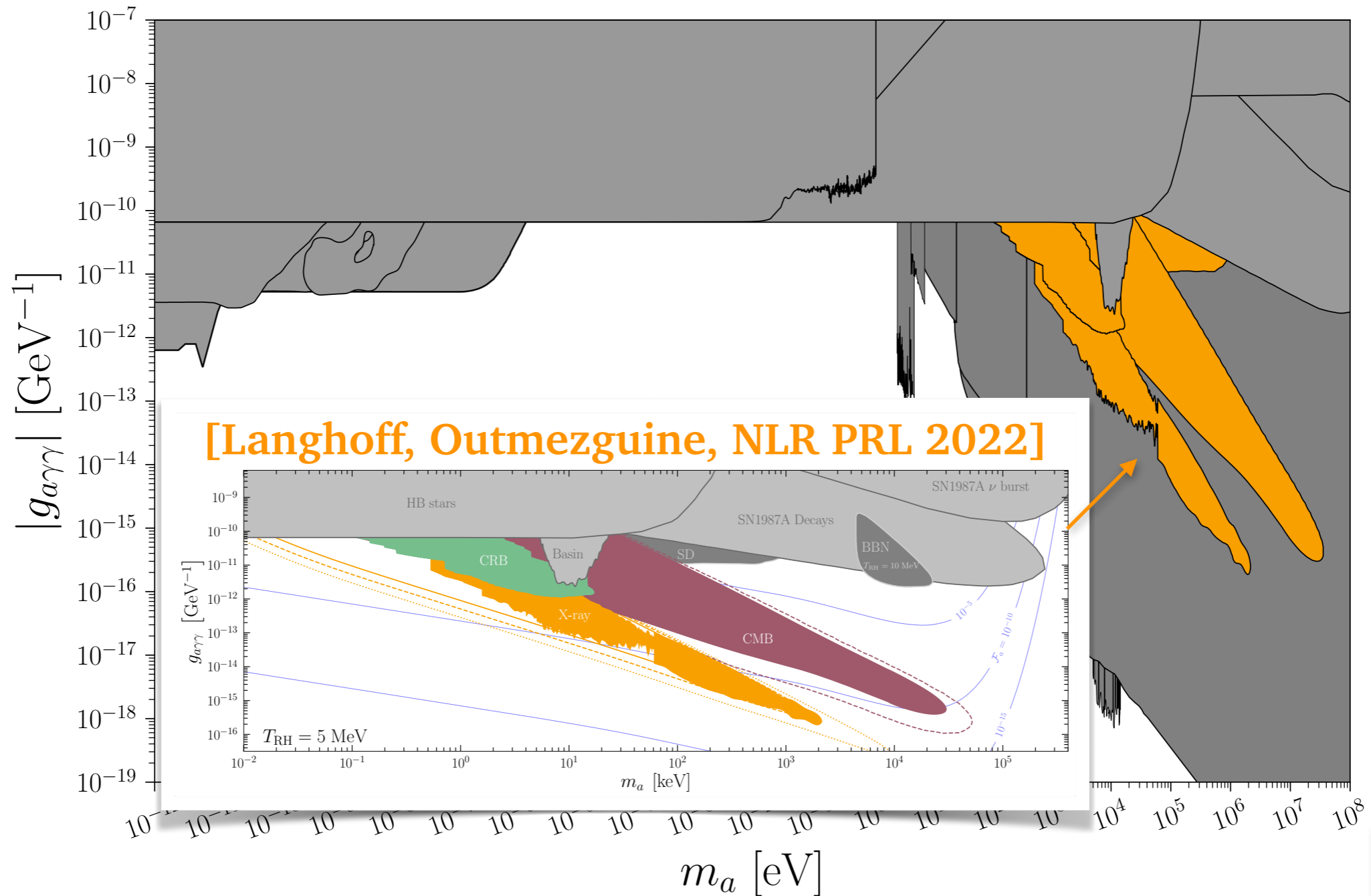
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Partial summary
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Motivation

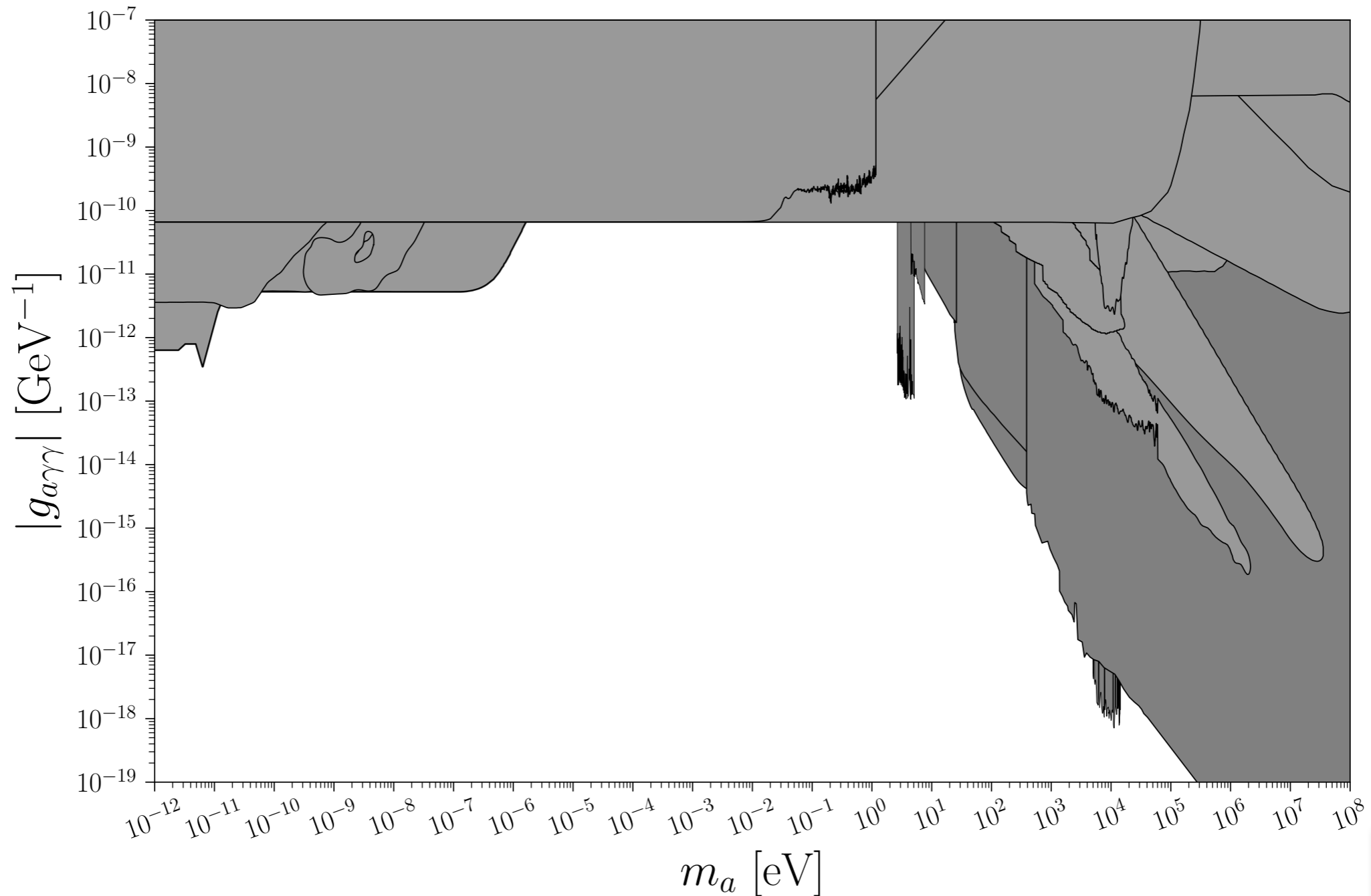
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Partial summary
[O'Hare github]

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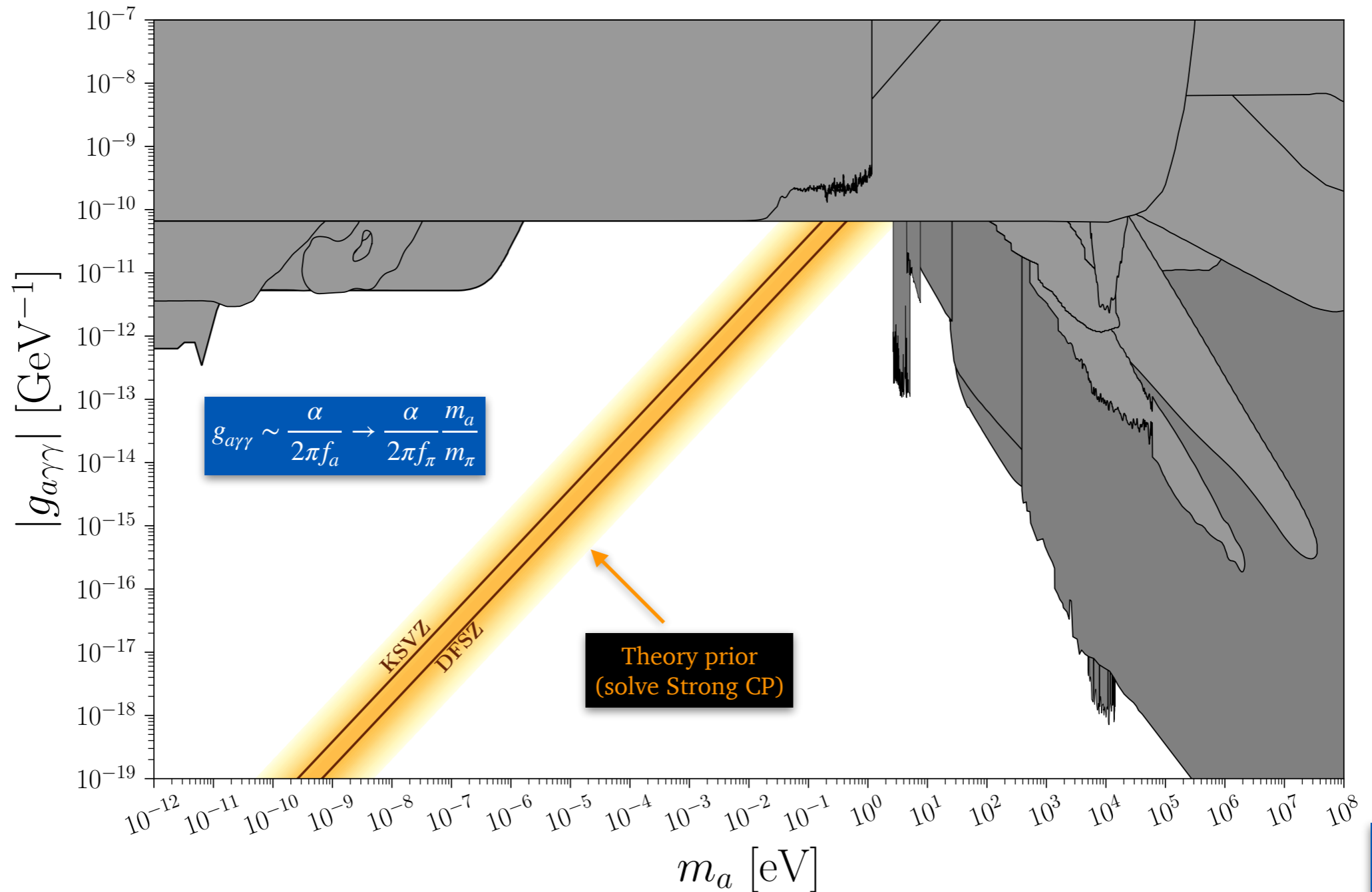
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Partial summary
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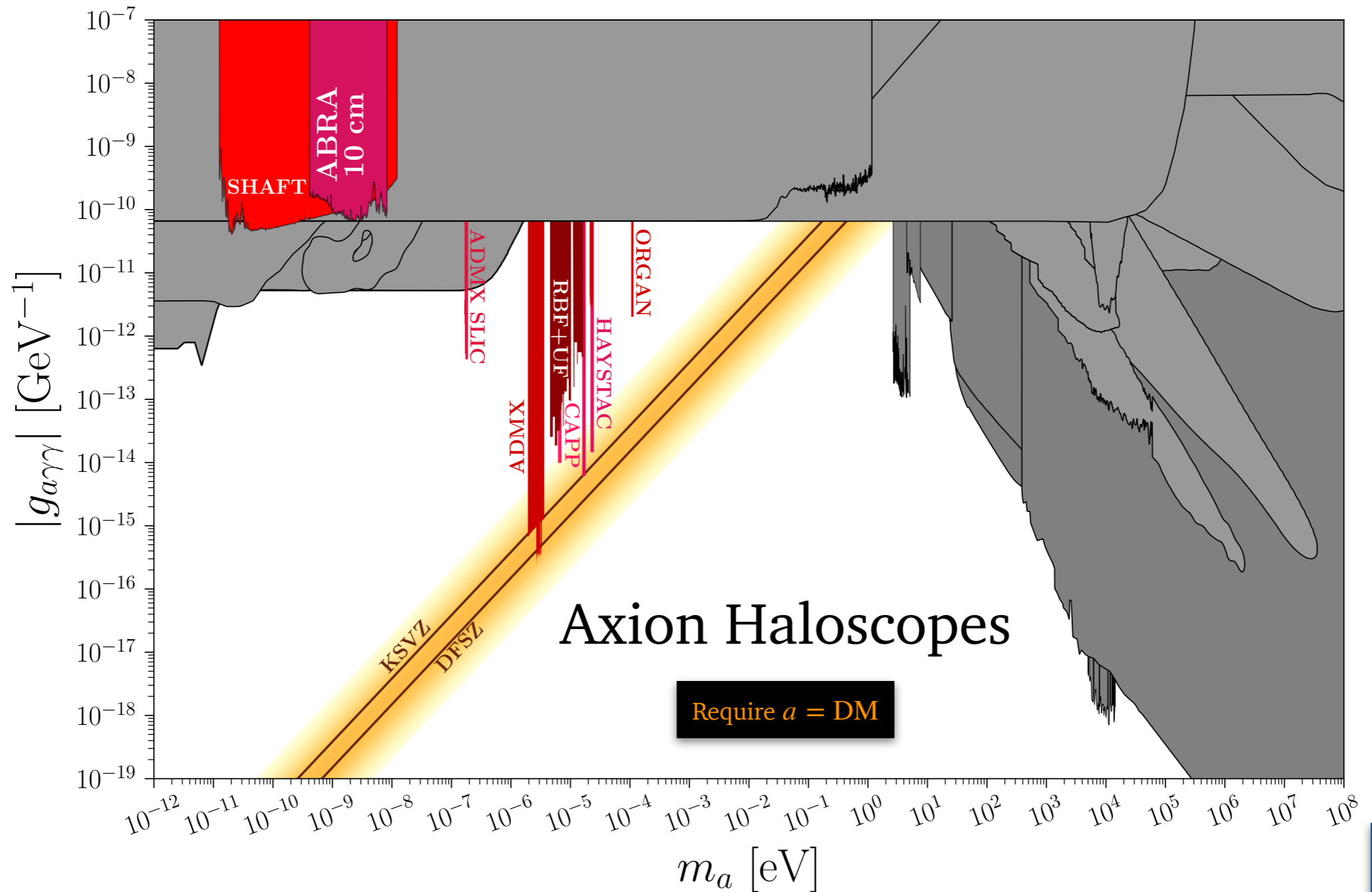
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Partial summary
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Motivation

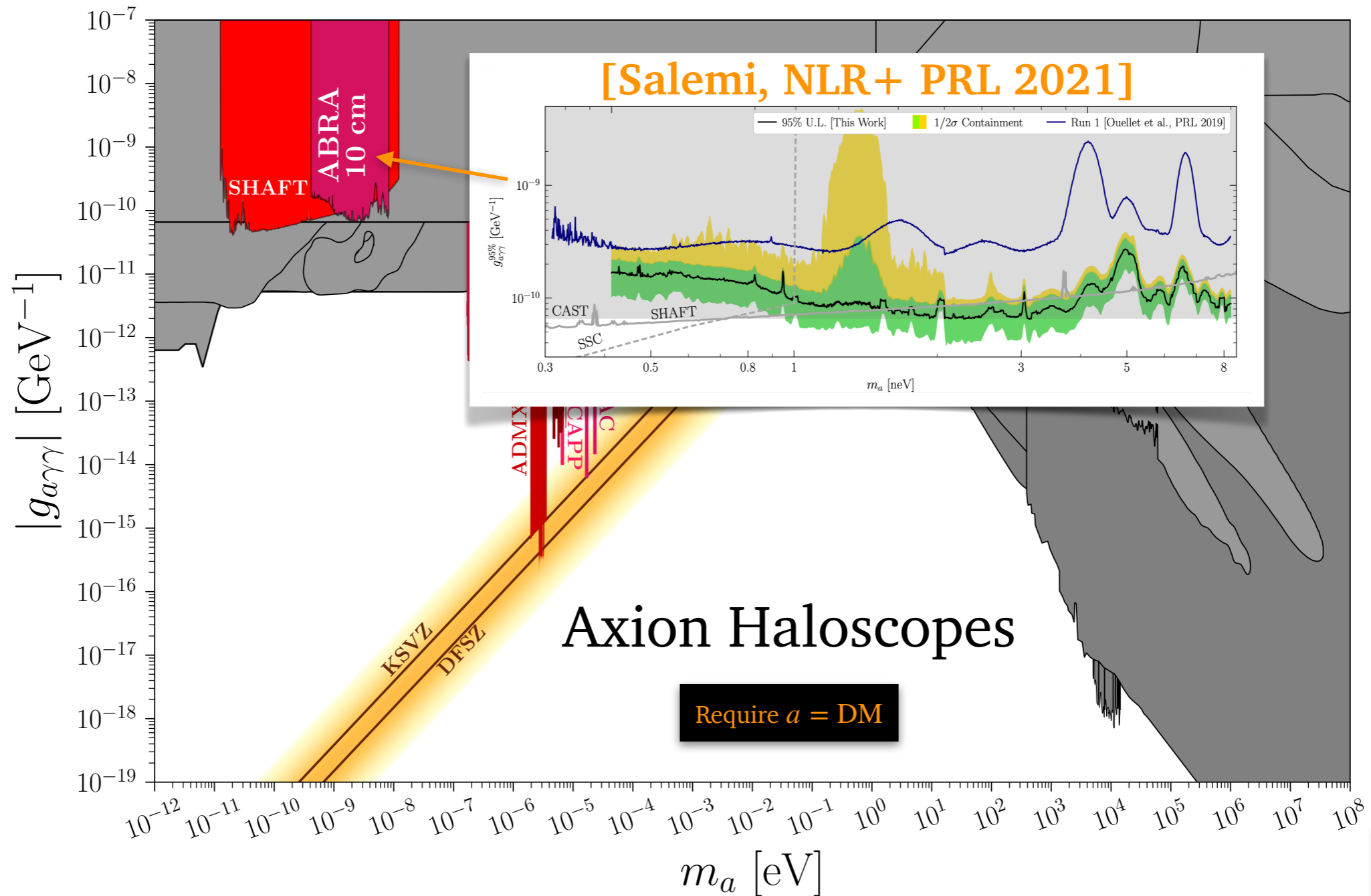
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Partial summary
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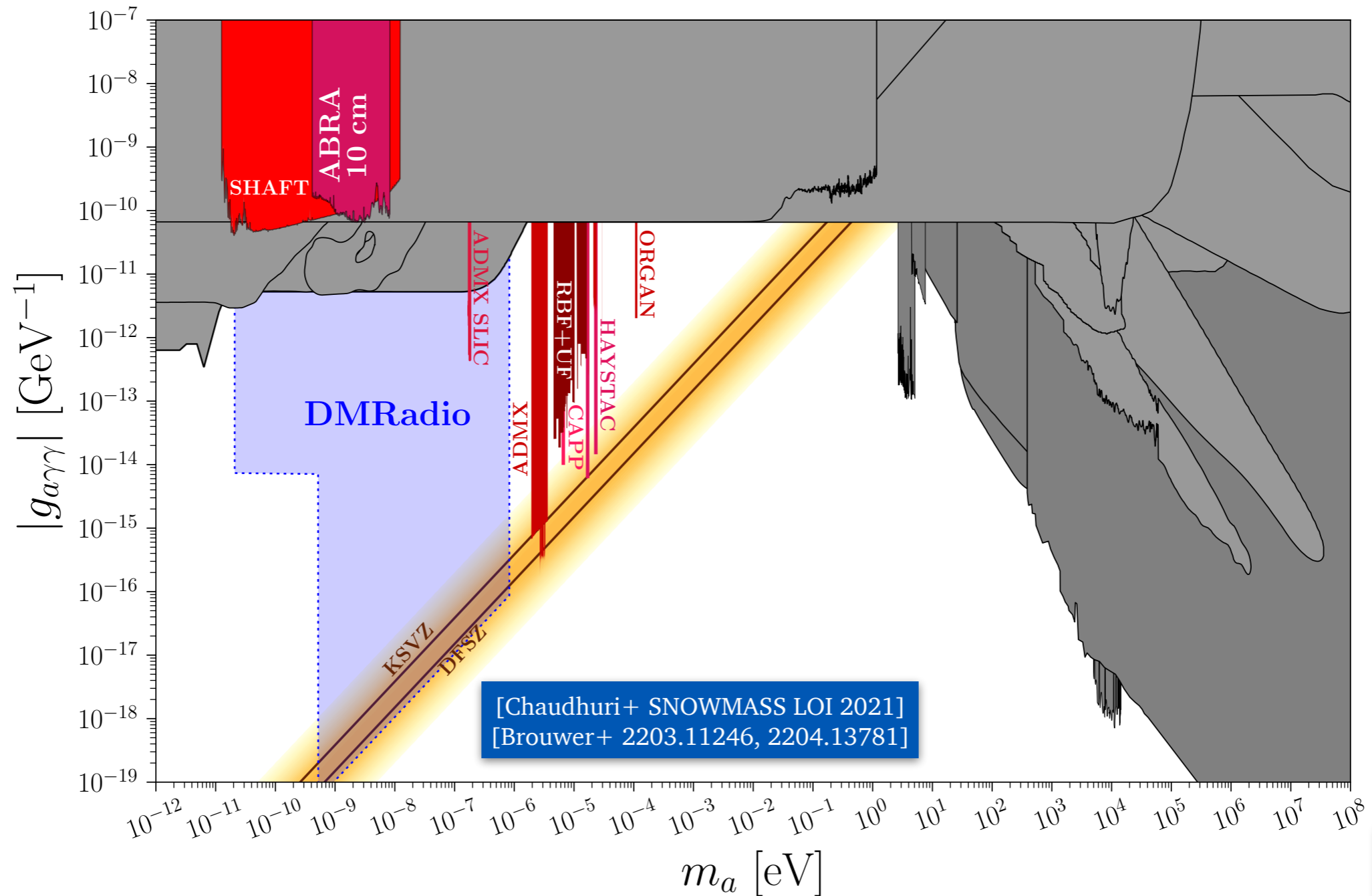
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Partial summary
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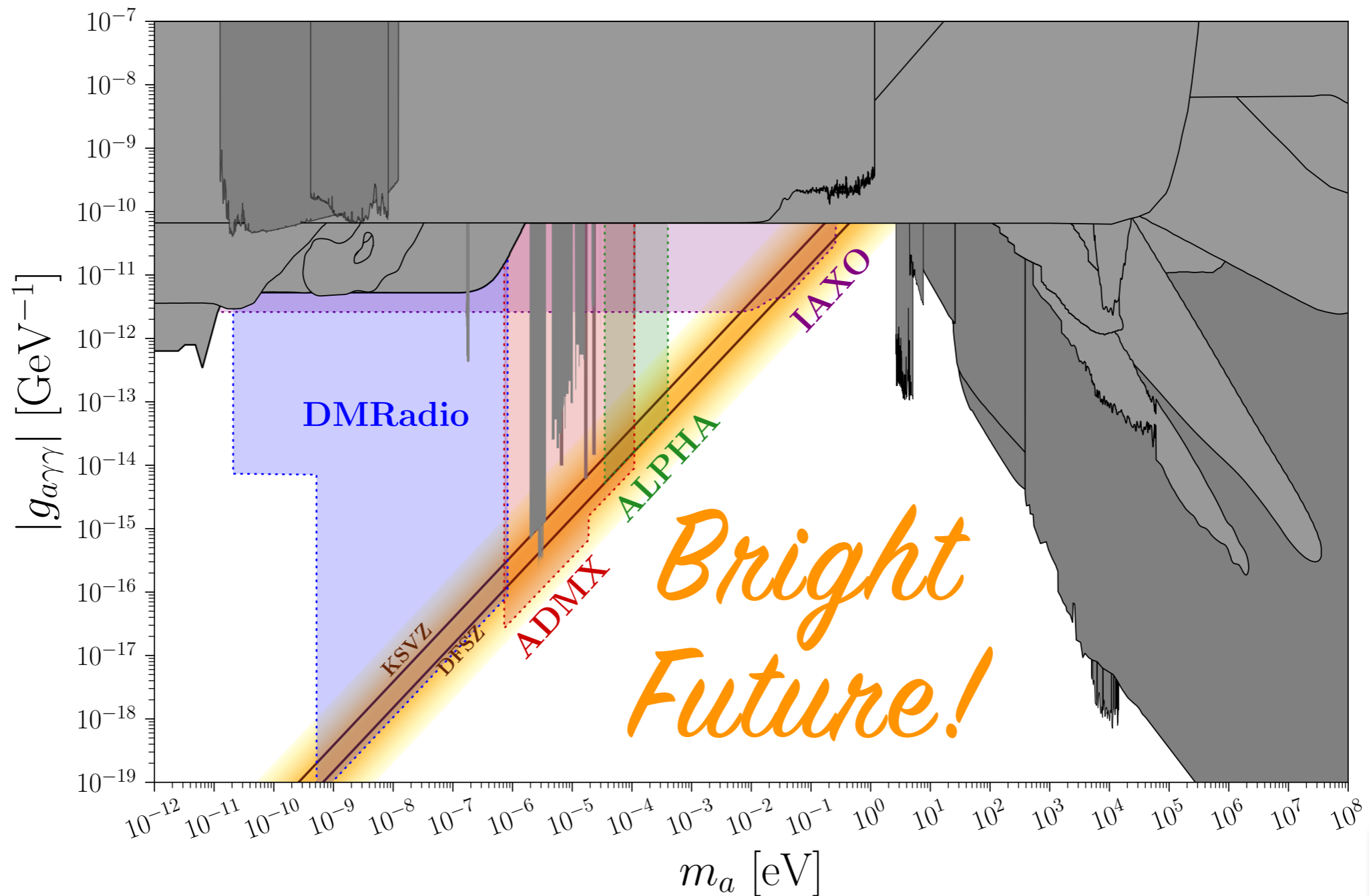
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Partial summary
[O'Hare github]

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Partial summary
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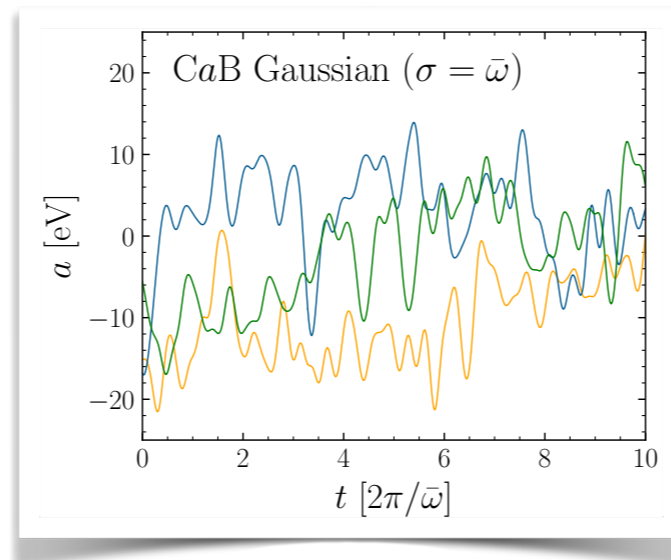
Outline

What else might we see with these instruments?

Outline

1. Could non-dark matter axions leave a signal?

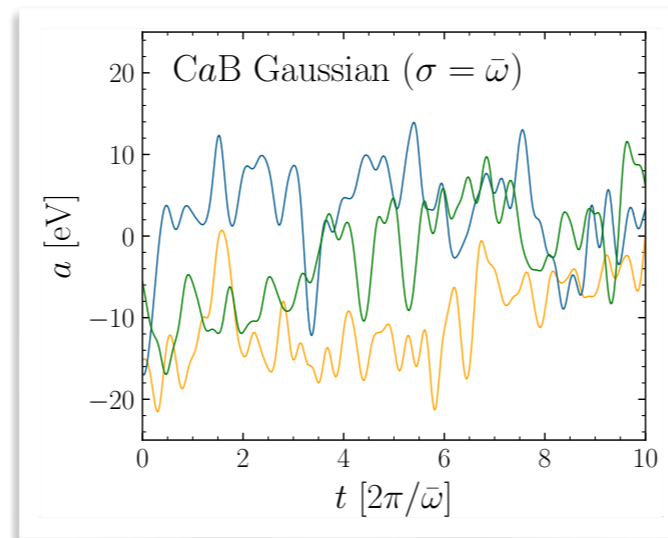
The Cosmic Axion Background



Outline

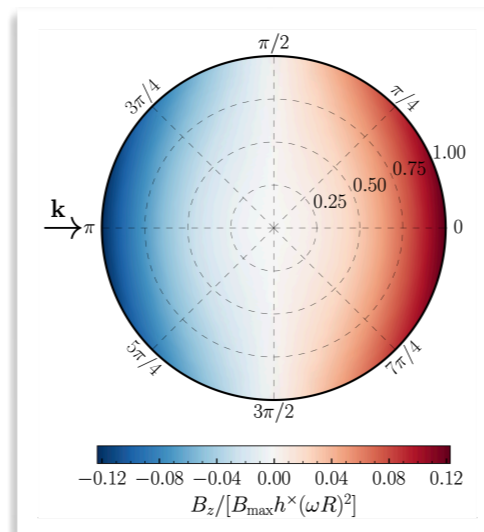
1. Could non-dark matter axions leave a signal?

The Cosmic Axion Background



2. What other passing waves could a haloscope detect?

High-Frequency Gravitational Waves



WARMUP

How to Discover Dark Matter

Axion Dark Matter

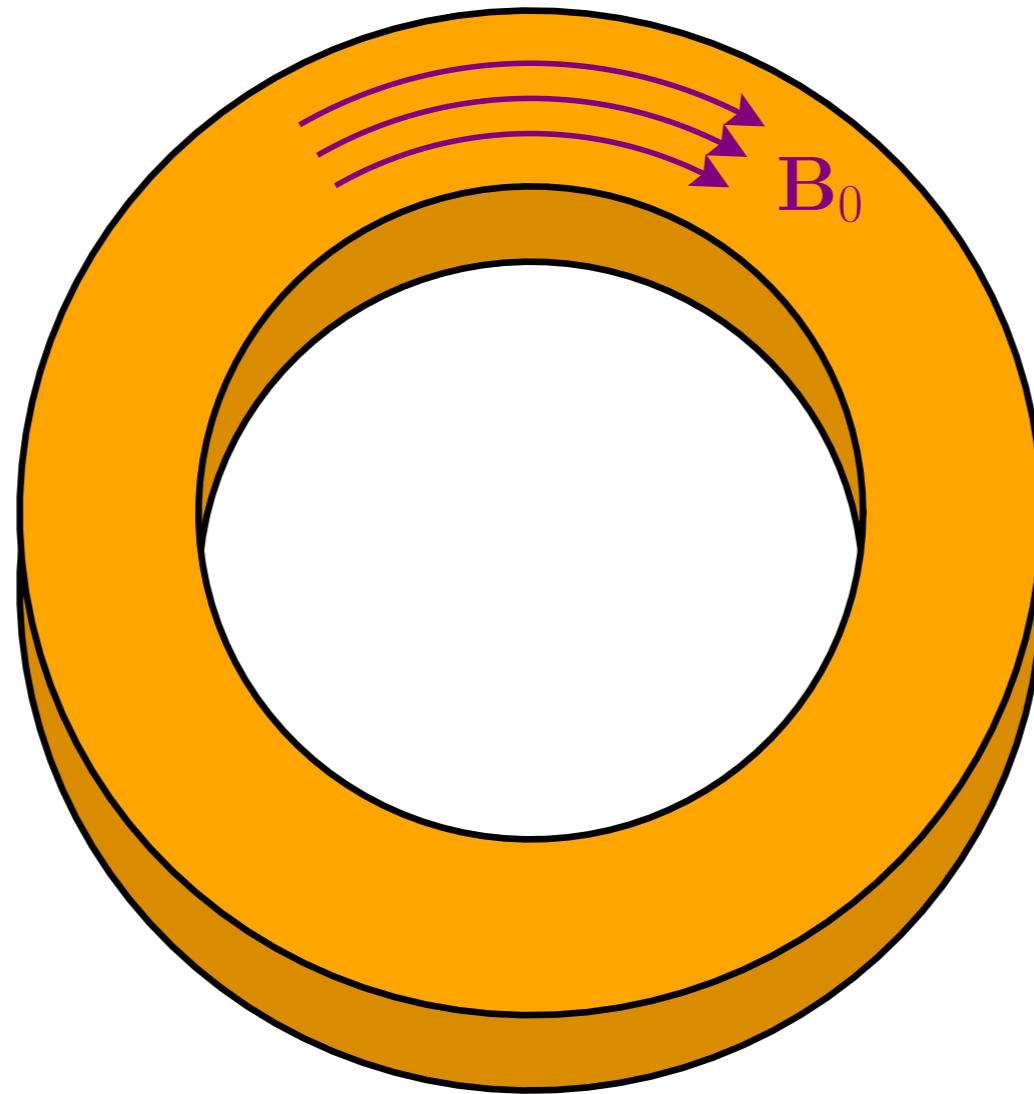
$$\mathcal{L} \supset -\frac{1}{4}g_{a\gamma\gamma}a(F\tilde{F})$$

Introduces a new source for E&M fields

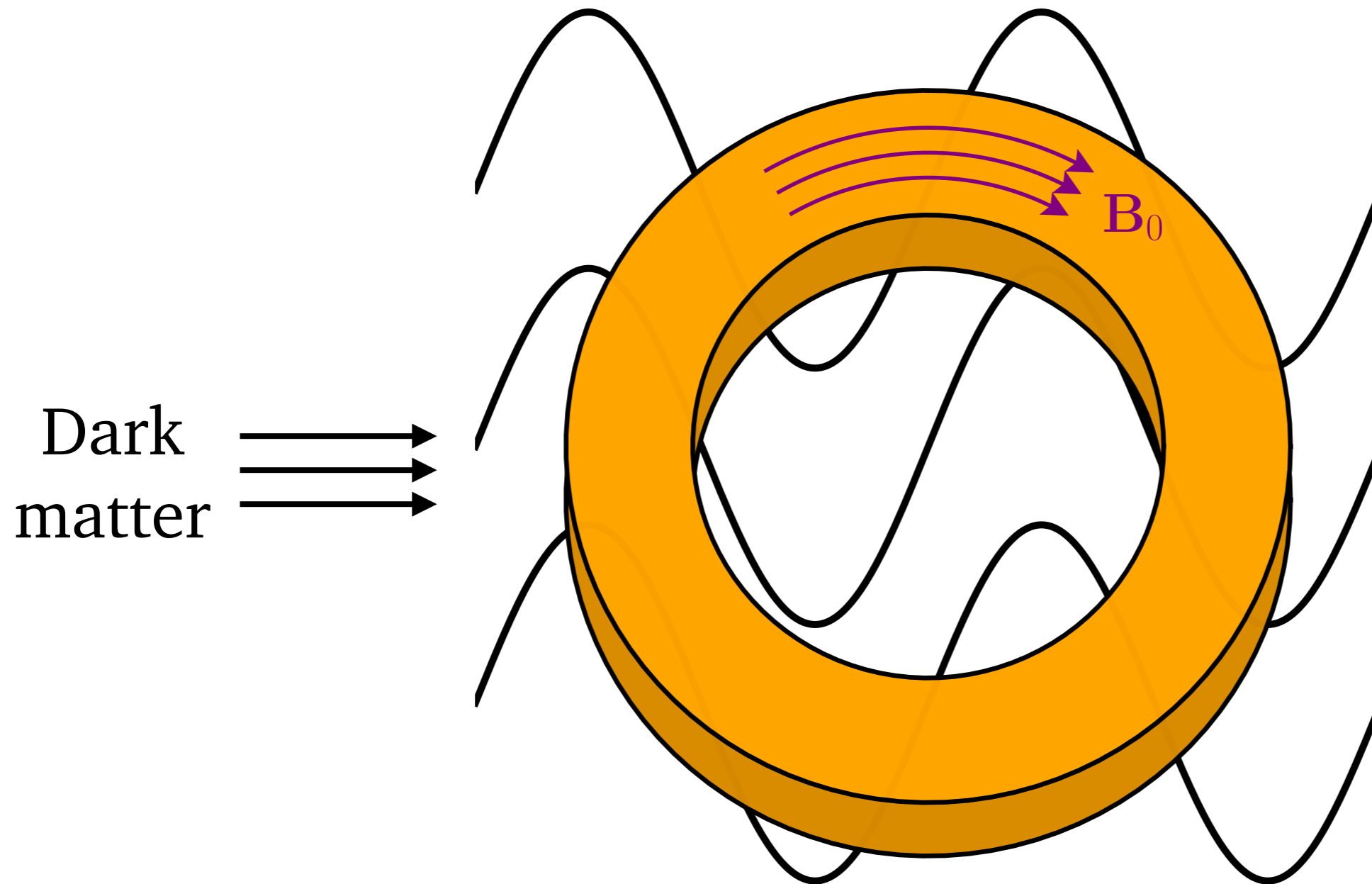
$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J} + g_{a\gamma\gamma} \mathbf{B} \partial_t a$$

Not shown: $\rho_{\text{eff}} = -g_{a\gamma\gamma} \mathbf{B} \cdot \nabla a$
and $\mathbf{J}_{\text{eff}} = -g_{a\gamma\gamma} \mathbf{E} \times \nabla a$

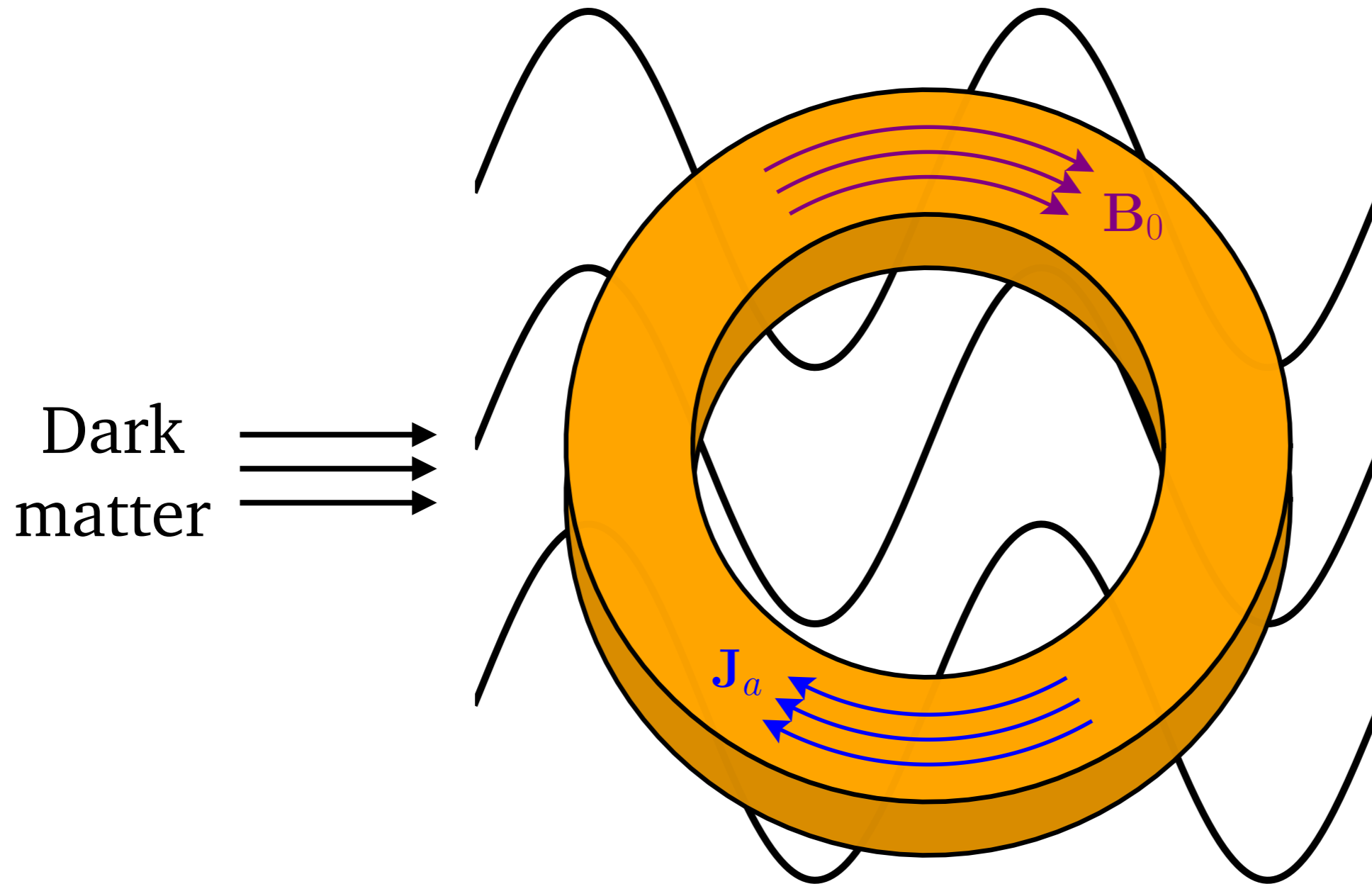
Detection with a Toroidal Magnet



Detection with a Toroidal Magnet

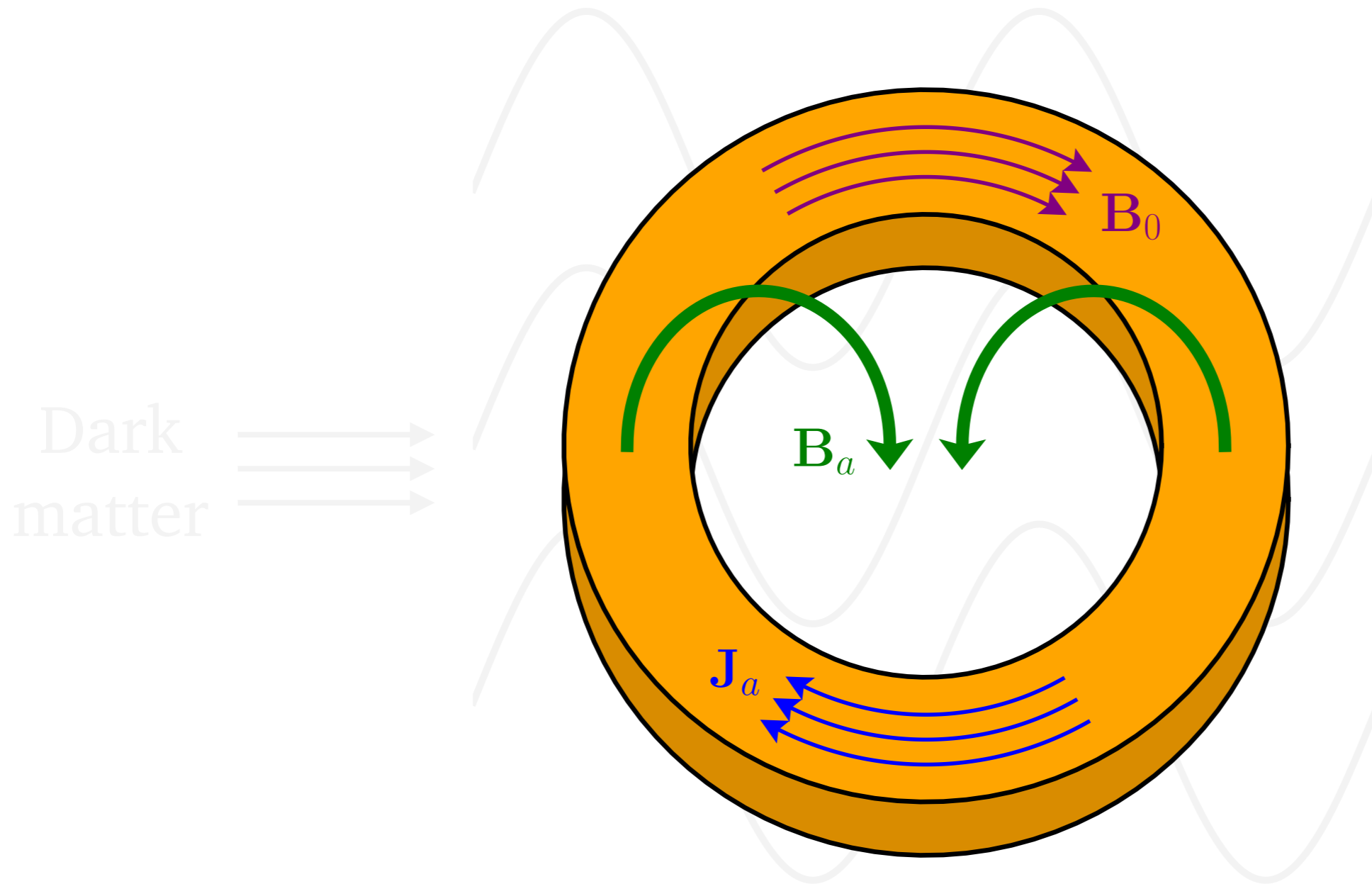


Detection with a Toroidal Magnet



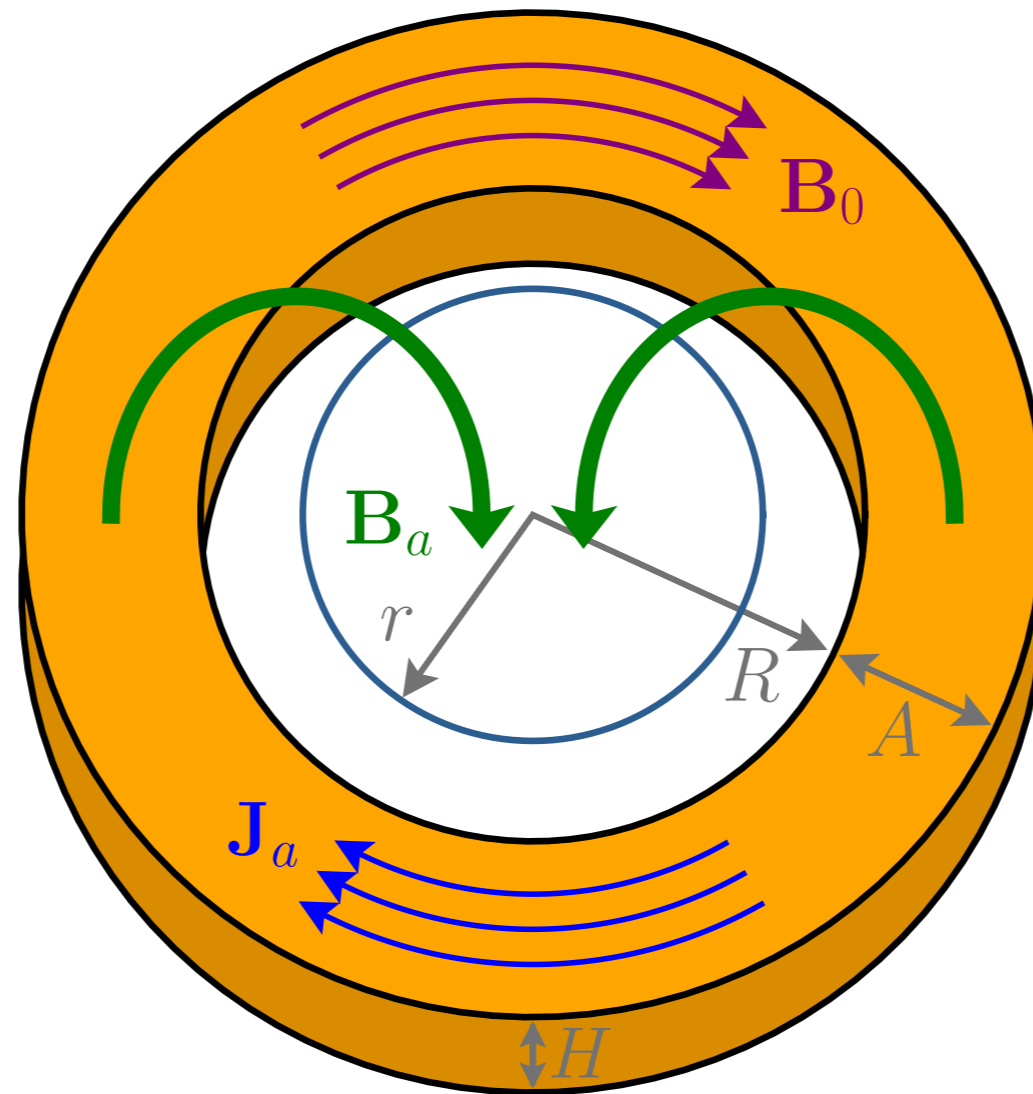
$$\mathbf{J}_a = g_{a\gamma\gamma} (\partial_t a) \mathbf{B}_0$$

Detection with a Toroidal Magnet



$$\mathbf{J}_a = g_{a\gamma\gamma} (\partial_t a) \mathbf{B}_0$$

Detection with a Toroidal Magnet



$$\Phi_a(t) \simeq g_{a\gamma\gamma}(\partial_t a) B_0 \pi r^2 R \ln(1 + A/R) \sim g_{a\gamma\gamma}(\partial_t a) B_0 V$$

$$H \gg R + A$$

Cosmic Axion Background

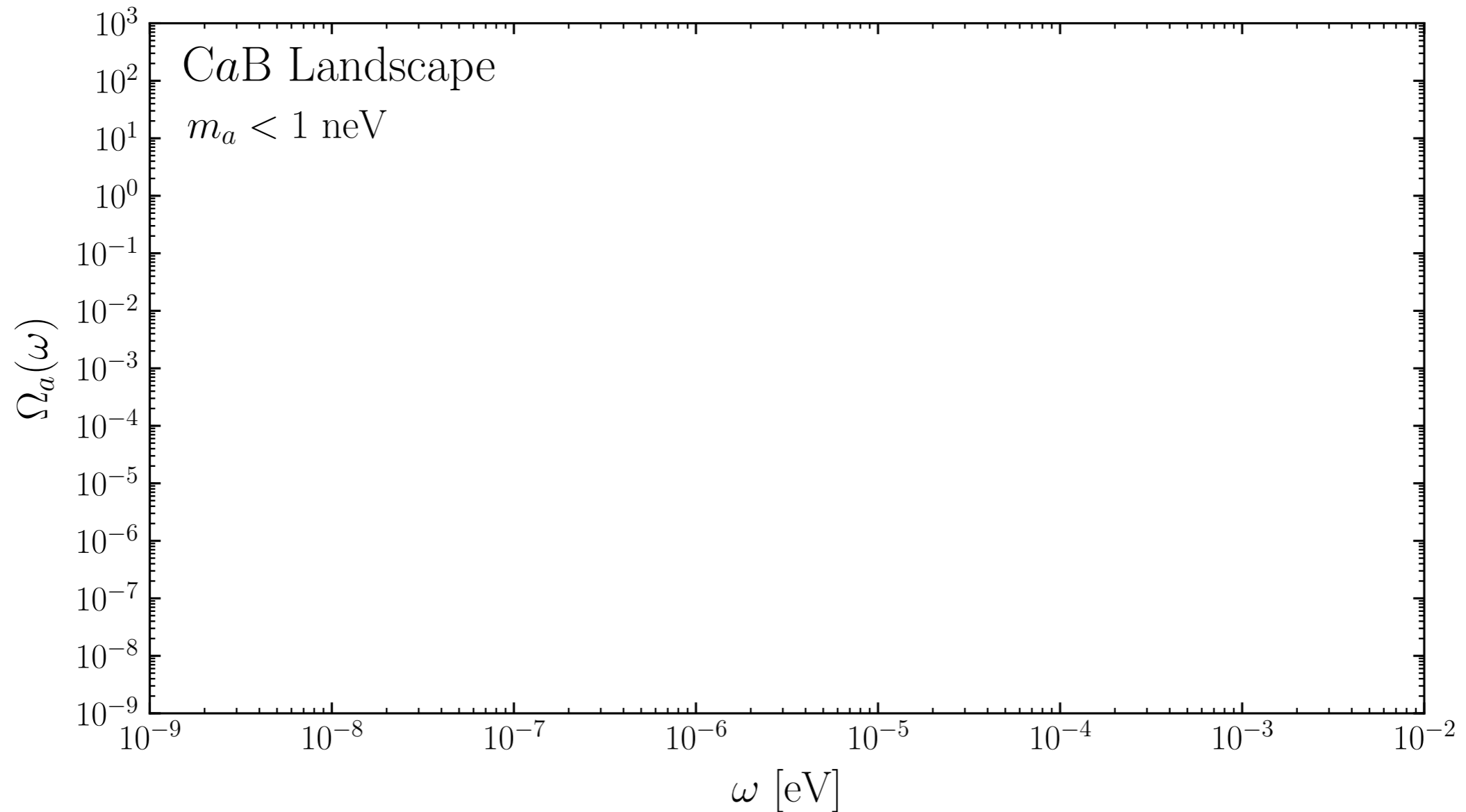
[Dror, Murayama, NLR PRD 2021]

The Cosmic Axion Background

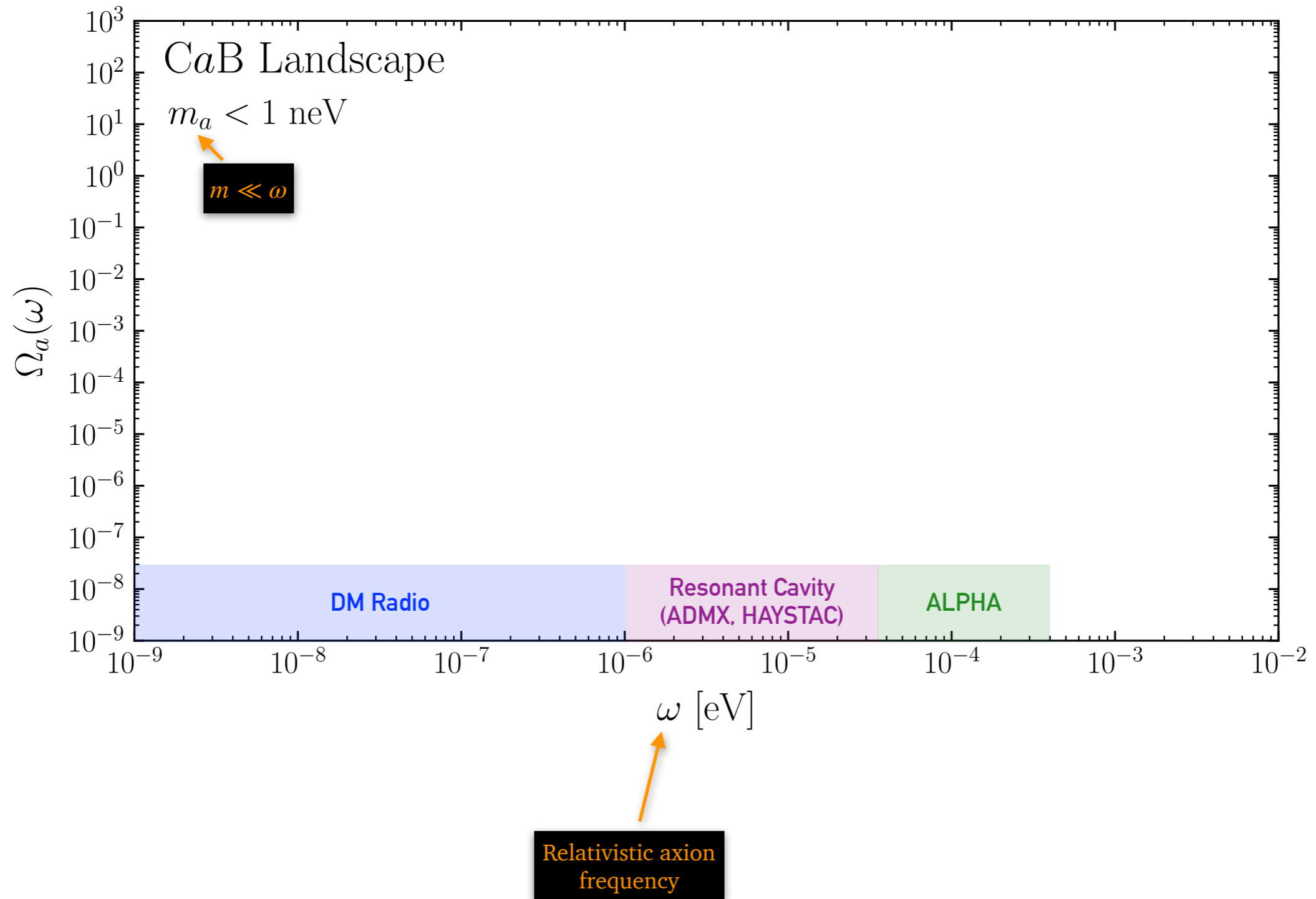
Can we detect relativistic axions that
are a relic of the early Universe?



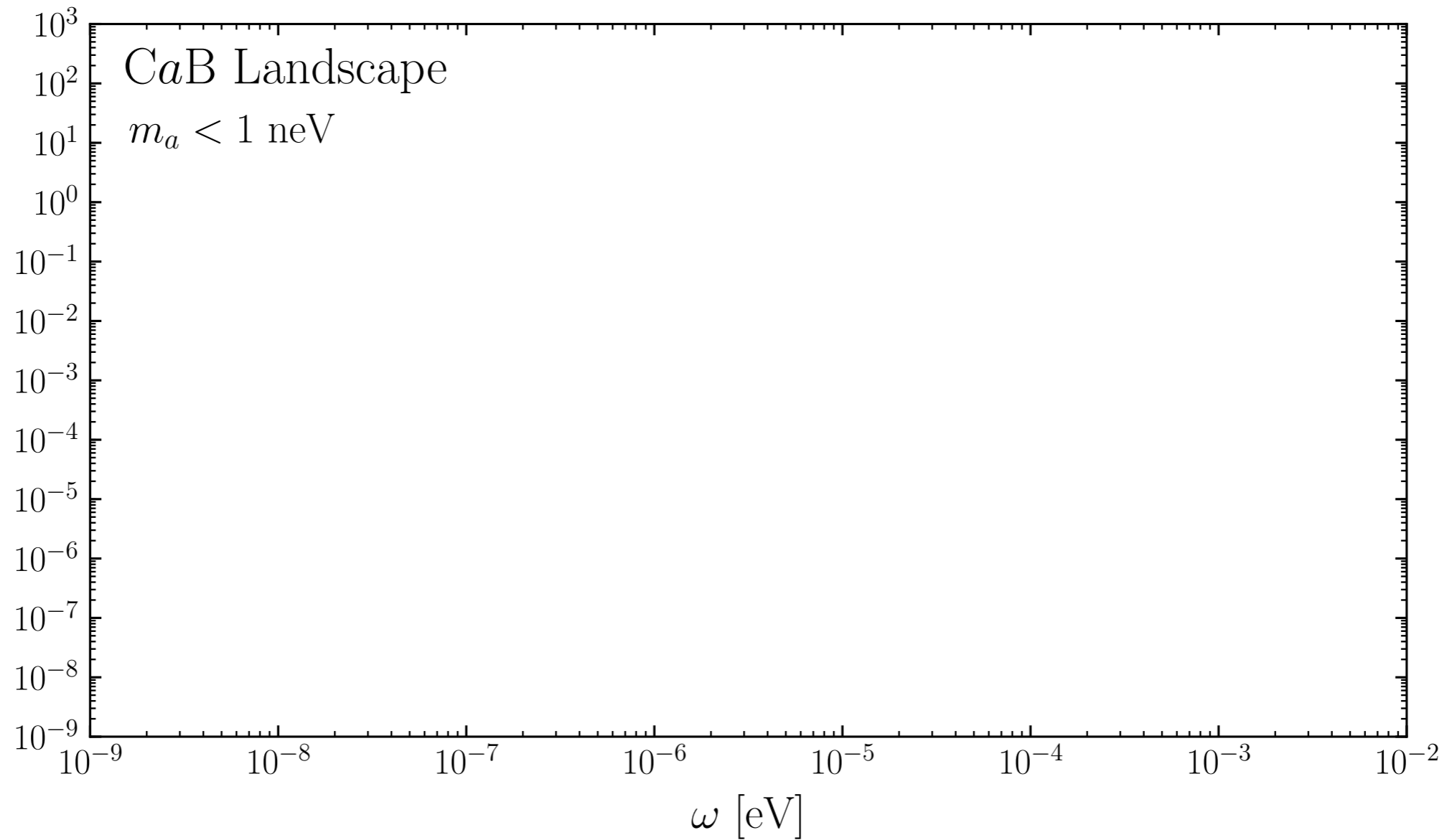
Landscape



Landscape



Landscape

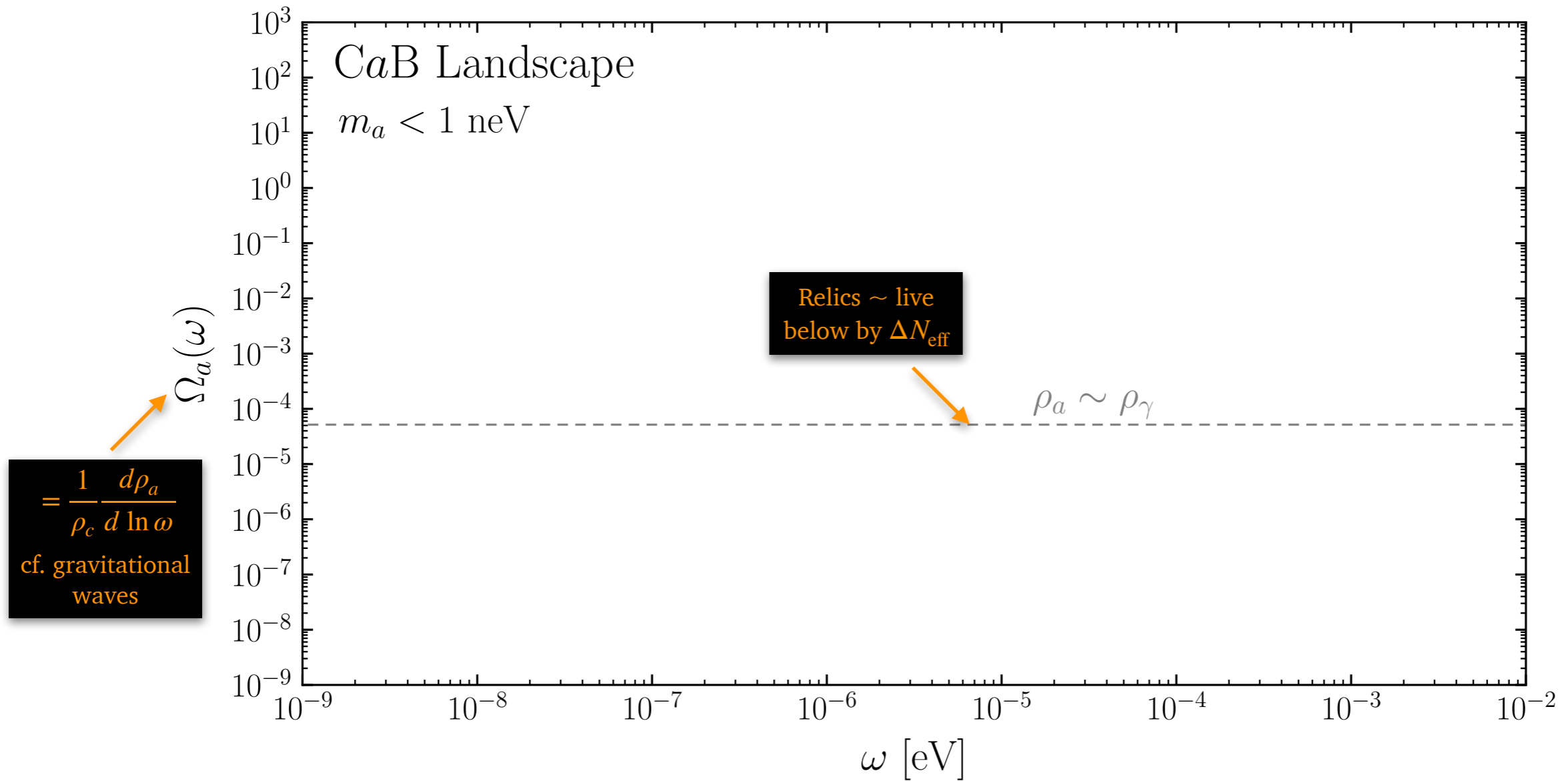


$$= \frac{1}{\rho_c} \frac{d\rho_a}{d \ln \omega}$$
 cf. gravitational waves

Where is $g_{a\gamma\gamma}$?
 Coming in a few slides

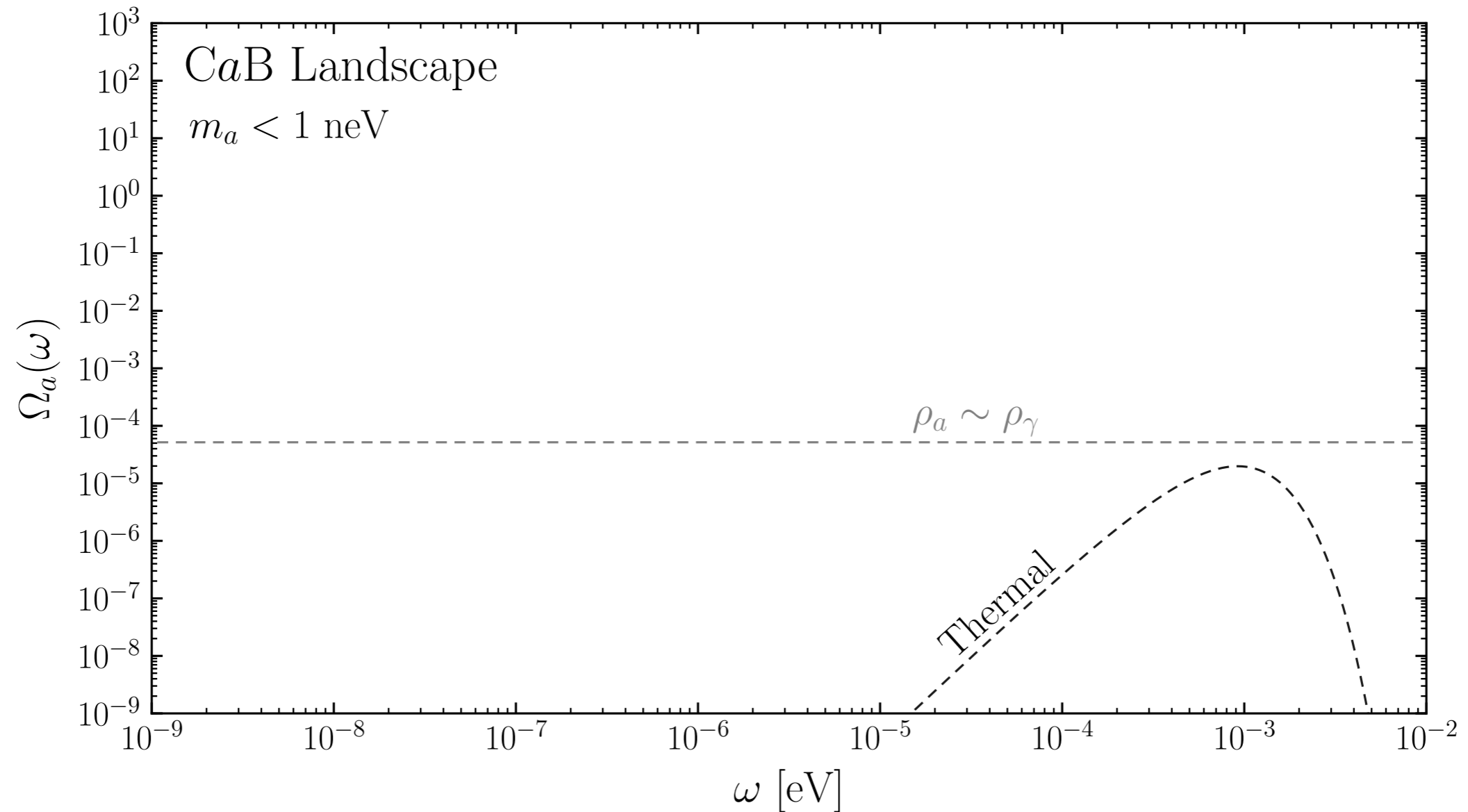


Landscape

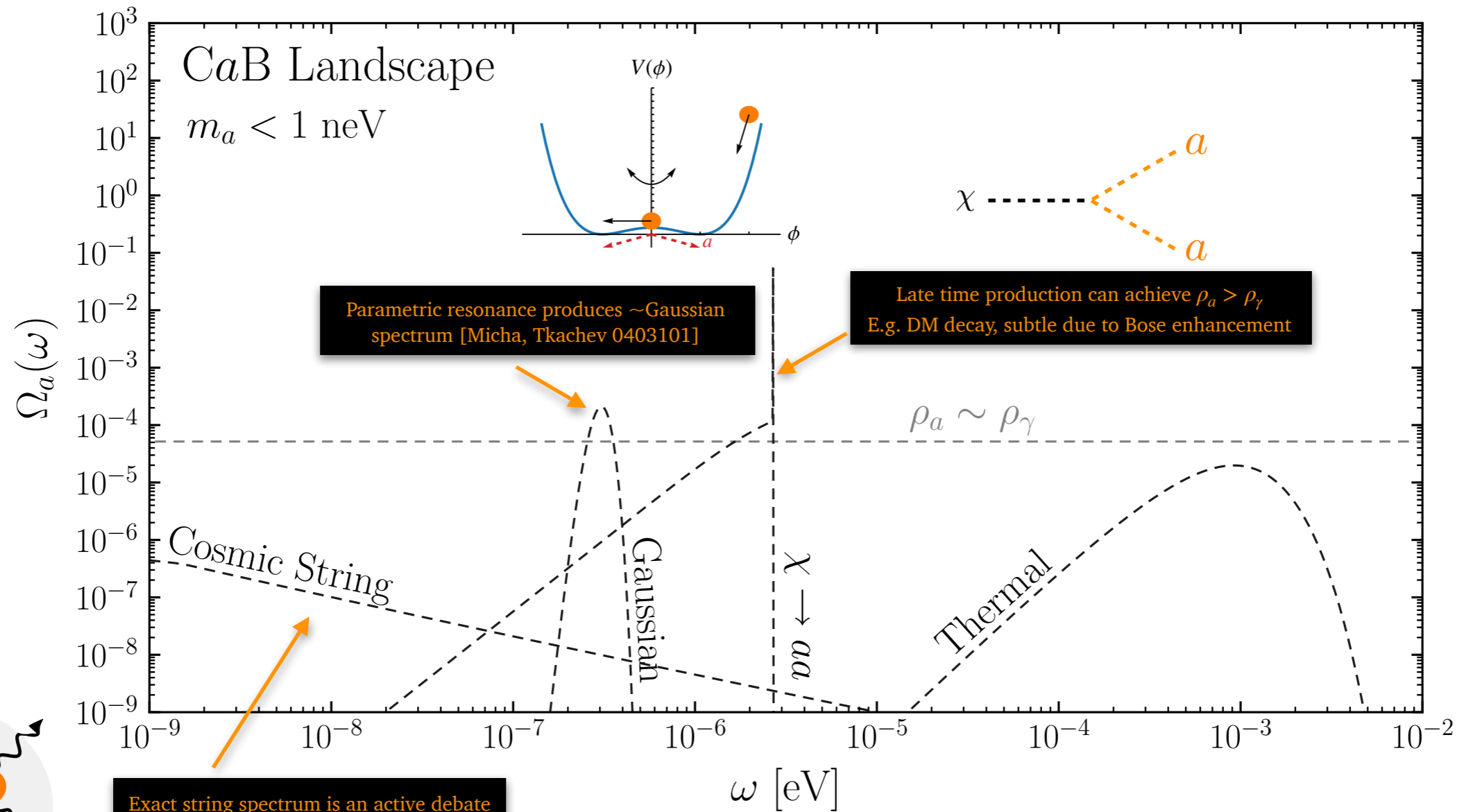


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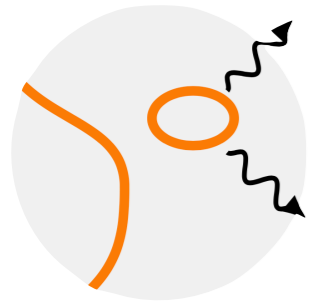


Landscape

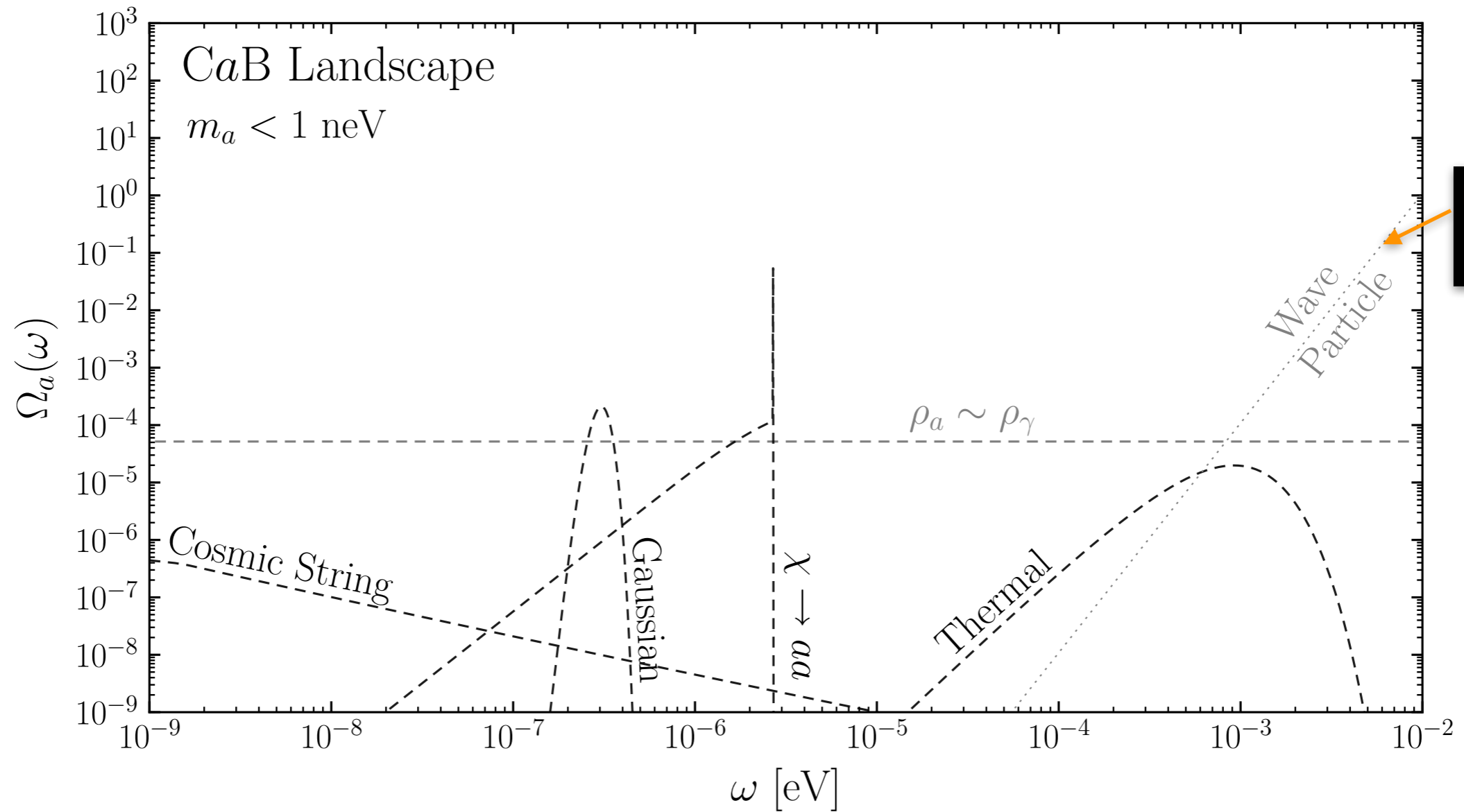


Exact string spectrum is an active debate
 Follow [Gorghetto+ 2018, 2020], see also [Buschmann+ 2022] & [Dine+ 2020]

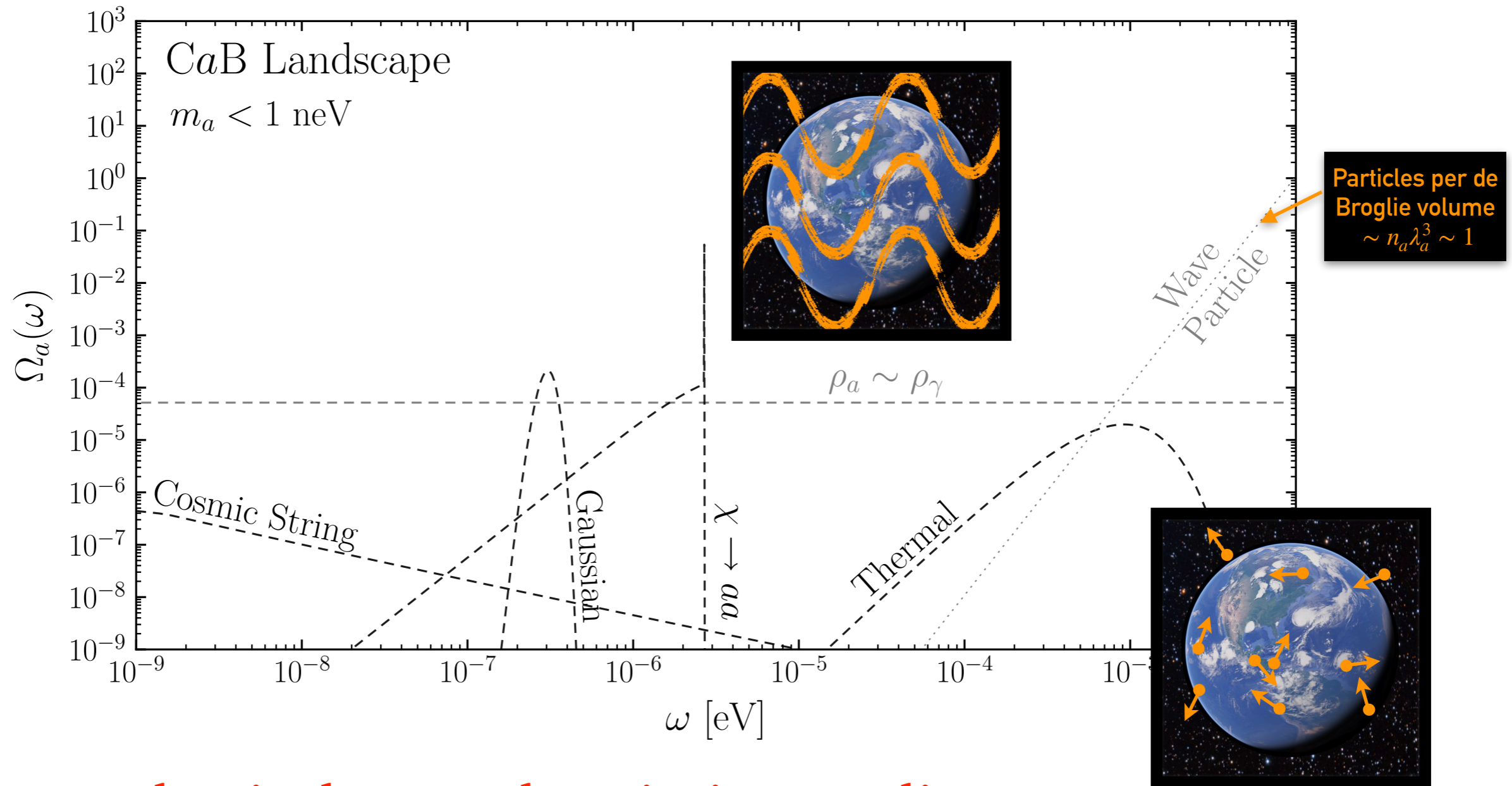
Additional Sources
 Moduli decay: [Conlon, Marsh 2013]
 Bosenova: [Eby+ 2022]



Landscape



Landscape



Classical wave description applies

Rough Sensitivity

$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Sampled from $p(\omega)$, the pdf of frequency/energy

Random phase

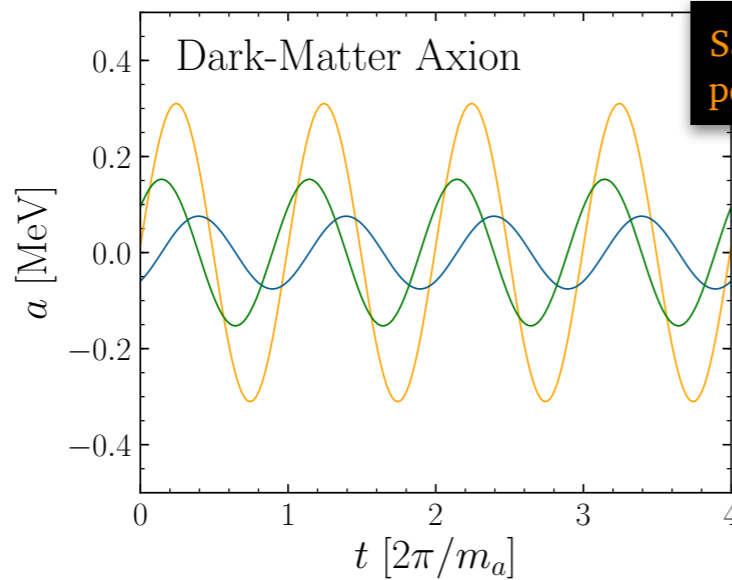
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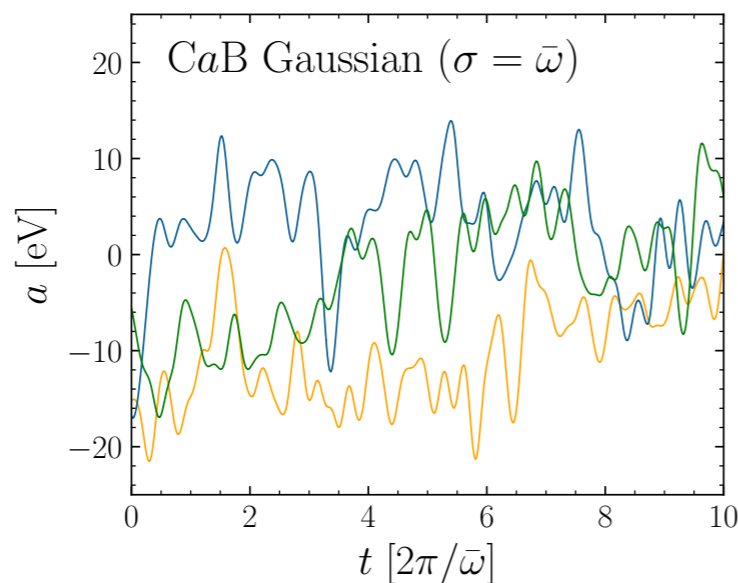
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Random phase

Dark Matter



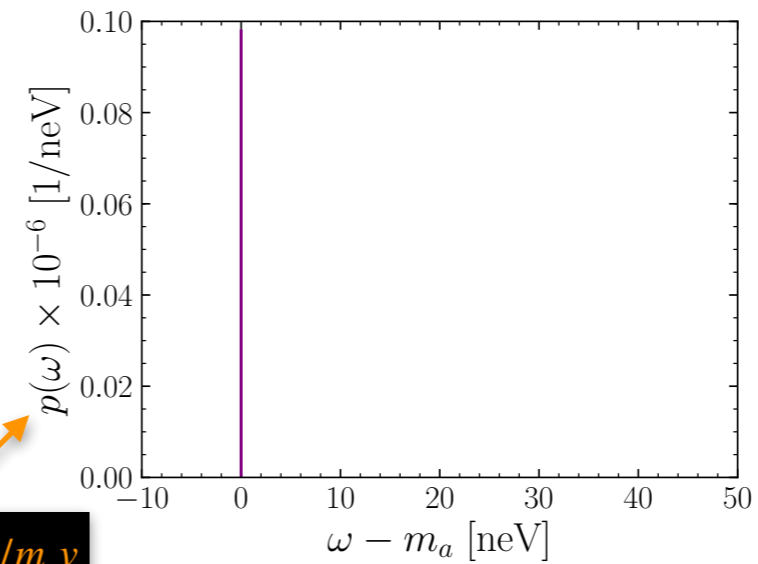
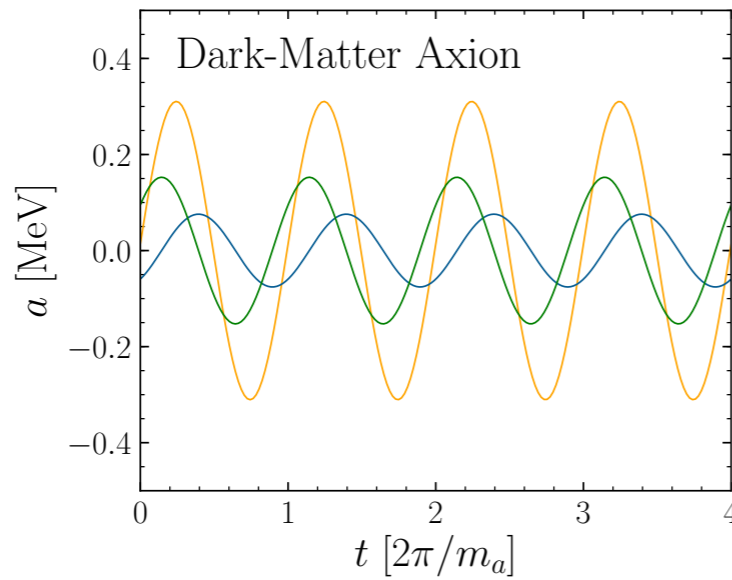
CaB



Rough Sensitivity

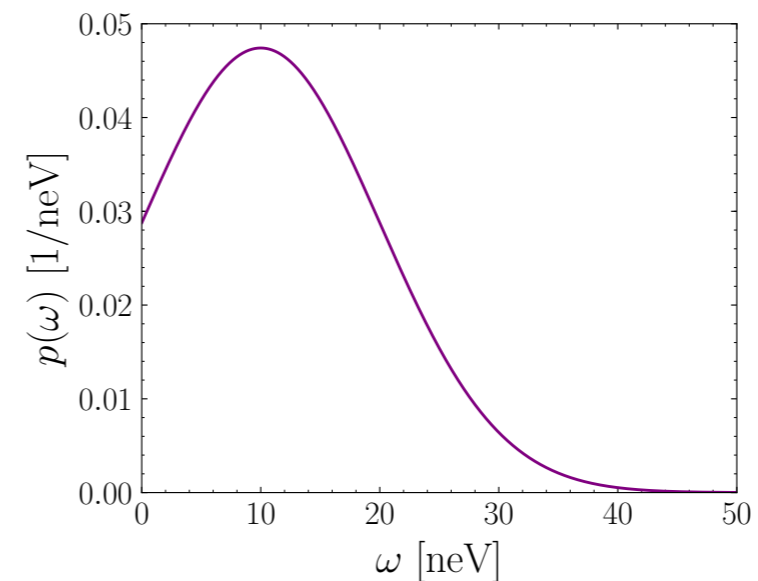
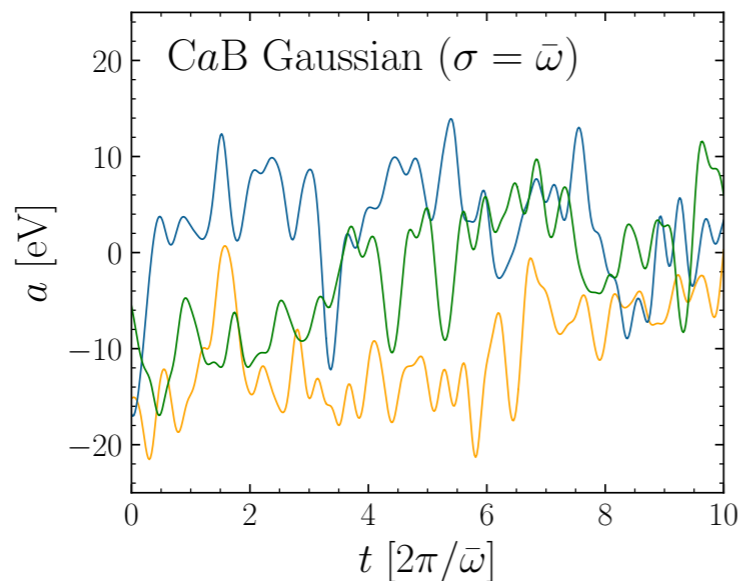
$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Dark Matter



$p(\omega) = f(v)/m_a v$

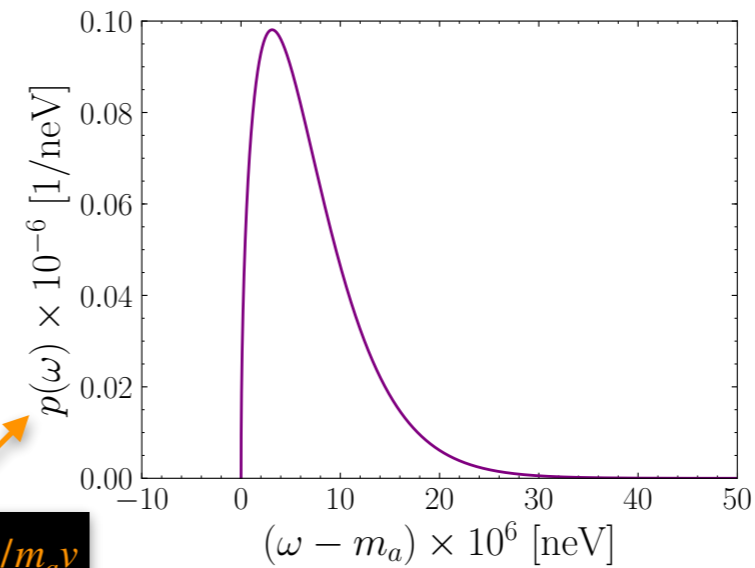
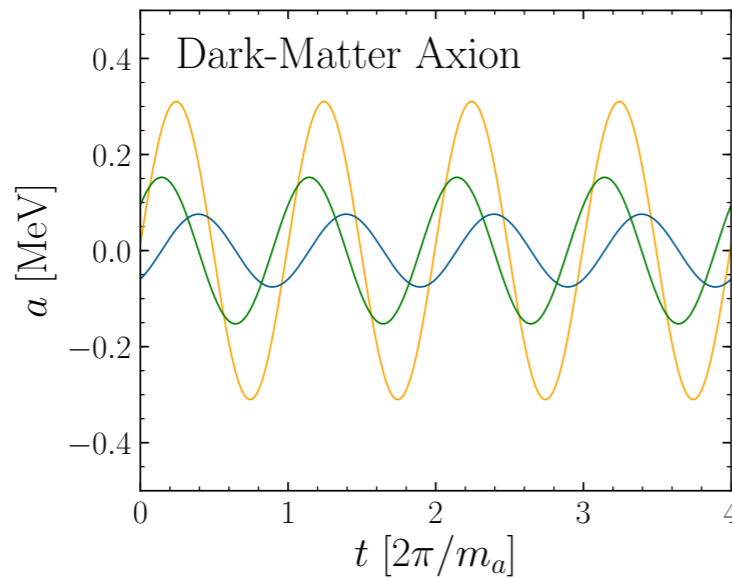
CaB



Rough Sensitivity

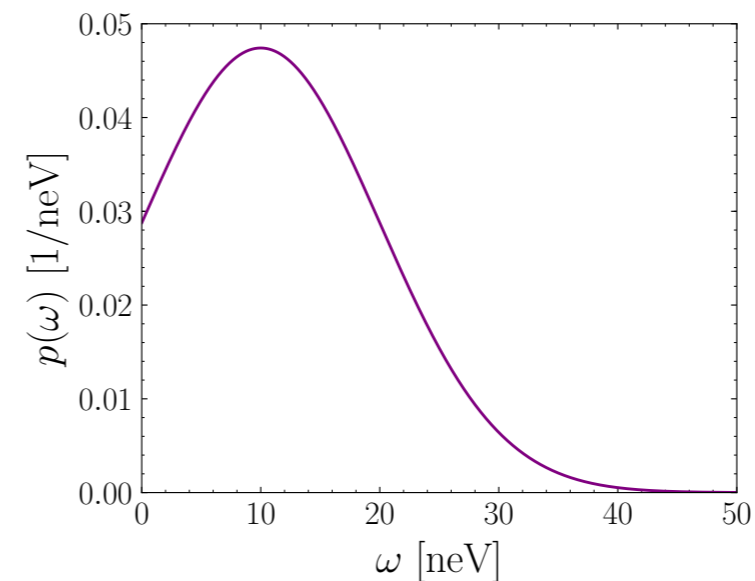
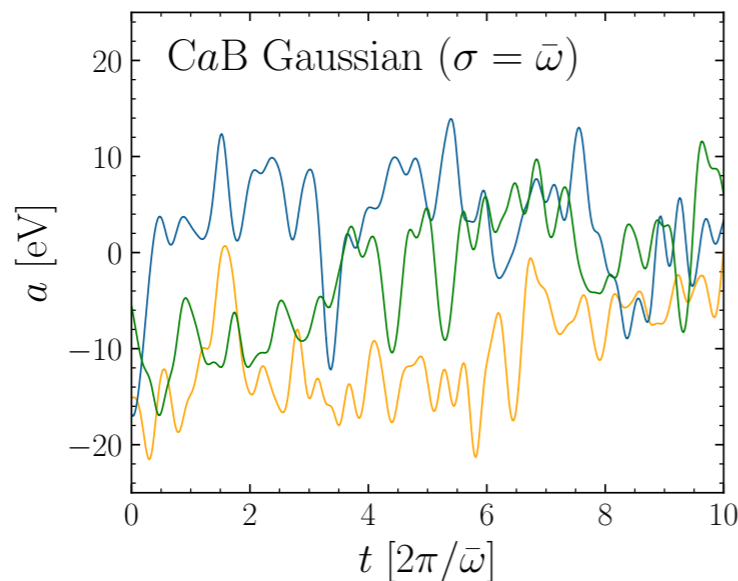
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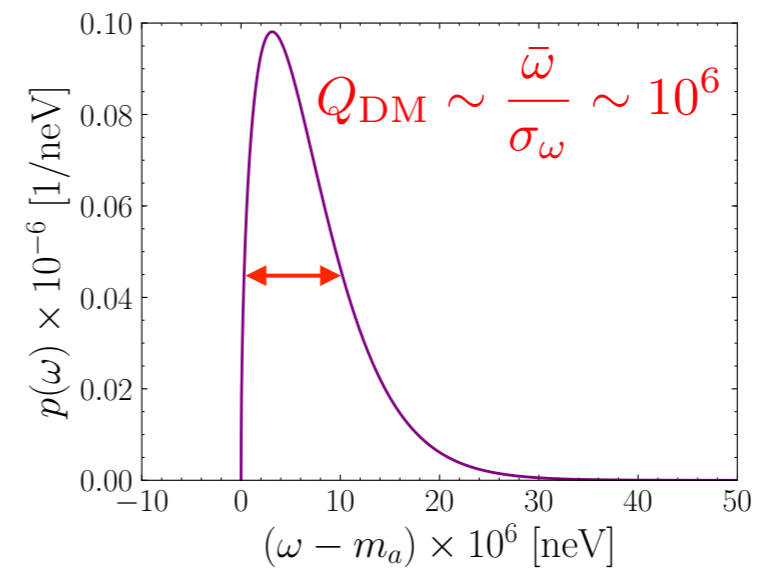
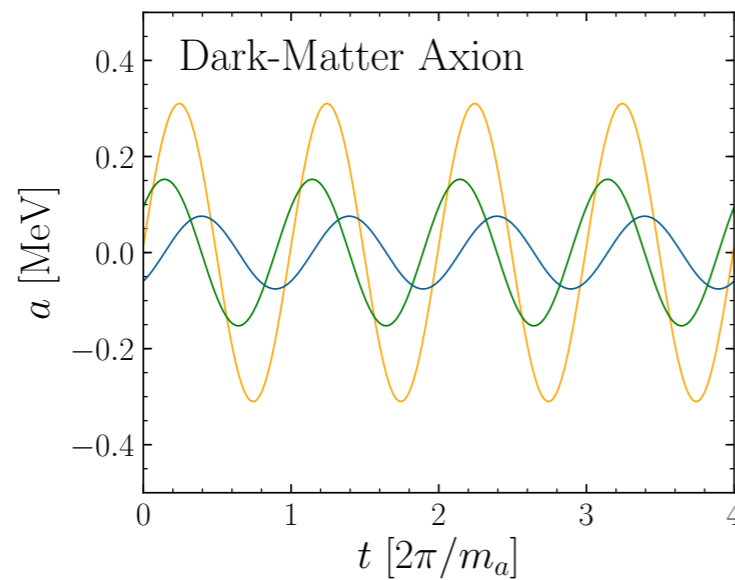
CaB



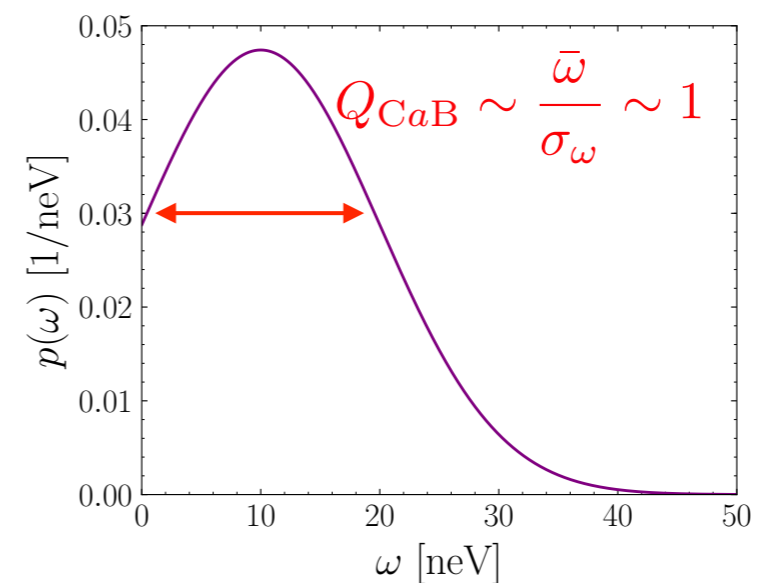
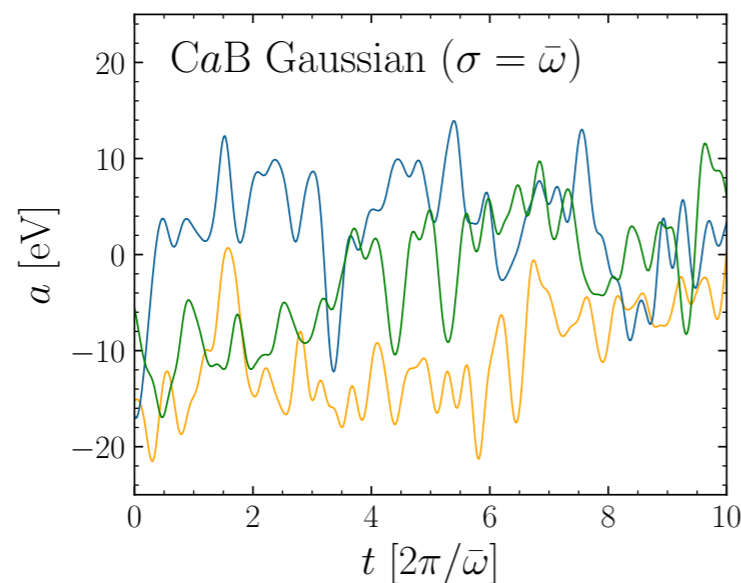
Rough Sensitivity

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Dark
Matter



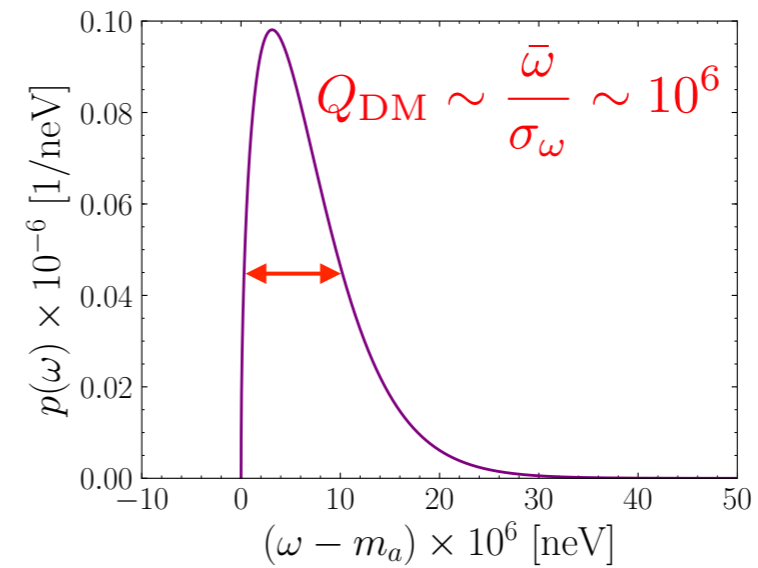
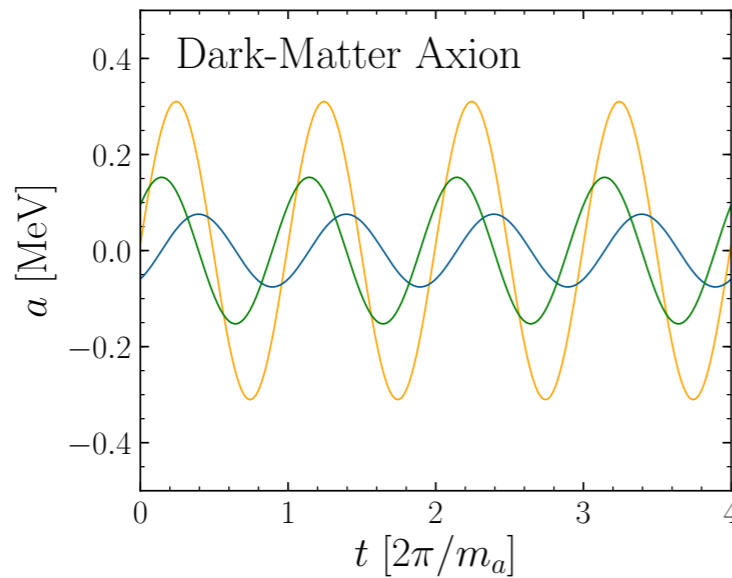
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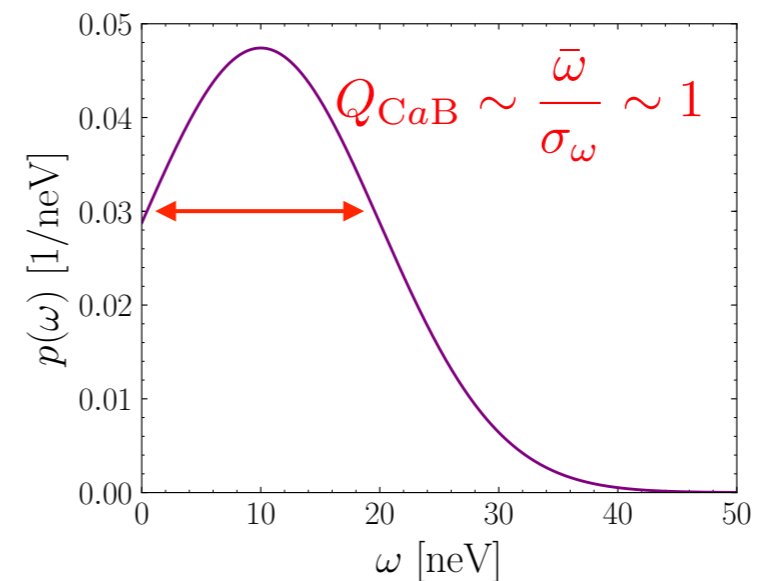
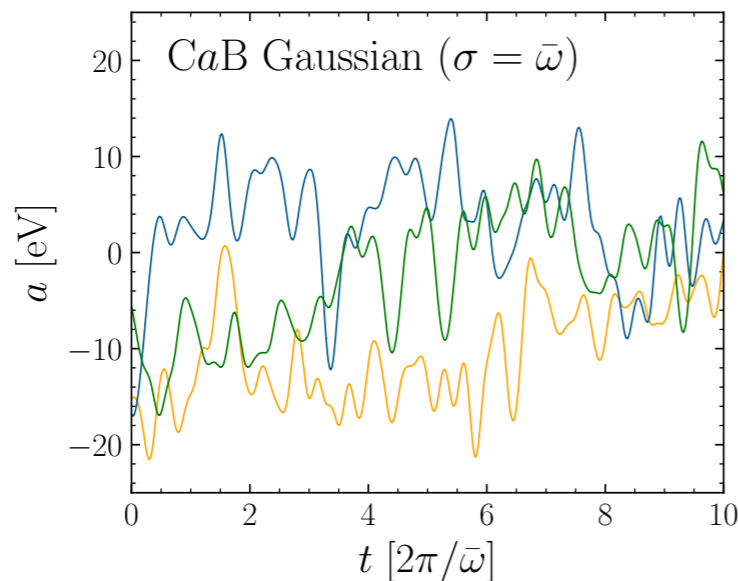
$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Dark Matter



Broad signal - existing searches remove as background
Resolved with ADMX

CaB



Rough Sensitivity

$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Accessible power in the axion field

Power spectral density -
measures power at a
given frequency

$$\langle S_{g\partial a}(\omega) \rangle = \pi g_{a\gamma\gamma}^2 \rho_a \frac{\omega}{\bar{\omega}} p(\omega)$$

Rough Sensitivity

$$a(t) \sim \sum_i \cos(\omega_i t + \phi_i)$$

Accessible power in the axion field

$$\langle S_{g\partial a}(\bar{\omega}) \rangle = \frac{\pi g_{a\gamma\gamma}^2 \rho_a Q_a}{\bar{\omega}}$$

Power spectral density -
measures power at a
given frequency

Approximate $p(\omega) \sim Q_a / \bar{\omega}$

Recall $Q_a \sim \bar{\omega} / \sigma_\omega$

Rough Sensitivity

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Approximate $p(\omega) \sim Q_a / \bar{\omega}$

Recall $Q_a \sim \bar{\omega} / \sigma_\omega$

Estimate sensitivity by matching power $P_{\text{DM}} = P_{\text{CaB}}$

Rough Sensitivity

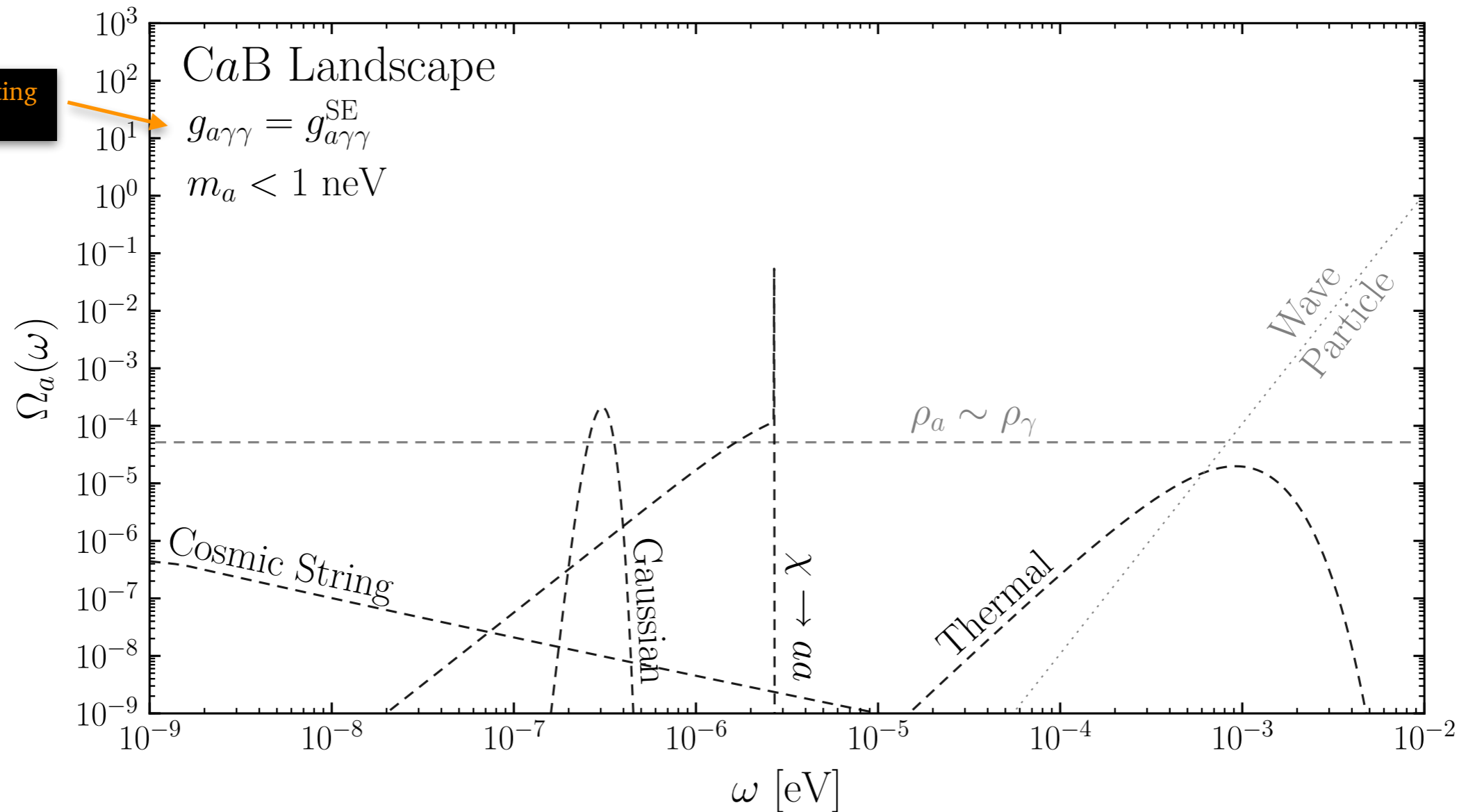
$$\rho_a = \rho_{\text{DM}} \left(\frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}^{\text{SE}}} \right)^2 \sqrt{\frac{Q_{\text{DM}}}{Q_{\text{CaB}}}}$$

Lose: $\rho_a \ll \rho_{\text{DM}}$ Win: $g_{a\gamma\gamma}^{\text{DM}} \ll g_{a\gamma\gamma}^{\text{SE}}$ Lose: $Q_{\text{CaB}} \ll Q_{\text{DM}}$

Parametric scaling confirmed by detailed calculations for both resonant and broadband instruments

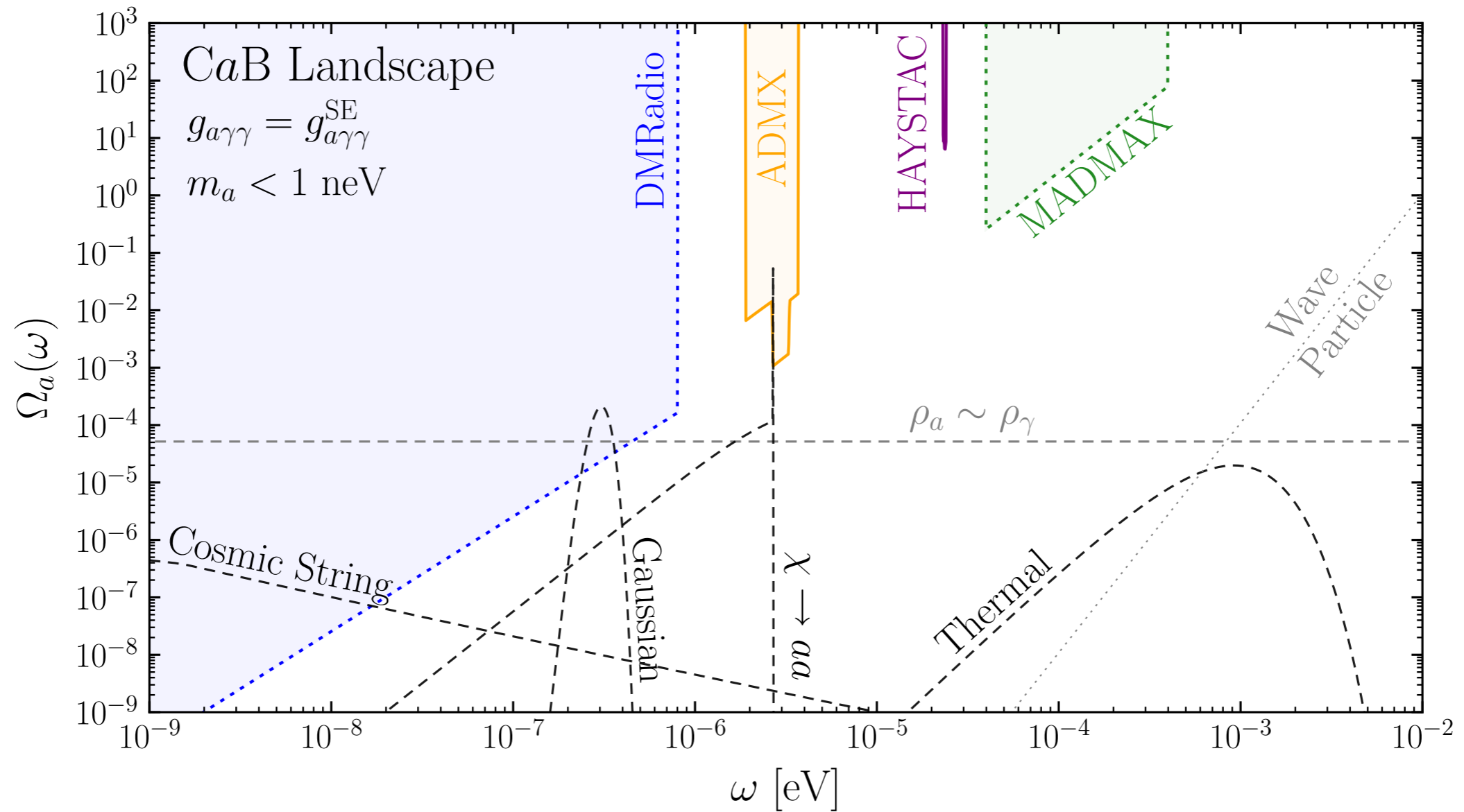
Detection requires enhanced $g_{a\gamma\gamma}$ e.g.
 [Choi, Im 2015], [Farina+ 2016],
 [Agrawal+ 2017], [Dror, Leedom 2020]

Experimental Landscape

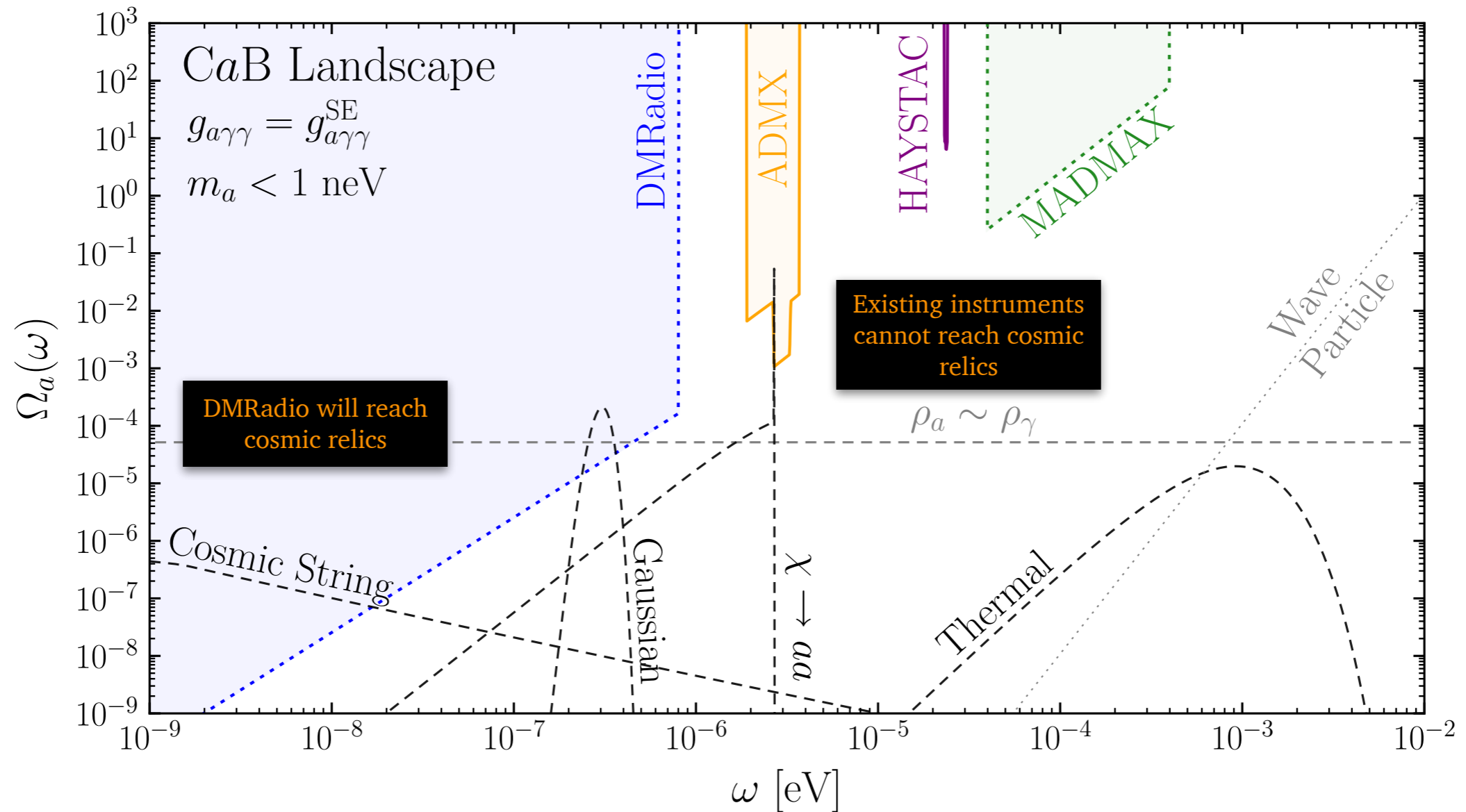


Saturate existing bounds

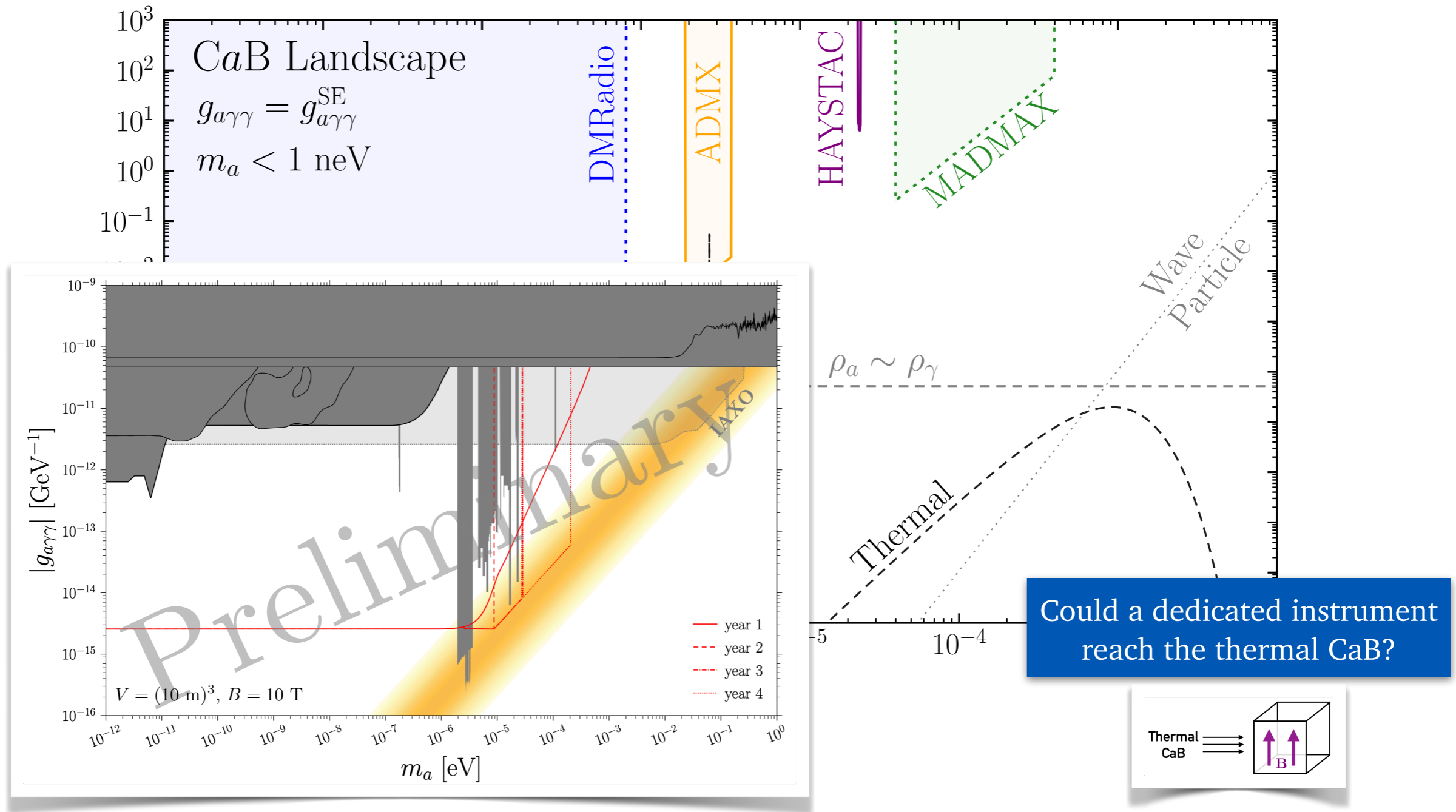
Experimental Landscape



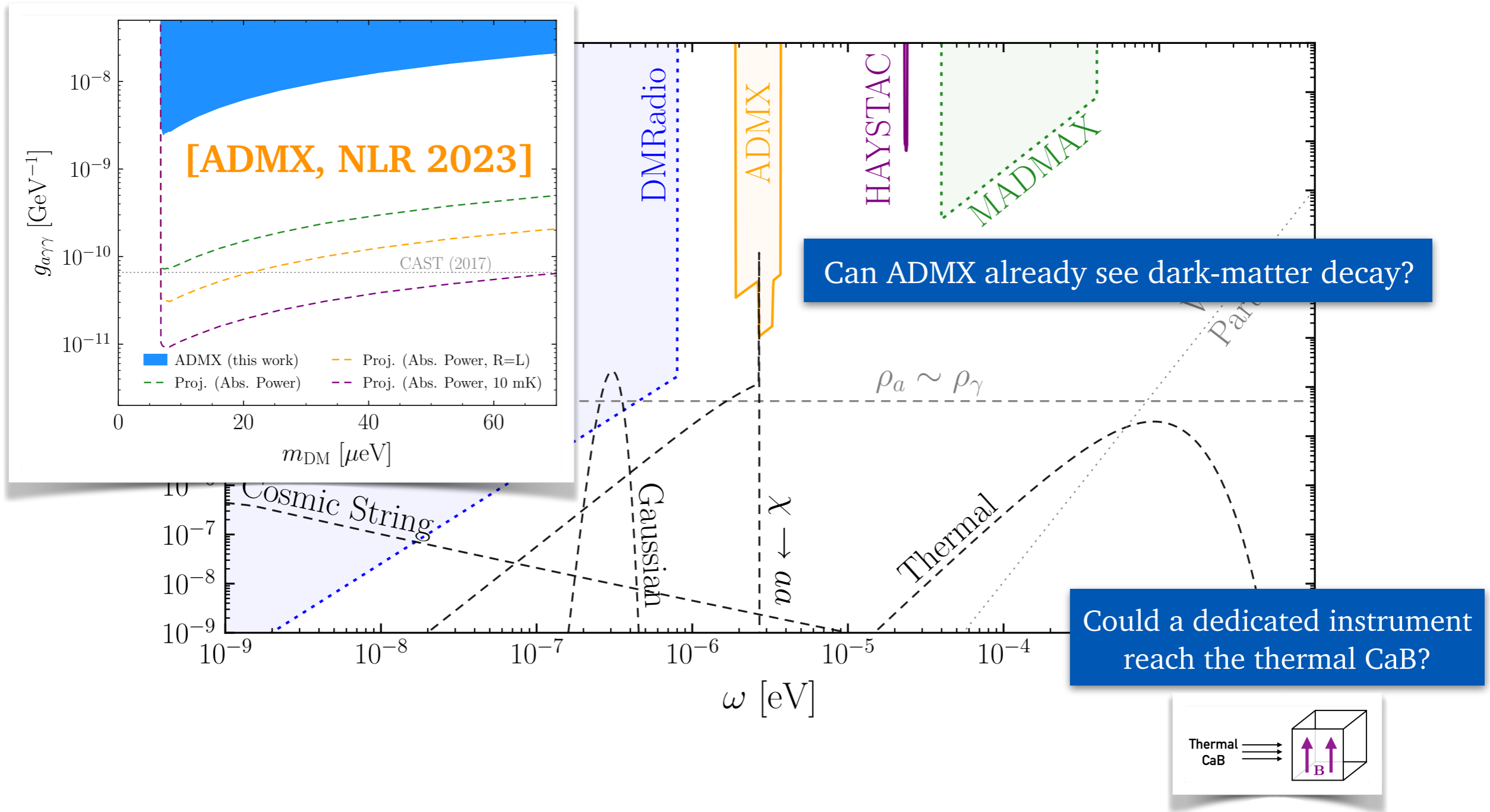
Experimental Landscape



Future Directions

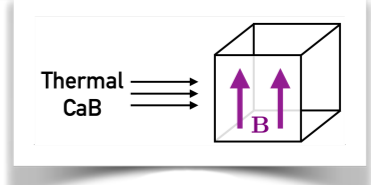
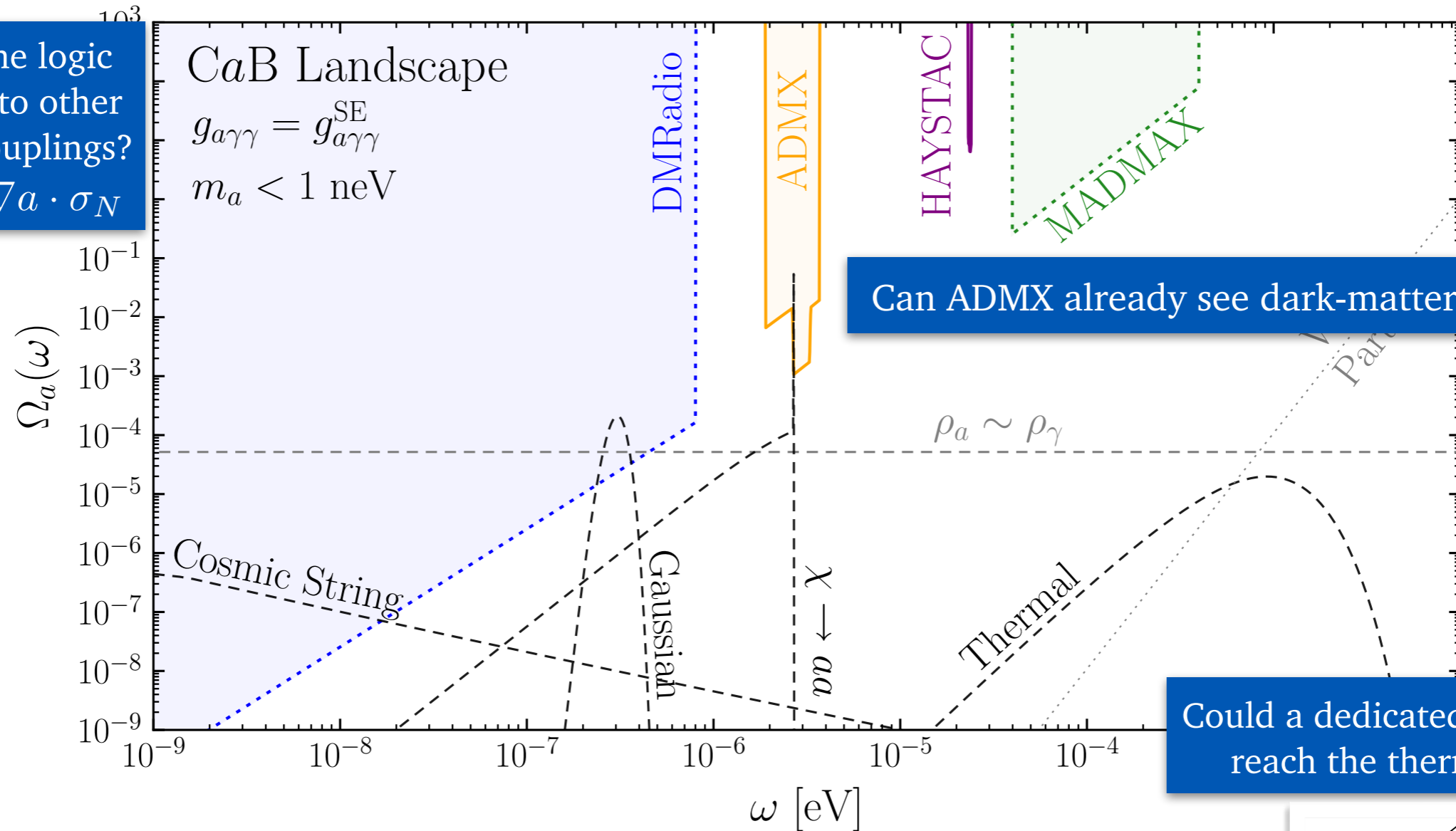


Future Directions



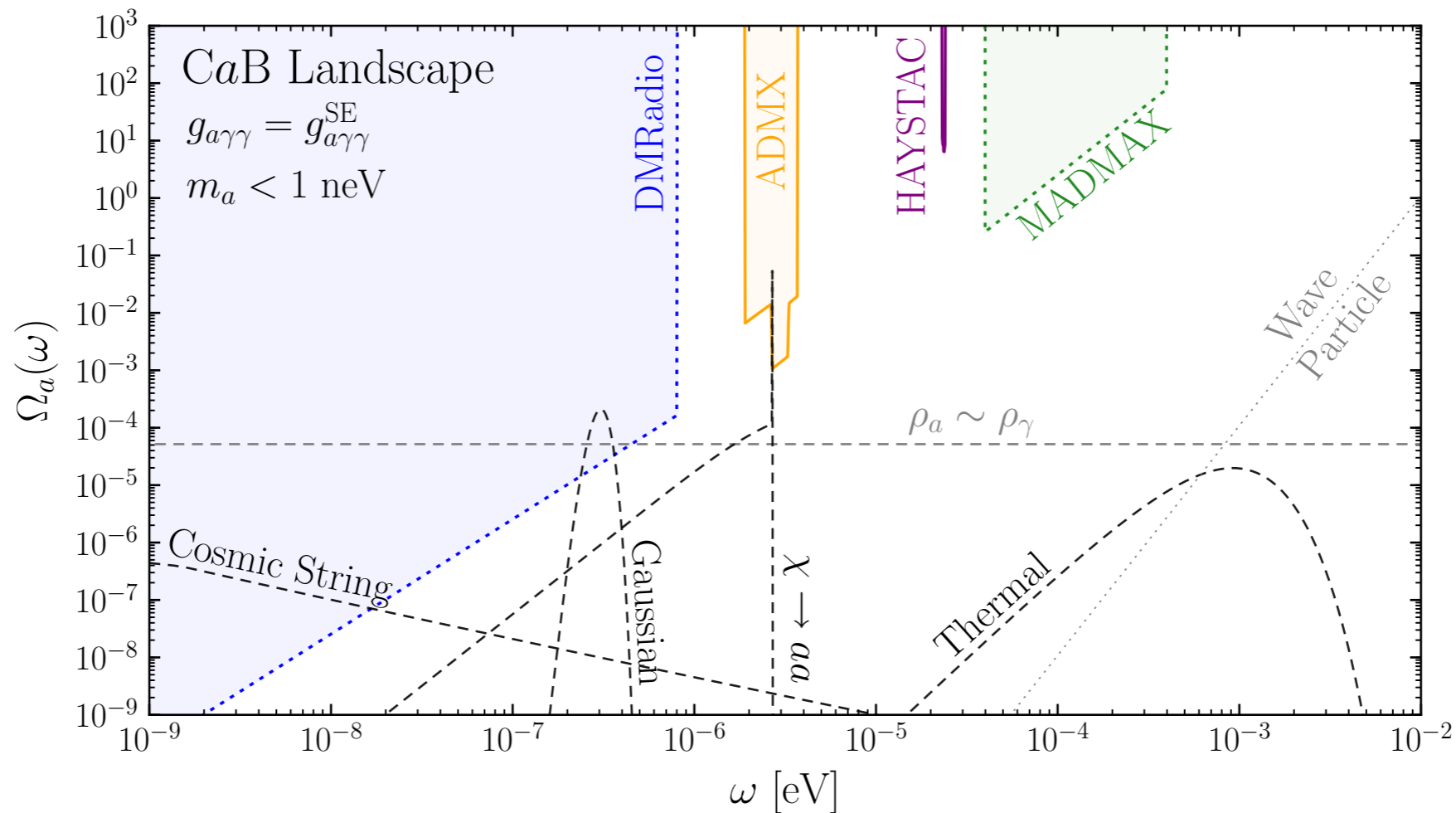
Future Directions

Does the logic extend to other axion couplings?
 $g_{aNN} \nabla a \cdot \sigma_N$



Summary

The Cosmic Axion Background could emerge as we drive towards dark matter



High-Frequency Gravitational Waves

[Domcke, Garcia-Cely, NLR PRL 2022]

High-Frequency Gravitational Waves

Is an axion haloscope also a gravitational-wave telescope?



Gravitational Wave Electrodynamics

$$S \supset \int d^4x \sqrt{-g} \left(-\frac{1}{4} F^2 \right)$$

Work with a linearized metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

Induces terms of the schematic form

$$hF^2$$

[Gertsenshtein 1962]
[Boccaletti+ 1970]
[Raffelt, Stodolsky 1988]

Gravitational Wave Electrodynamics

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Induces terms of the schematic form

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

[Gertsenshtein 1962]
[Boccaletti+ 1970]
[Raffelt, Stodolsky 1988]

Clear analogy with
axion electrodynamics

Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]



Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a$$

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]

Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_a f_a}$$

See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]

Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_a f_a} \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_\pi f_\pi}$$

QCD Axion


See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]

Gravitational Wave Electrodynamics

$$hF^2 \leftrightarrow g_{a\gamma\gamma} a F \tilde{F}$$

0th order estimate by pushing this analogy

$$h \sim g_{a\gamma\gamma} a \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_a f_a} \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_\pi f_\pi} \sim 10^{-22}$$



See also [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]

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Far from bound on cosmological sources

$$h \lesssim 10^{-27} \left(\frac{1 \text{ MHz}}{f} \right) \Delta N_{\text{eff}}^{1/2}$$

Only sensitive to late time sources
e.g. PBH Binary signal within reach

See also [Berlin, Blas, Tito D'Agnolo, Ellis,
Harnik, Kahn, Schutte-Engel 2021]

Gravitational Wave Electrodynamics

As for the axion, induce new E&M sources

$$\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P}$$

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \nabla \times \mathbf{M} + \partial_t \mathbf{P}$$

although with a more complicated form

$$P_i = -h_{ij} E_j + \frac{1}{2} h E_i + h_{00} E_i - \epsilon_{ijk} h_{0j} B_k$$

$$M_i = -h_{ij} B_j - \frac{1}{2} h B_i + h_{jj} B_i + \epsilon_{ijk} h_{0j} E_k$$

Axion equivalent

$$\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$$

Gravitational Wave Electrodynamics

As for the axion, induce new E&M sources

$$\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P}$$

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$$M_i = -h_{ij} B_j - \frac{1}{2} h B_i + h_{jj} B_i + \epsilon_{ijk} h_{0j} E_k$$

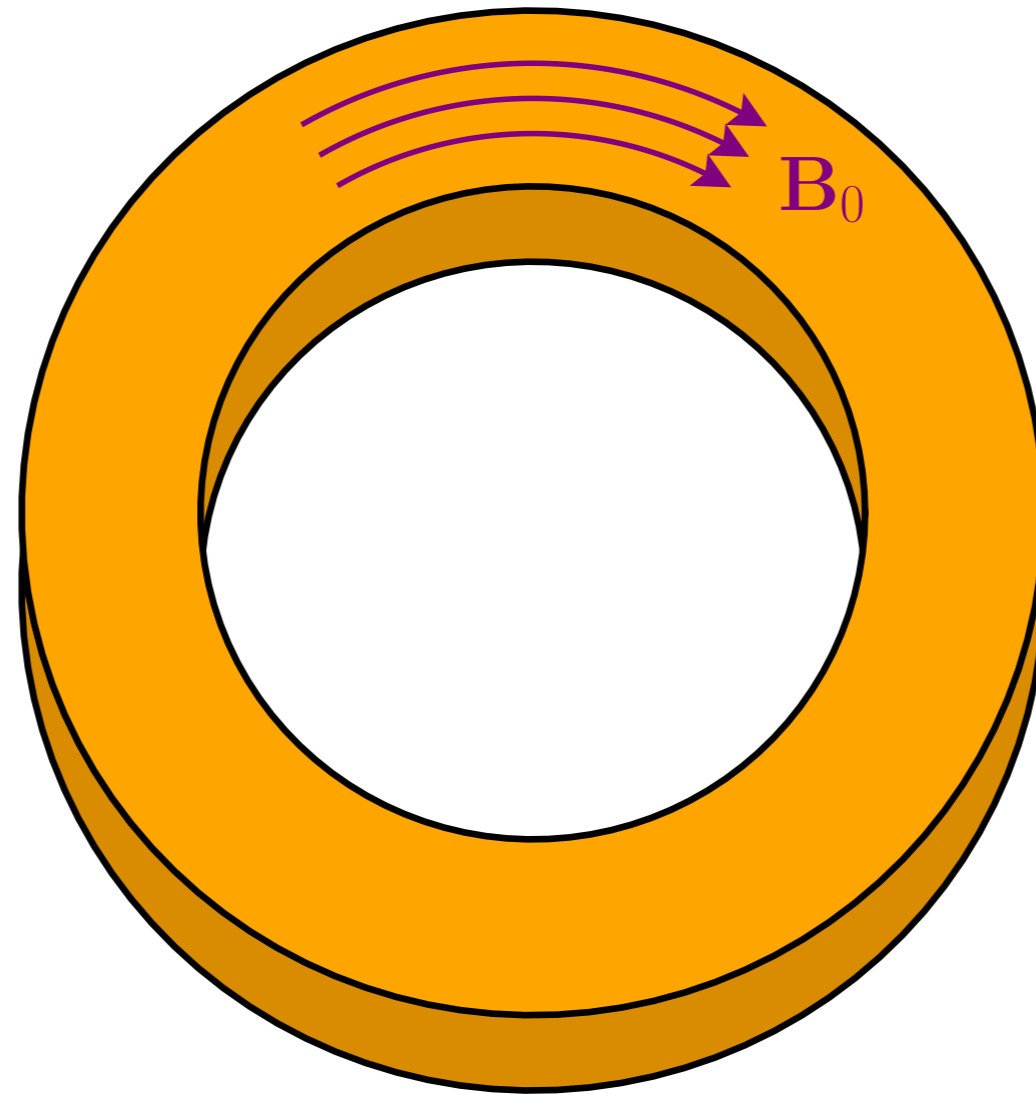
Following [Berlin+ 2021] work in proper detector frame, where

Axion equivalent
 $\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$

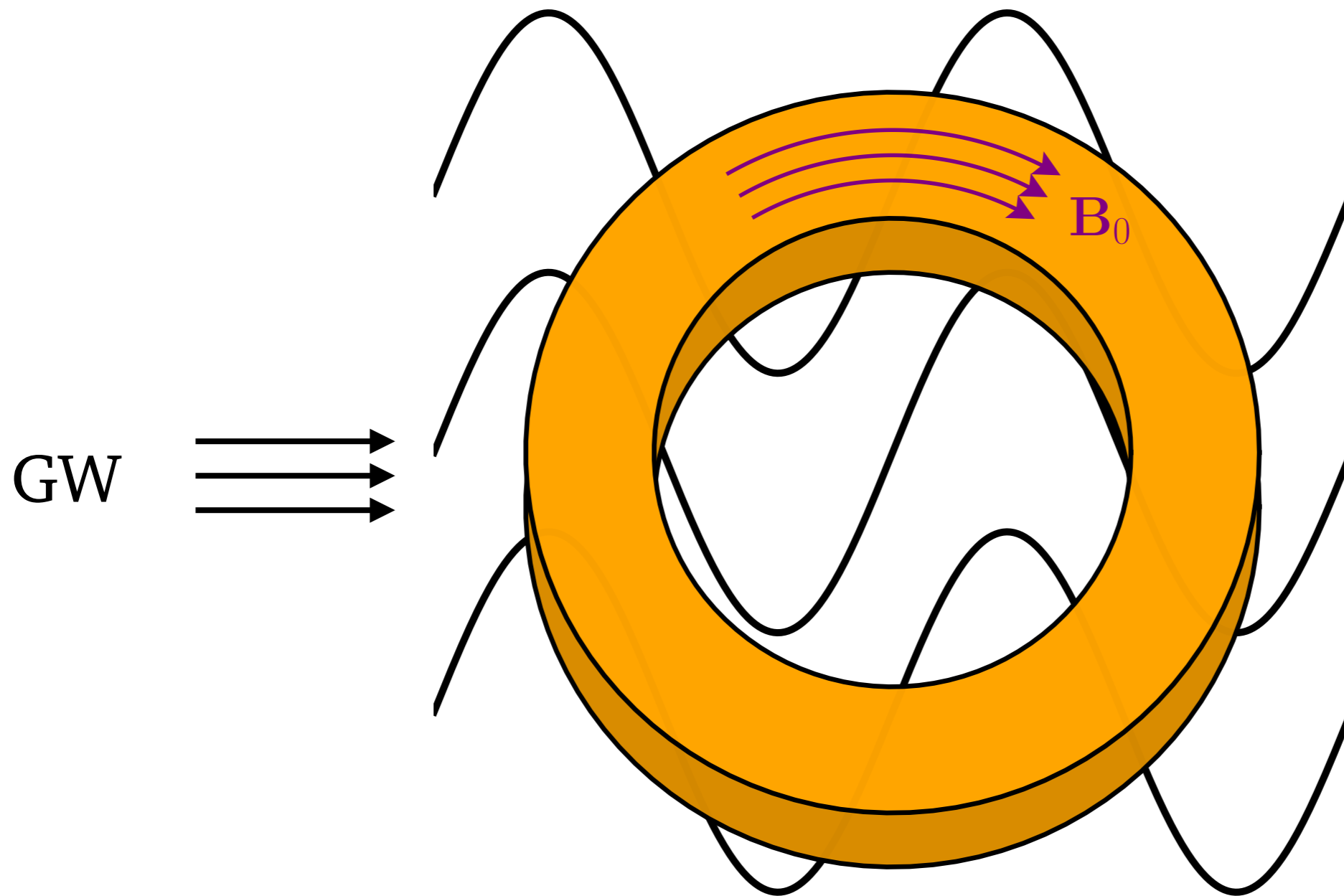
$$h \sim \omega^2 + \mathcal{O}(\omega^3)$$

See also [Fortini and Gualdi 1982],
 [Marzlin 1994], [Rakhmanov 2014]

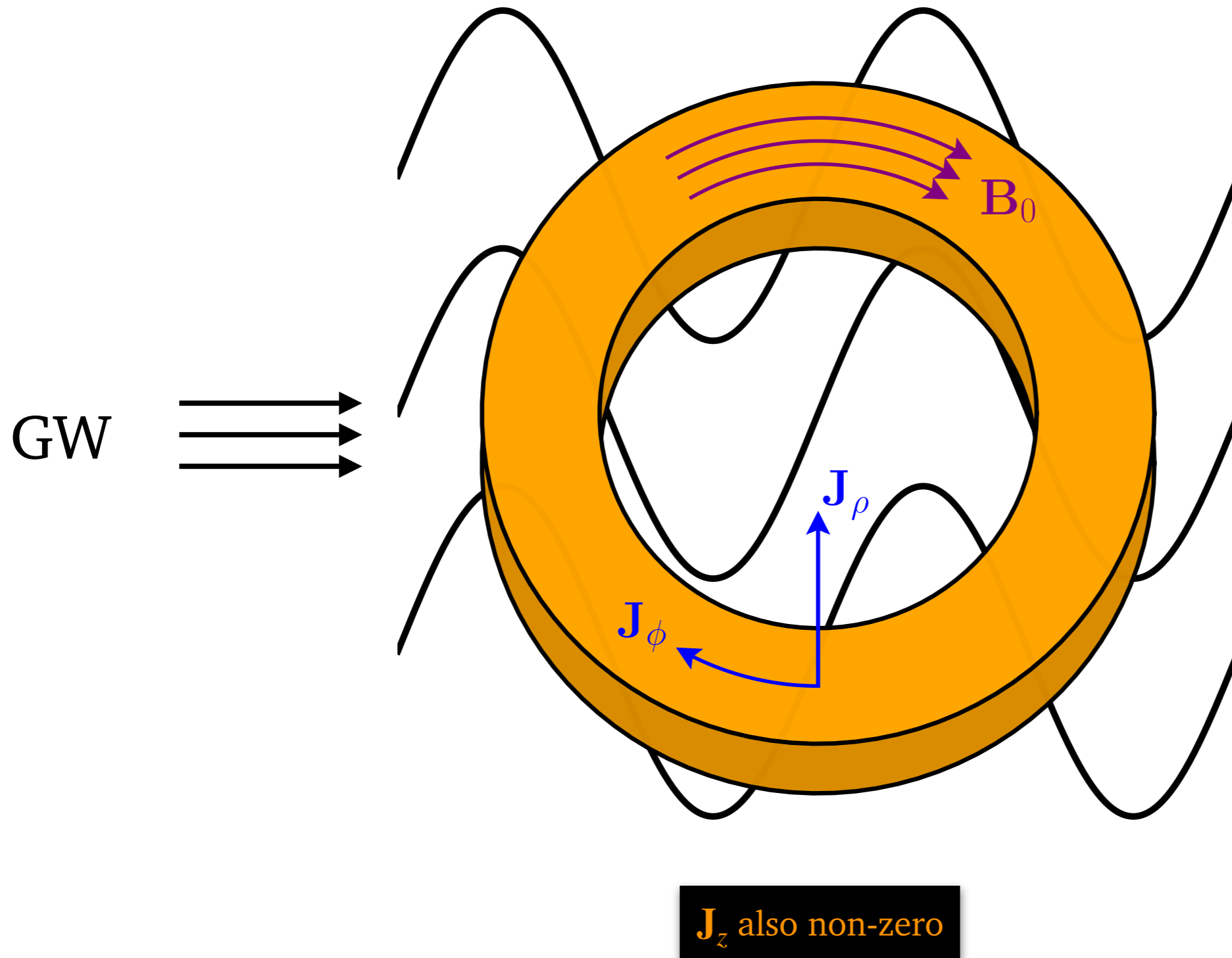
Detection Strategy



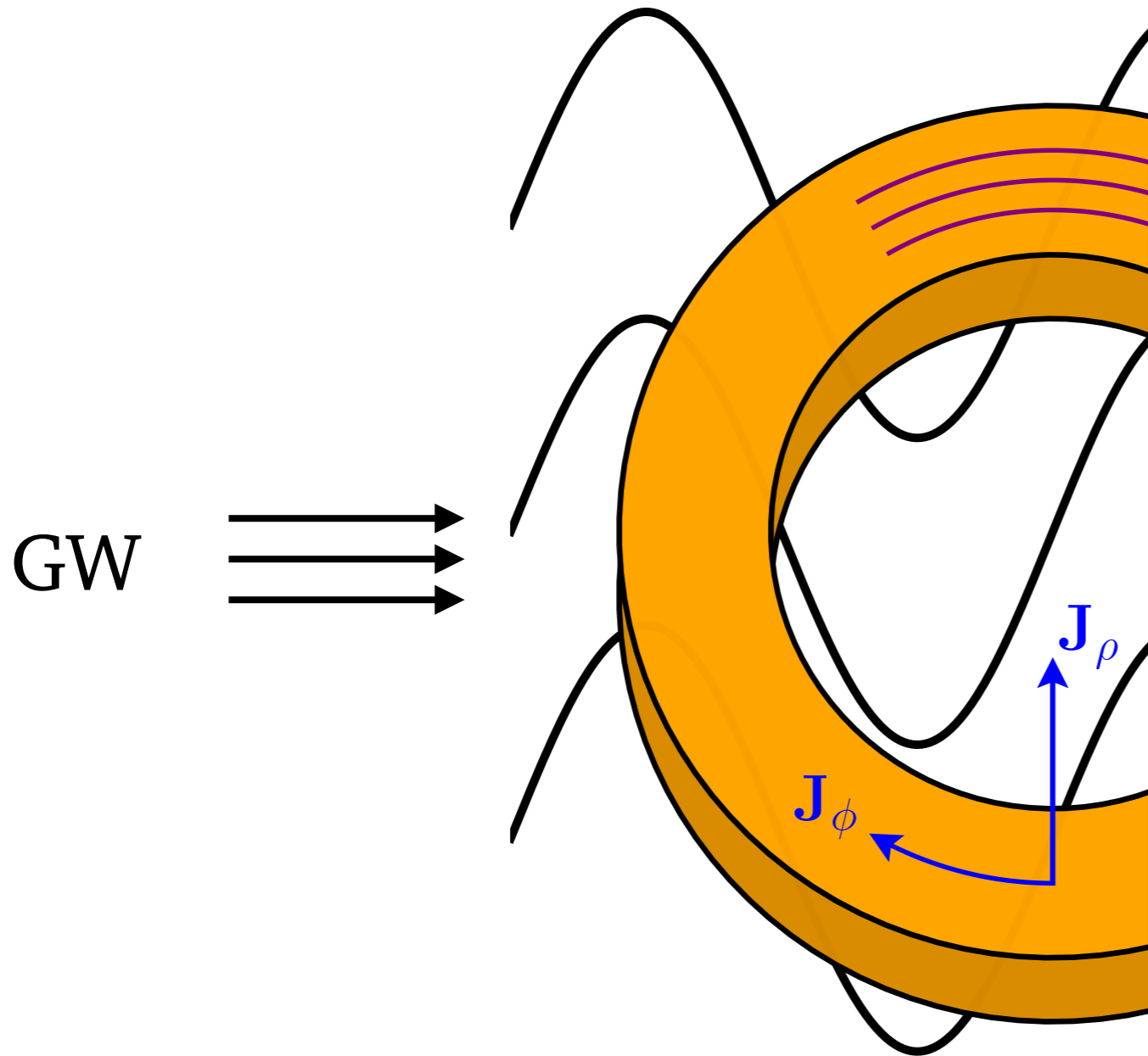
Detection Strategy



Detection Strategy



Detection Strategy



$$j_\phi = \frac{\omega^2 B_{\max} R}{\rho} \left[\frac{e^\kappa}{\kappa} - \frac{2e^\kappa}{\kappa^2} + \frac{2(e^\kappa - 1)}{\kappa^3} \right] (z h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} - \rho h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0}),$$

$$j_\rho = \frac{\omega^2 B_{\max} R}{\rho} \left(\left[-\frac{1}{2} - \frac{1}{\kappa} + \frac{2e^\kappa}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (\rho h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} + z h_{z z}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + \left[\frac{e^\kappa}{\kappa} + \frac{2}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (z h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} + z h_{z z}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + i k_z \left[\frac{1}{2\kappa} + \frac{1}{2\kappa^2} - \frac{1 + 2e^\kappa}{\kappa^3} + \frac{3(e^\kappa - 1)}{\kappa^4} \right] r_i r_j h_{ij}^{\text{TT}}|_{\mathbf{r}=0} \right),$$

$\kappa = i\mathbf{k} \cdot \mathbf{r}$

$$h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left(-h^+ (\sin^2(\phi - \phi_h) - \cos^2(\phi - \phi_h) \cos^2 \theta_h) + 2h^\times \cos \theta_h \cos(\phi - \phi_h) \sin(\phi - \phi_h) \right),$$

$$h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left(-h^+ (1 + \cos^2 \theta_h) \sin(\phi - \phi_h) \cos(\phi - \phi_h) + h^\times \cos(2(\phi - \phi_h)) \cos \theta_h \right),$$

$$h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} = -\frac{e^{-i\omega t}}{\sqrt{2}} \left(h^+ \cos \theta_h \sin \theta_h \cos(\phi - \phi_h) + h^\times \sin \theta_h \sin(\phi - \phi_h) \right),$$

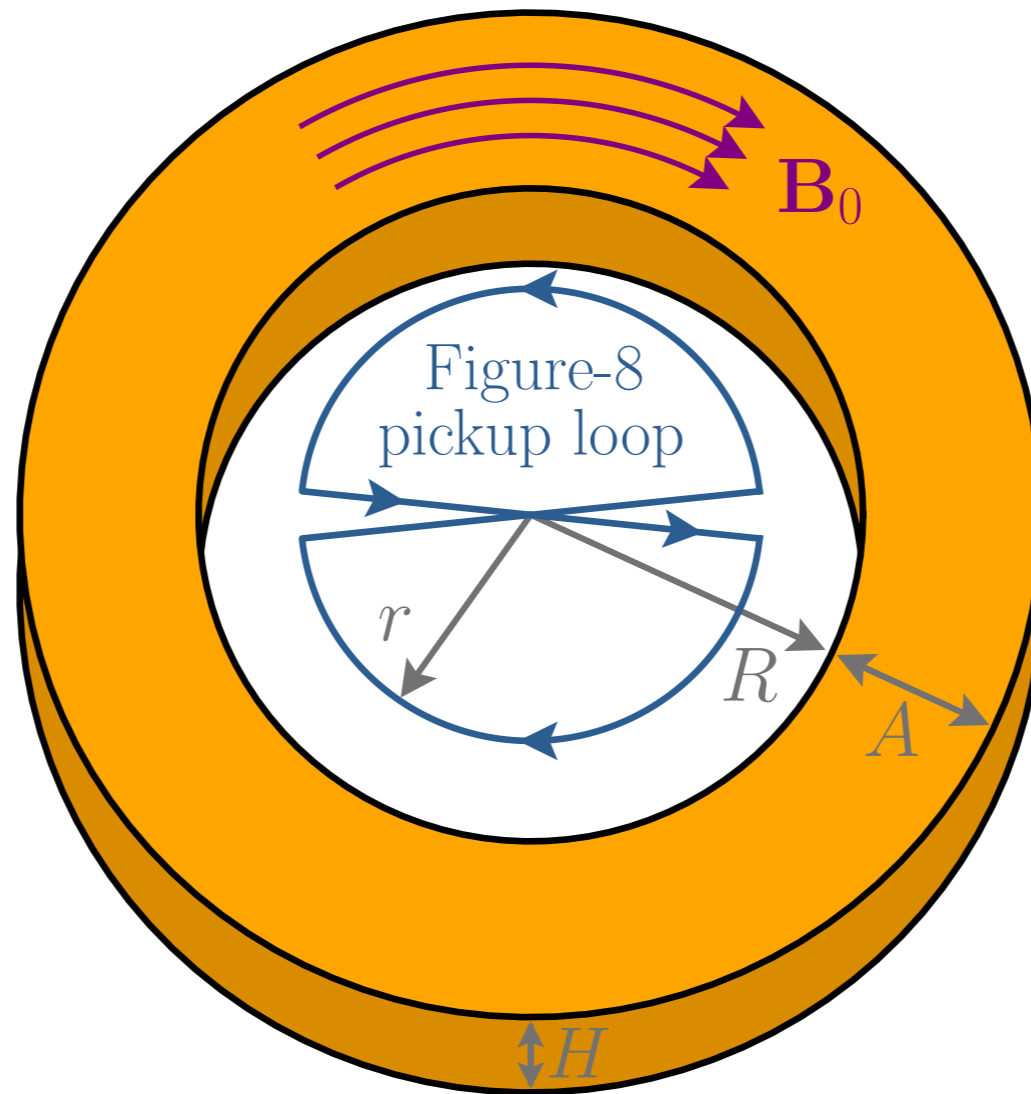
$$h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left(h^+ \cos \theta_h \sin \theta_h \sin(\phi - \phi_h) - h^\times \sin \theta_h \cos(\phi - \phi_h) \right),$$

$$h_{z z}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} h^+ \sin^2 \theta_h.$$

Cf. axion: $\mathbf{J}_a = g_{a\gamma\gamma}(\partial_t a)\mathbf{B}_0$

\mathbf{J}_z also non-zero

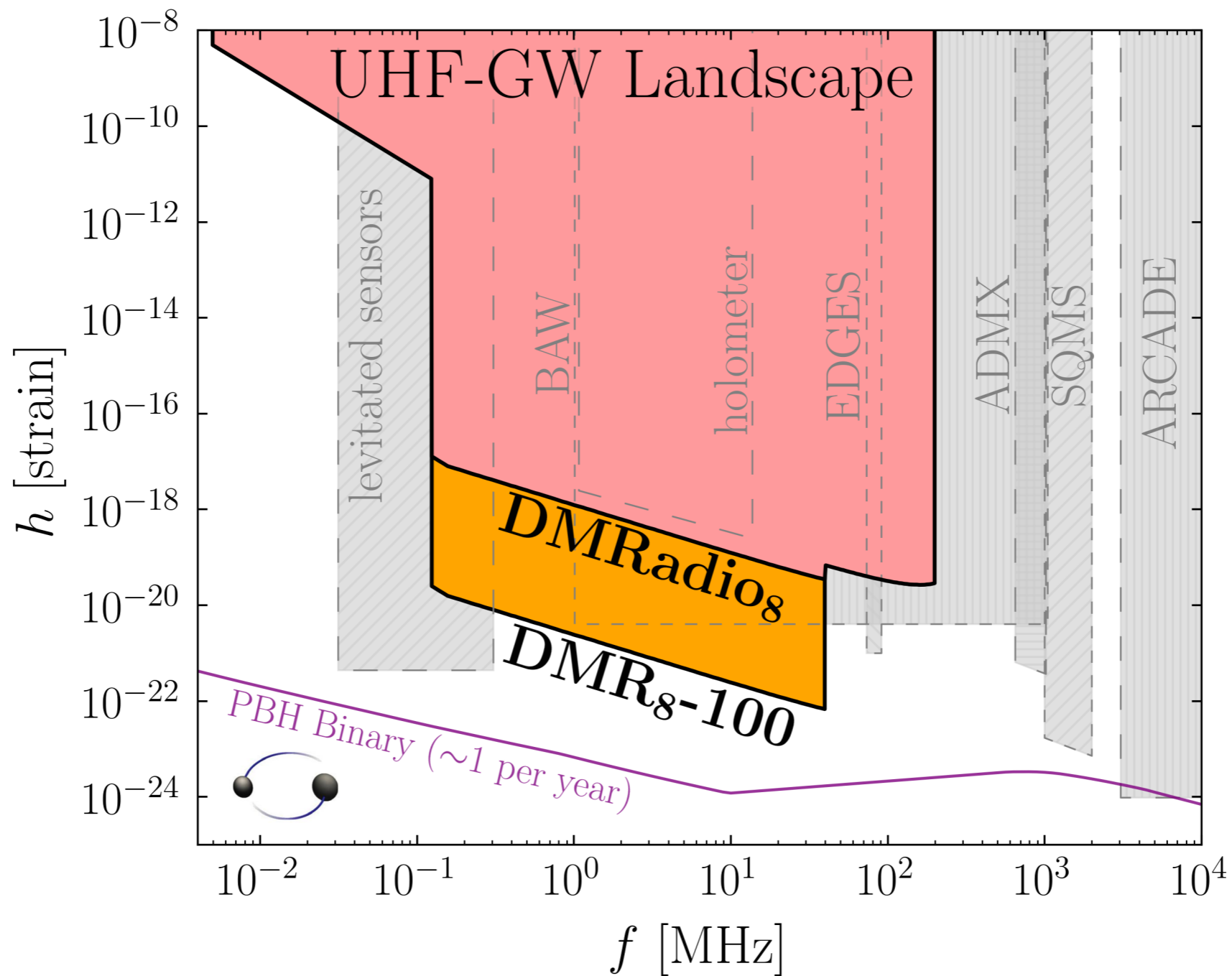
Optimized Reach



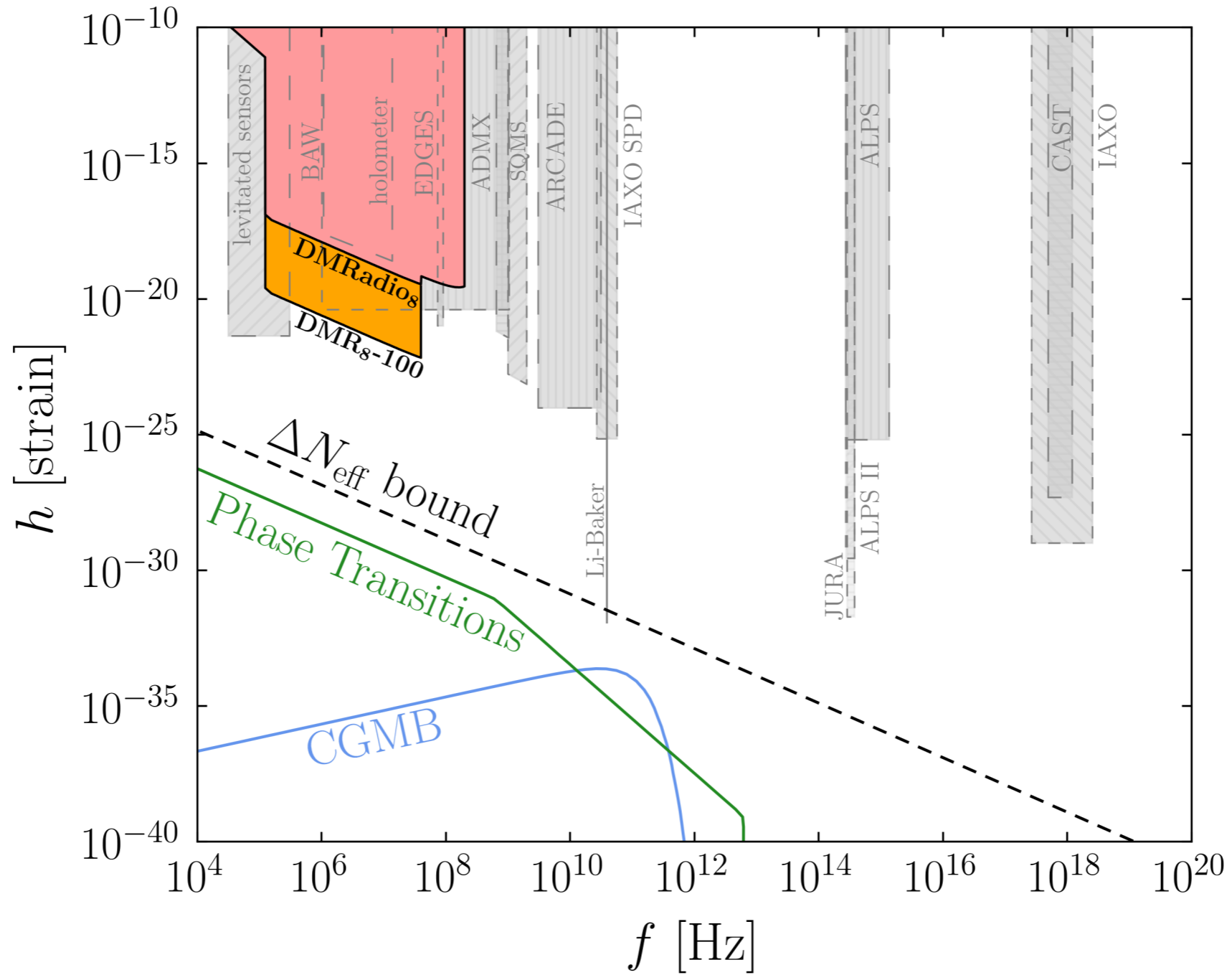
Circular loop is not optimal
Why? Symmetry implies
selection rules for GW
[Domcke, Garcia-Cely, Lee,
NLR 2306.03125]

$$\Phi_h(t) \simeq \frac{e^{-i\omega t}}{3\sqrt{2}} \omega^2 B_0 r^3 R \ln(1 + A/R) s_{\theta_h} (h^\times s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h}) \sim \omega^2 h B_0 V^{4/3}$$

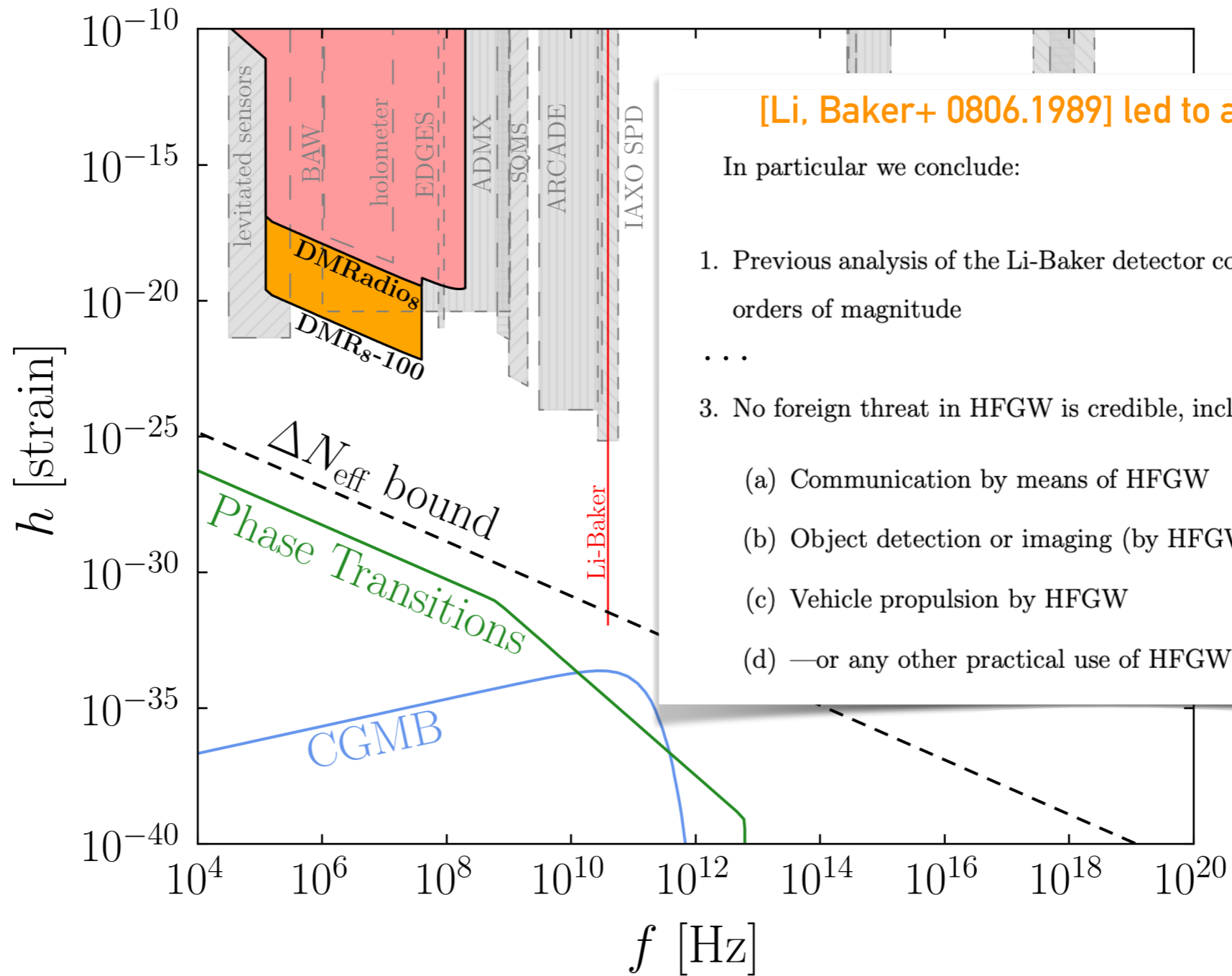
Optimized Reach



Future Directions



Future Directions

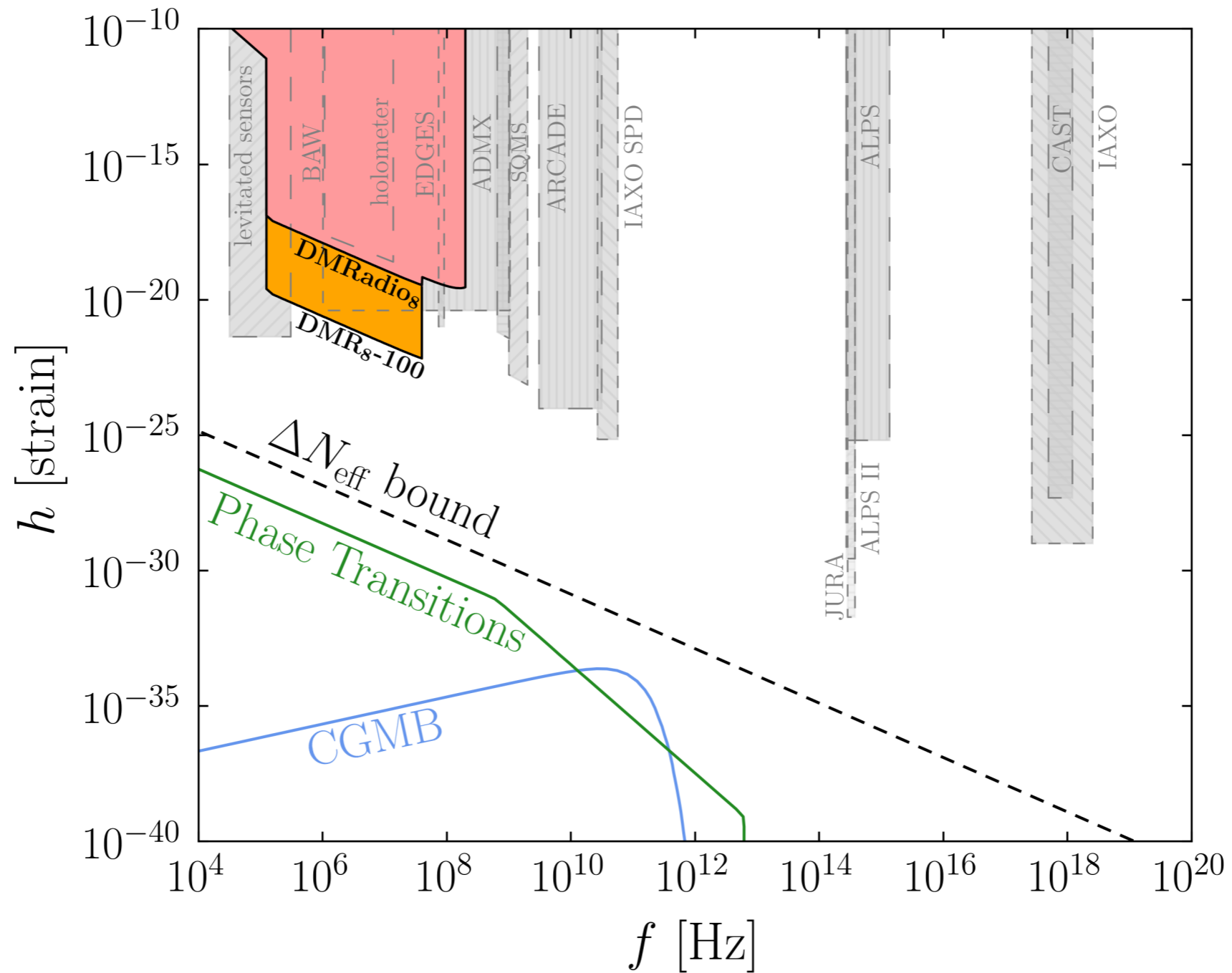


[Li, Baker+ 0806.1989] led to a JASON report

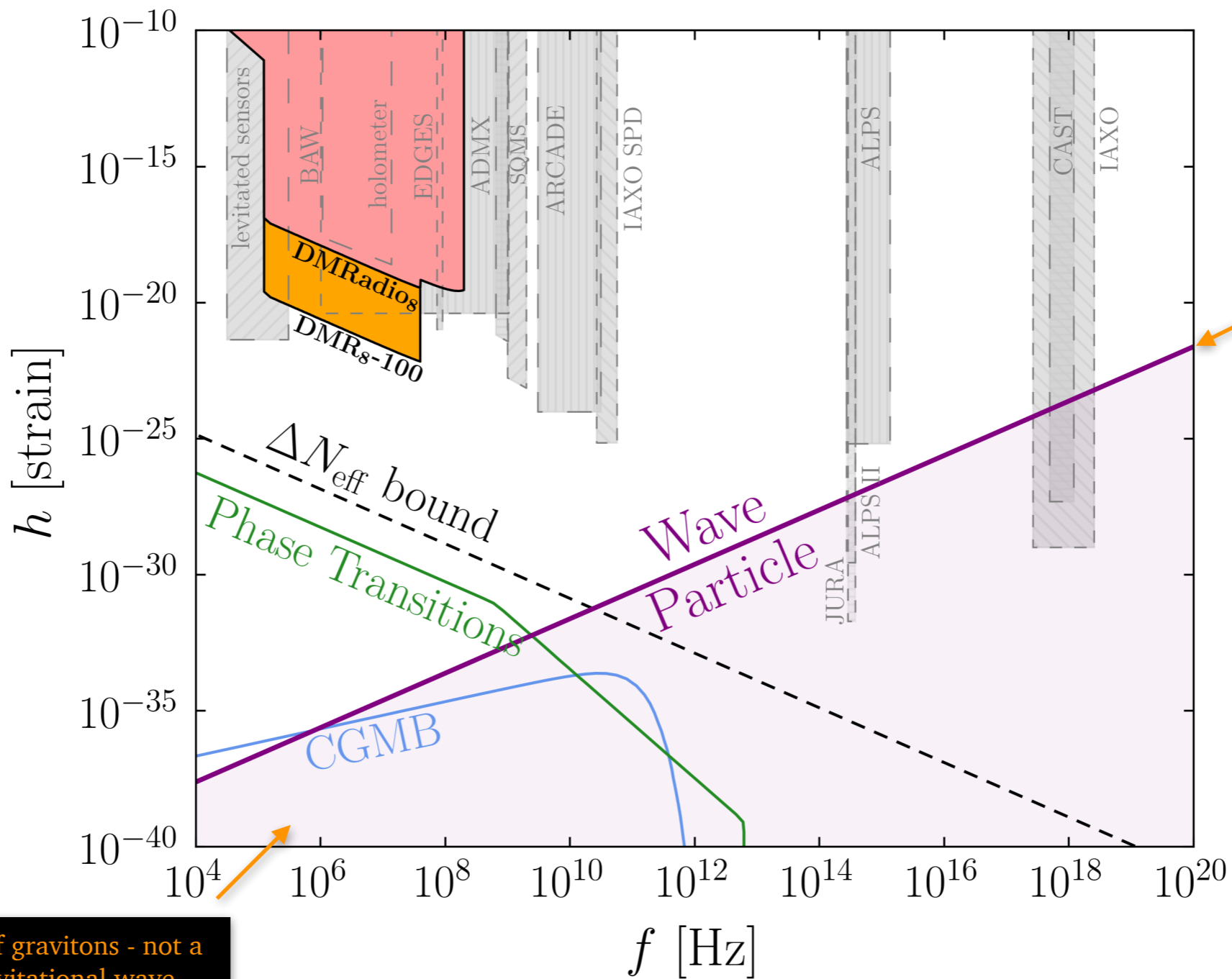
In particular we conclude:

1. Previous analysis of the Li-Baker detector concept is incorrect by many orders of magnitude
- ...
3. No foreign threat in HFGW is credible, including:
 - (a) Communication by means of HFGW
 - (b) Object detection or imaging (by HFGW radar or tomography)
 - (c) Vehicle propulsion by HFGW
 - (d) —or any other practical use of HFGW.

Future Directions



Future Directions

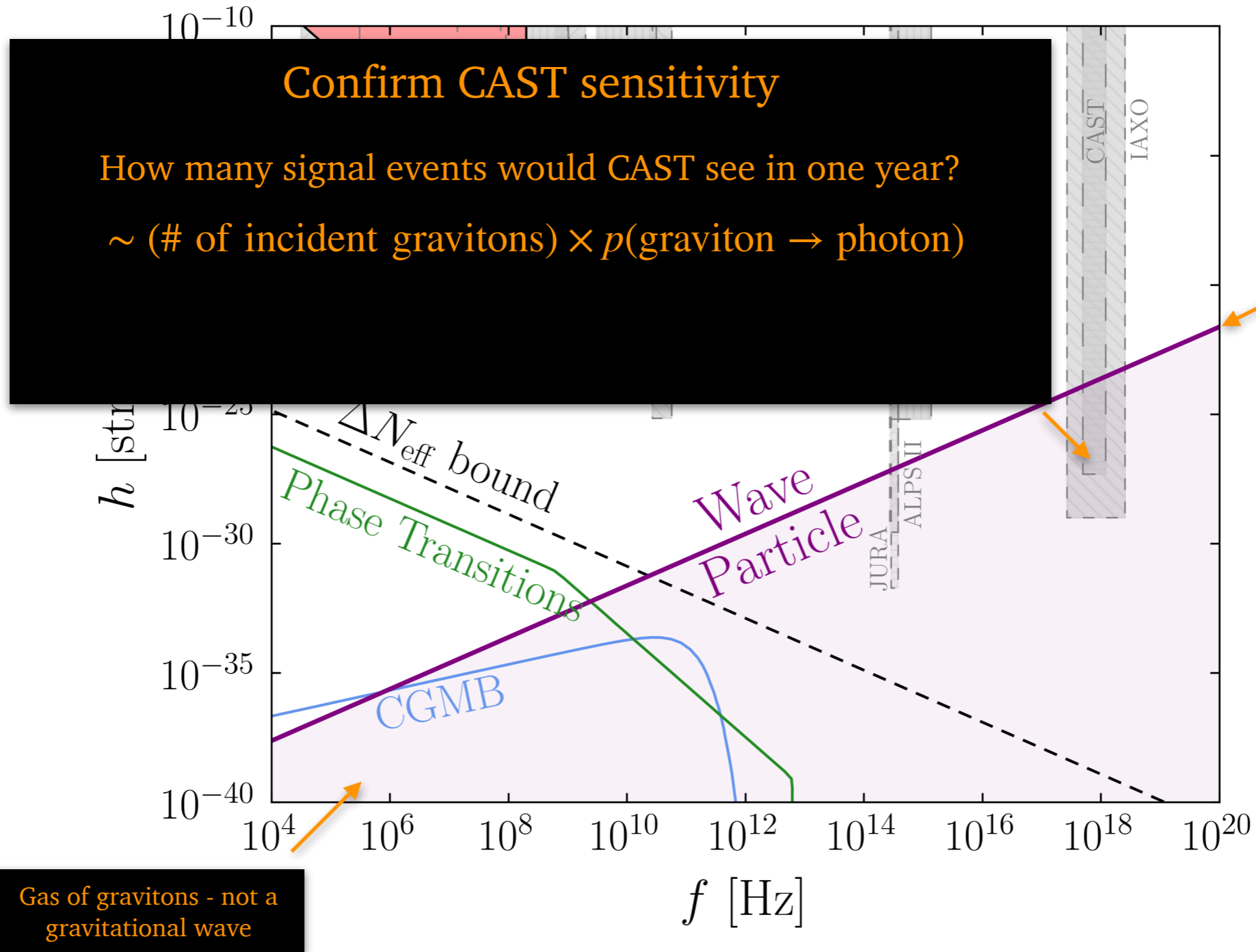


$$n_h \lambda_h^3 \sim \frac{\rho}{\omega^4}$$

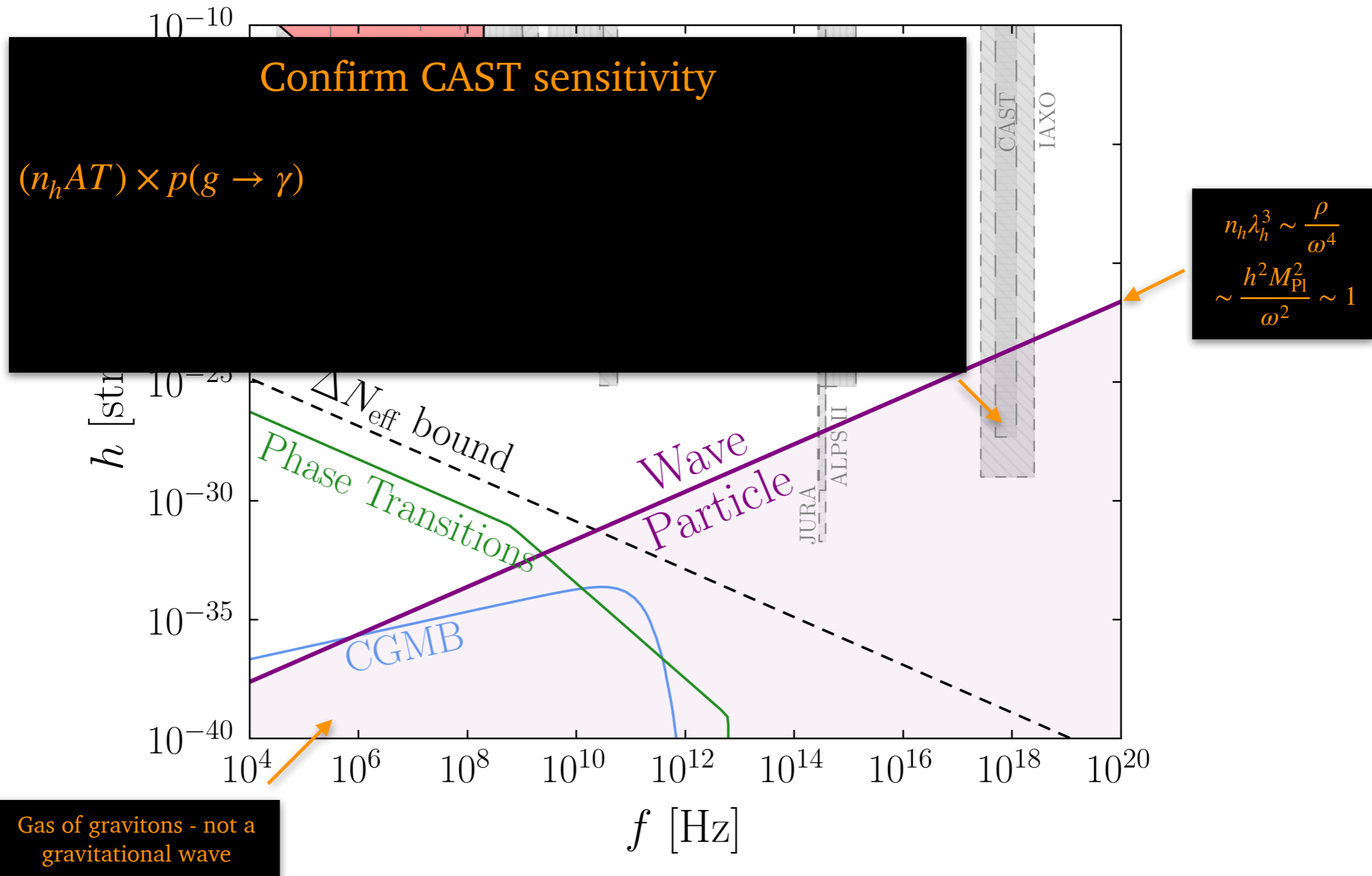
$$\sim \frac{h^2 M_{\text{Pl}}^2}{\omega^2} \sim 1$$

Gas of gravitons - not a gravitational wave

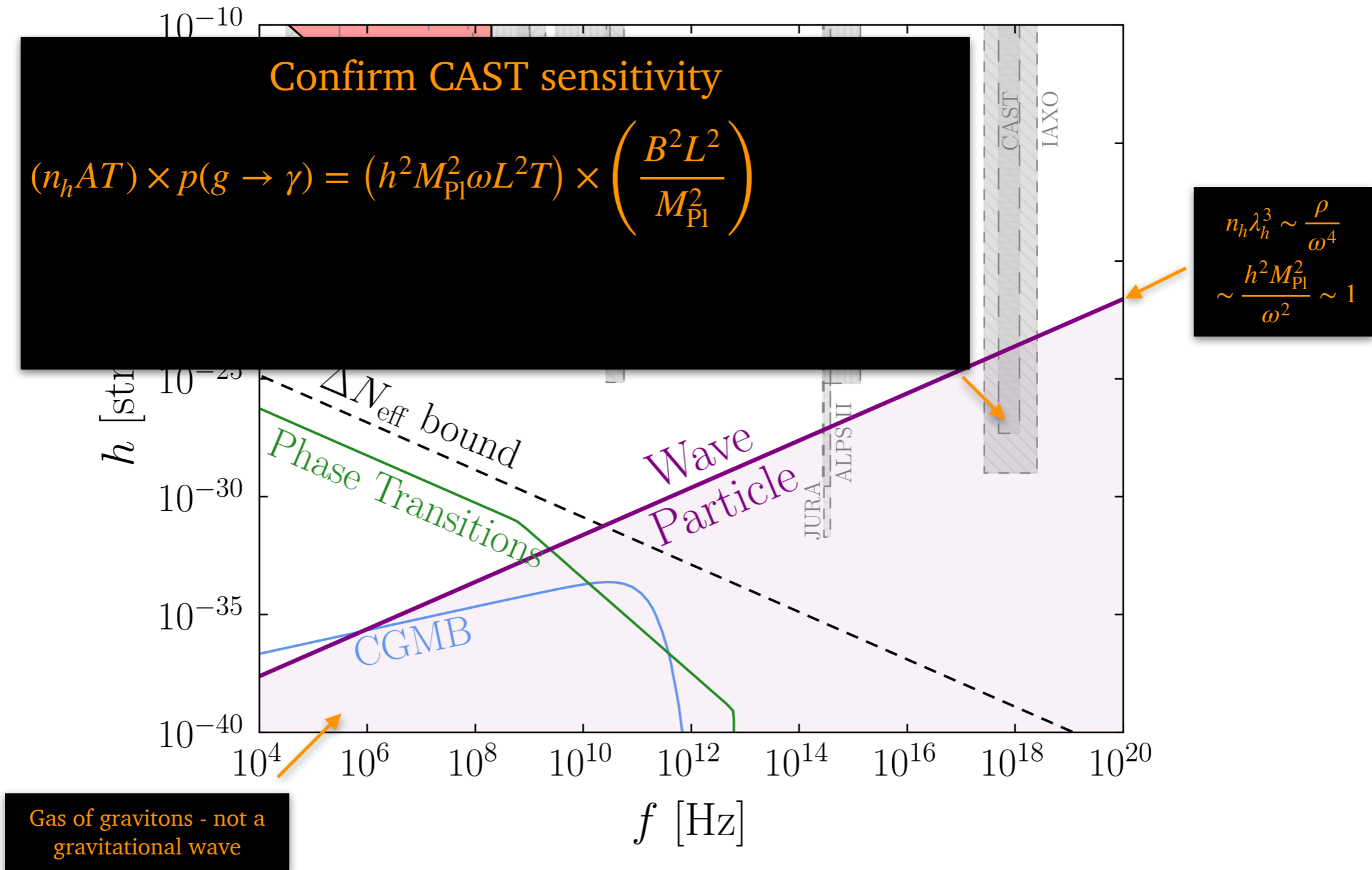
Future Directions



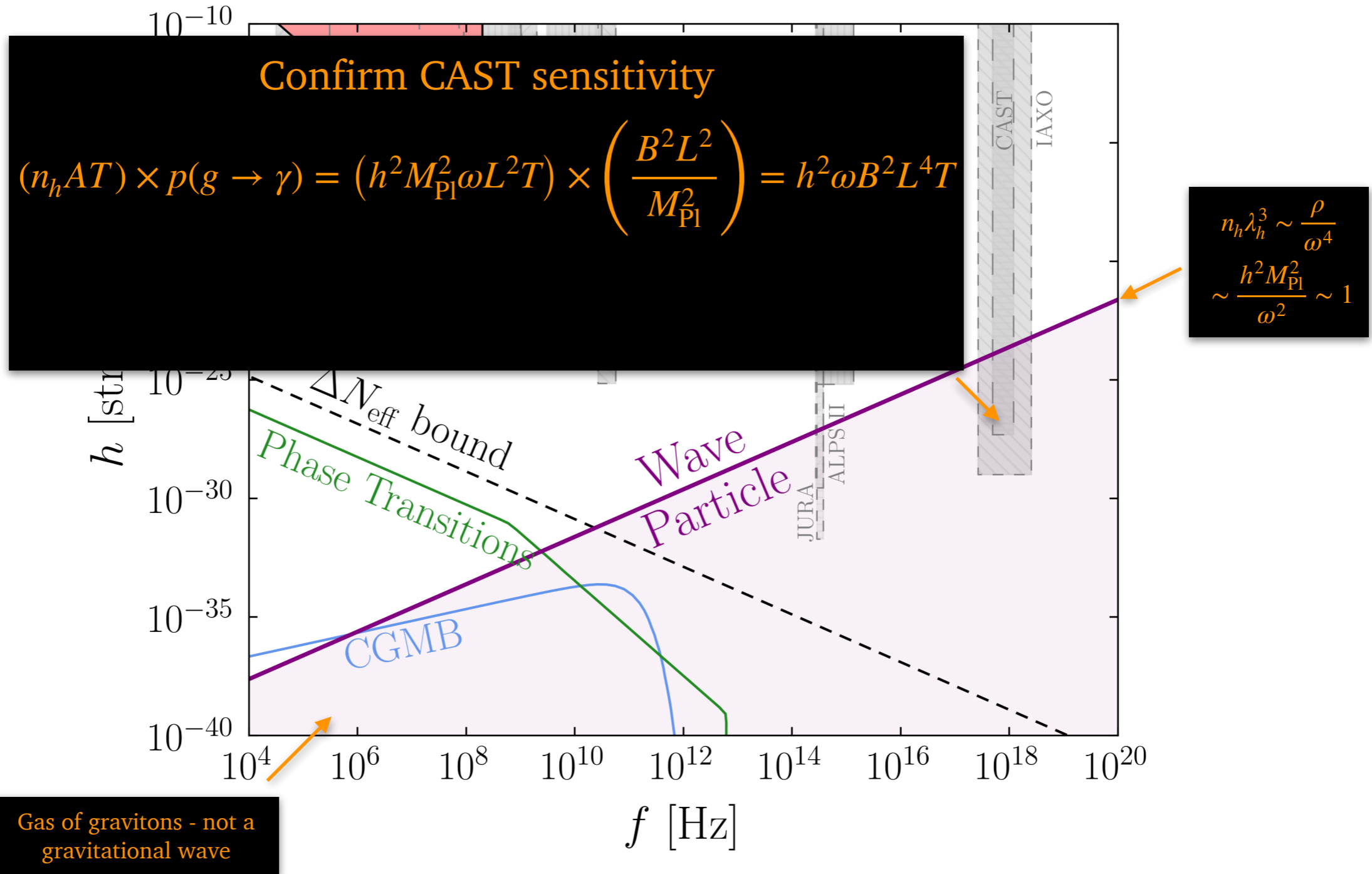
Future Directions



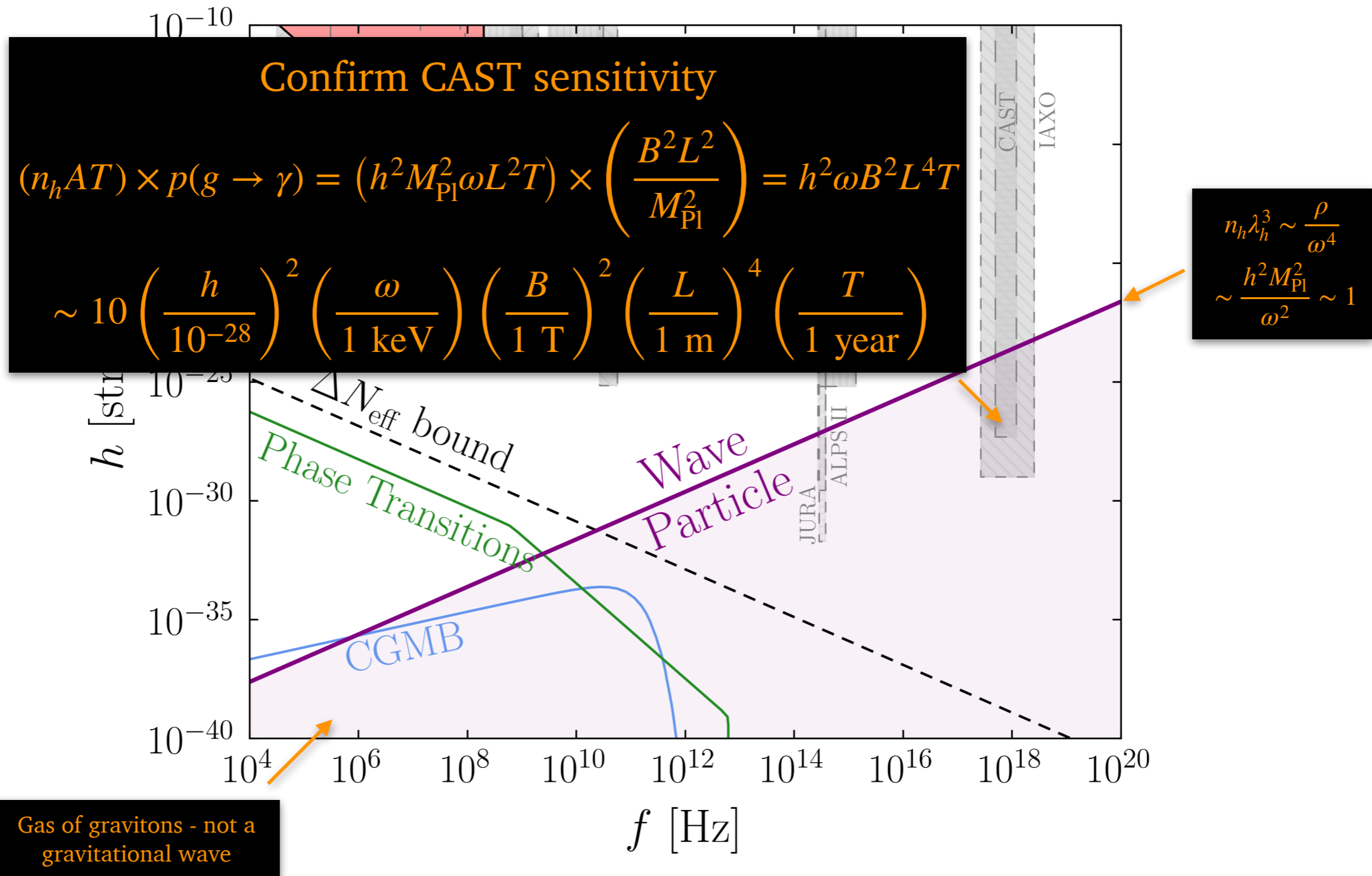
Future Directions



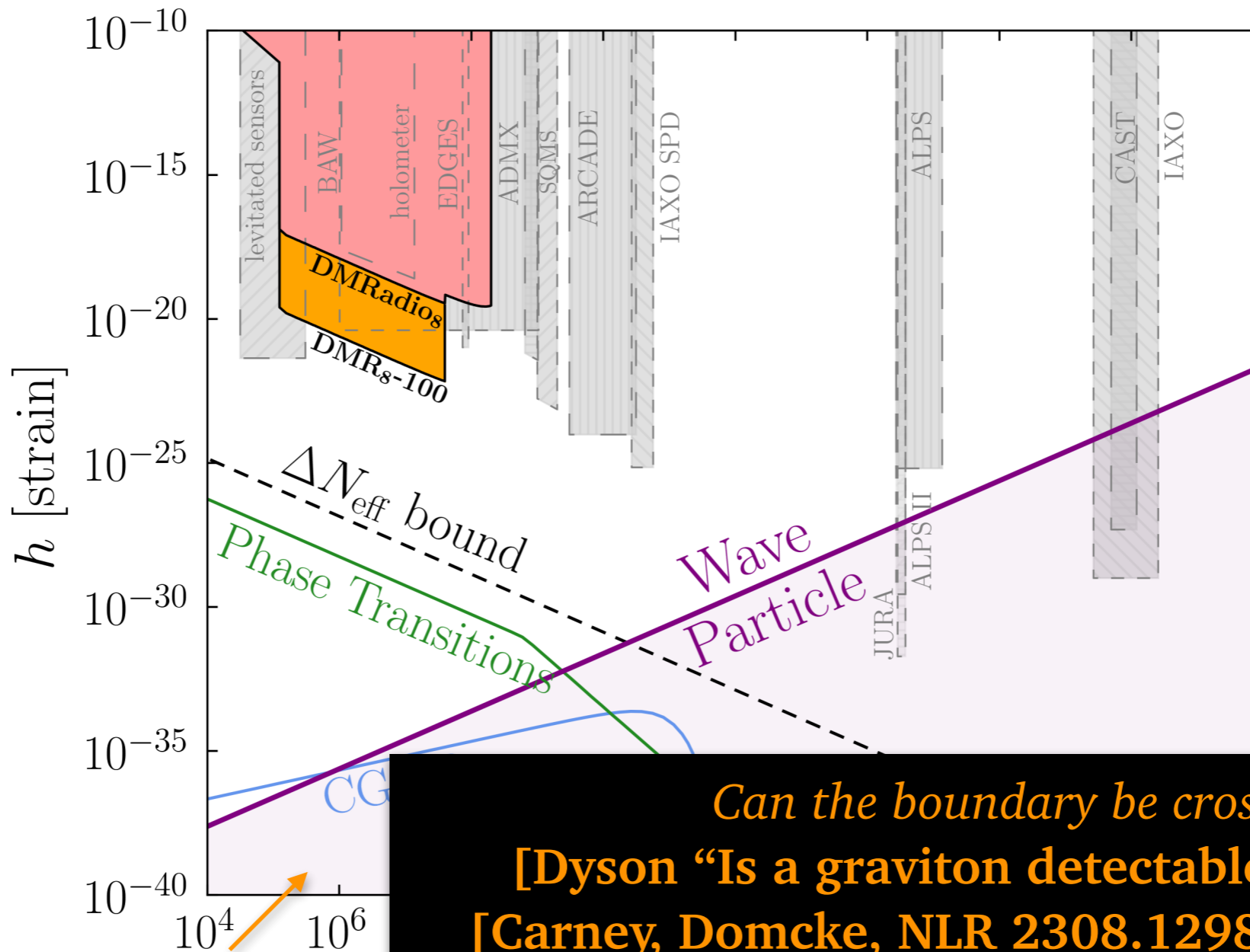
Future Directions



Future Directions



Future Directions



$$n_h \lambda_h^3 \sim \frac{\rho}{\omega^4}$$

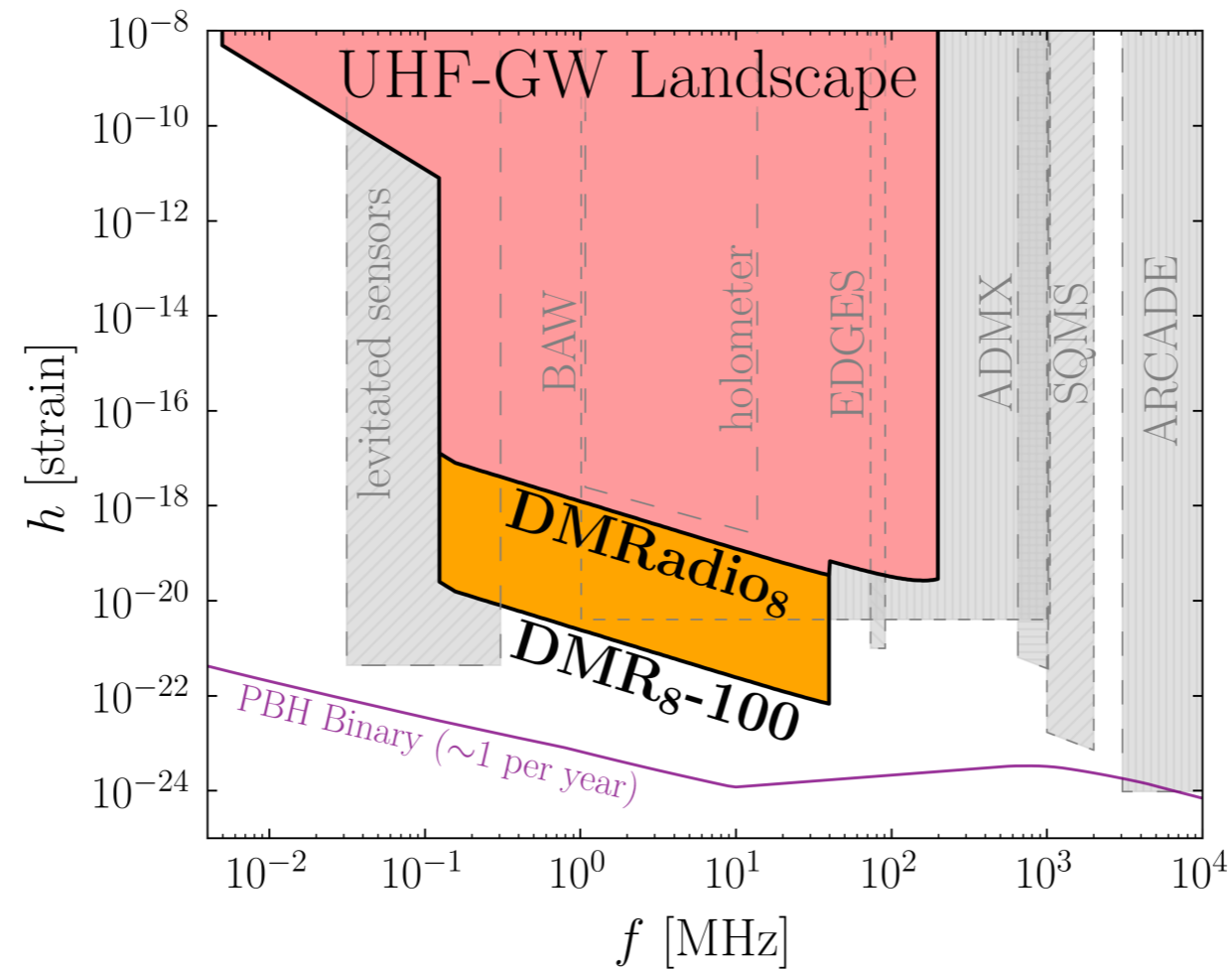
$$\sim \frac{h^2 M_{\text{Pl}}^2}{\omega^2} \sim 1$$

Gas of gravitons - not a gravitational wave

Can the boundary be crossed?
[Dyson "Is a graviton detectable?" 2013] No
[Carney, Domcke, NLR 2308.12988] Yes, but this wouldn't prove gravity was quantised

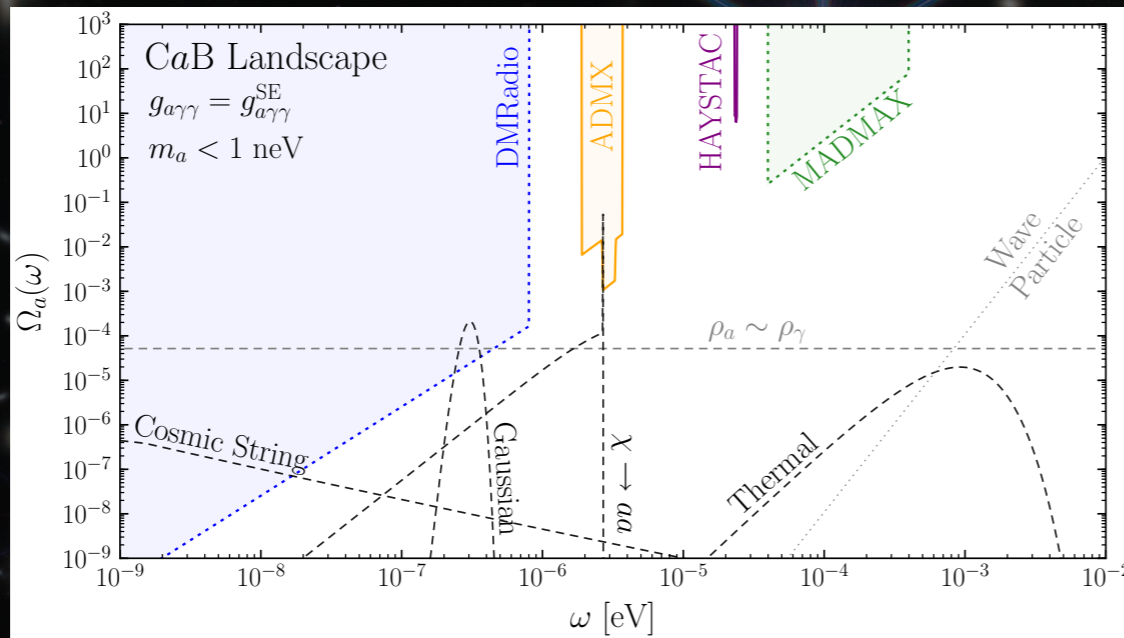
Summary

Axion haloscopes are
gravitational wave telescopes

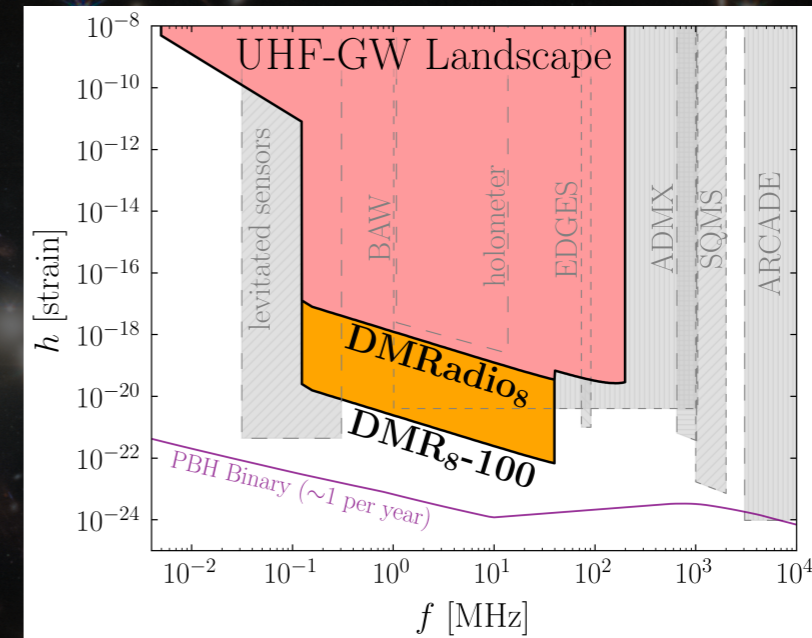


Conclusion

Our deepening search for dark matter opens a path to many new discoveries



[Dror, Murayama, NLR PRD 2021]



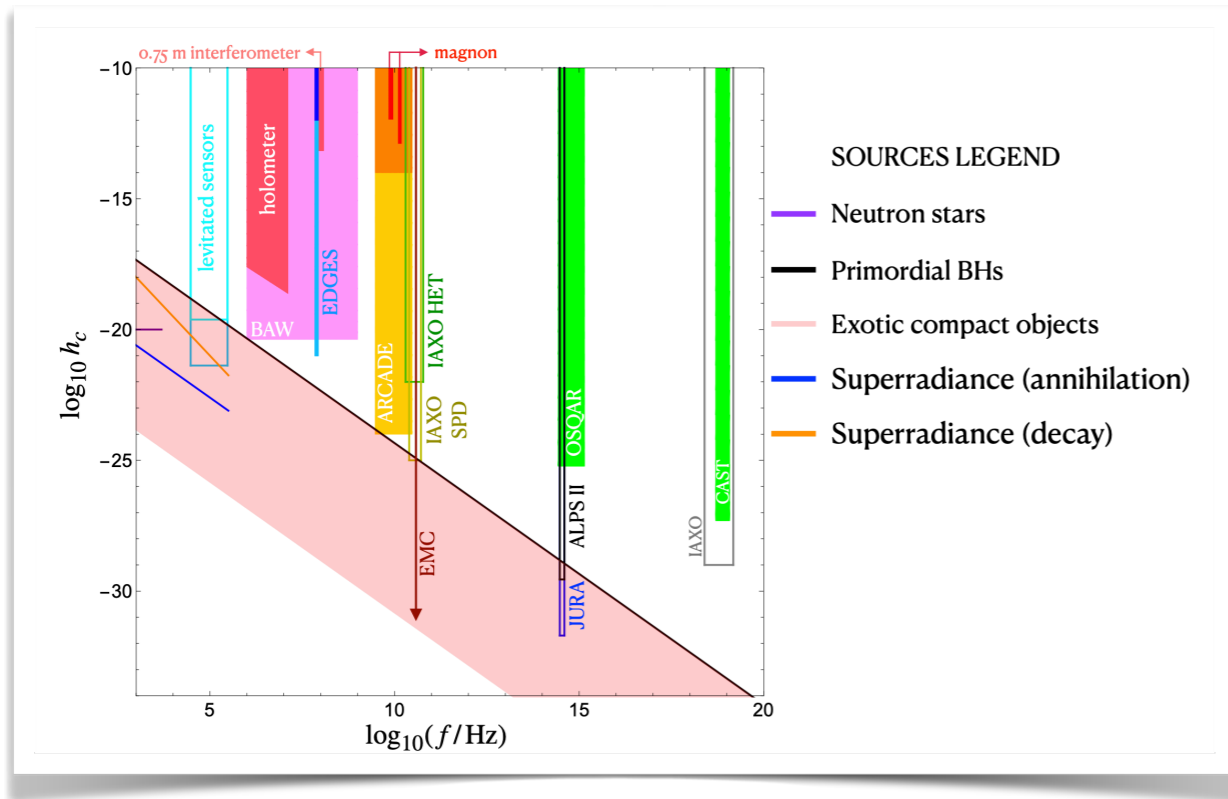
[Domcke, Garcia-Cely, NLR PRL 2022]



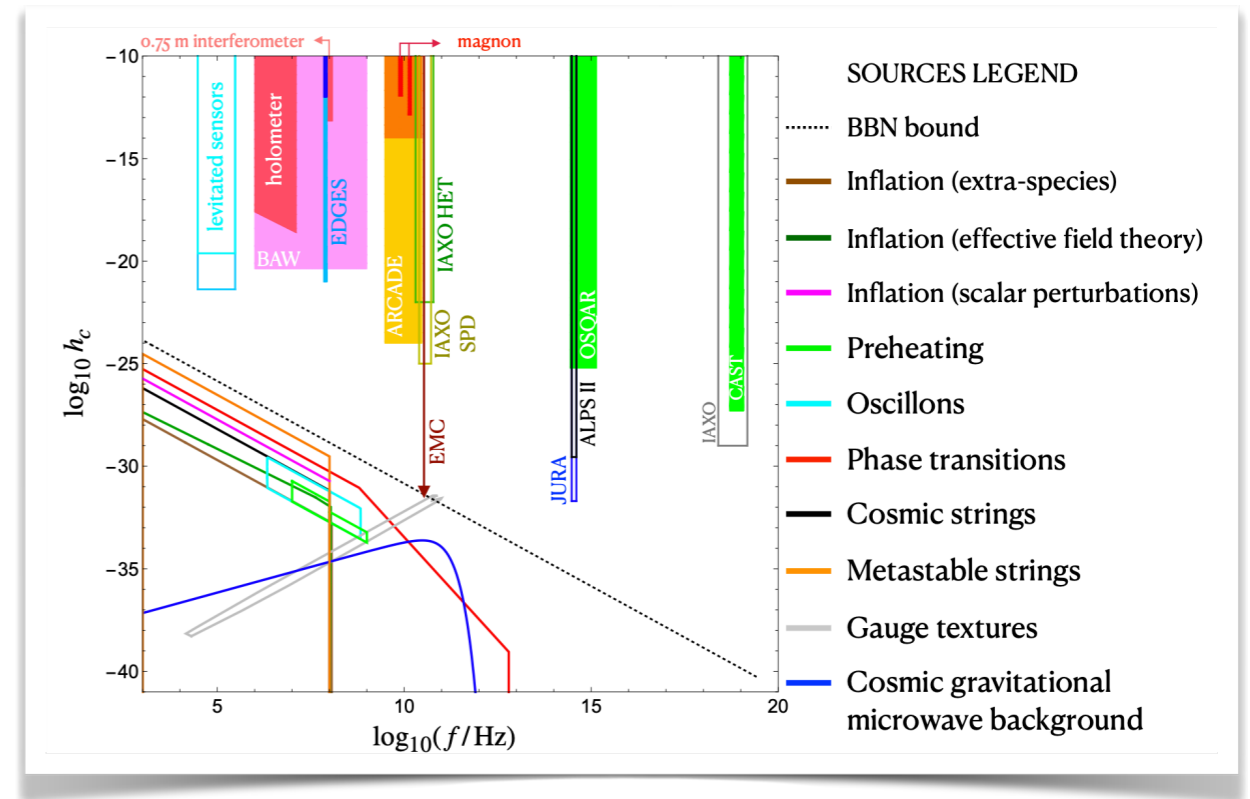
Backup Slides

Sources

Late Universe

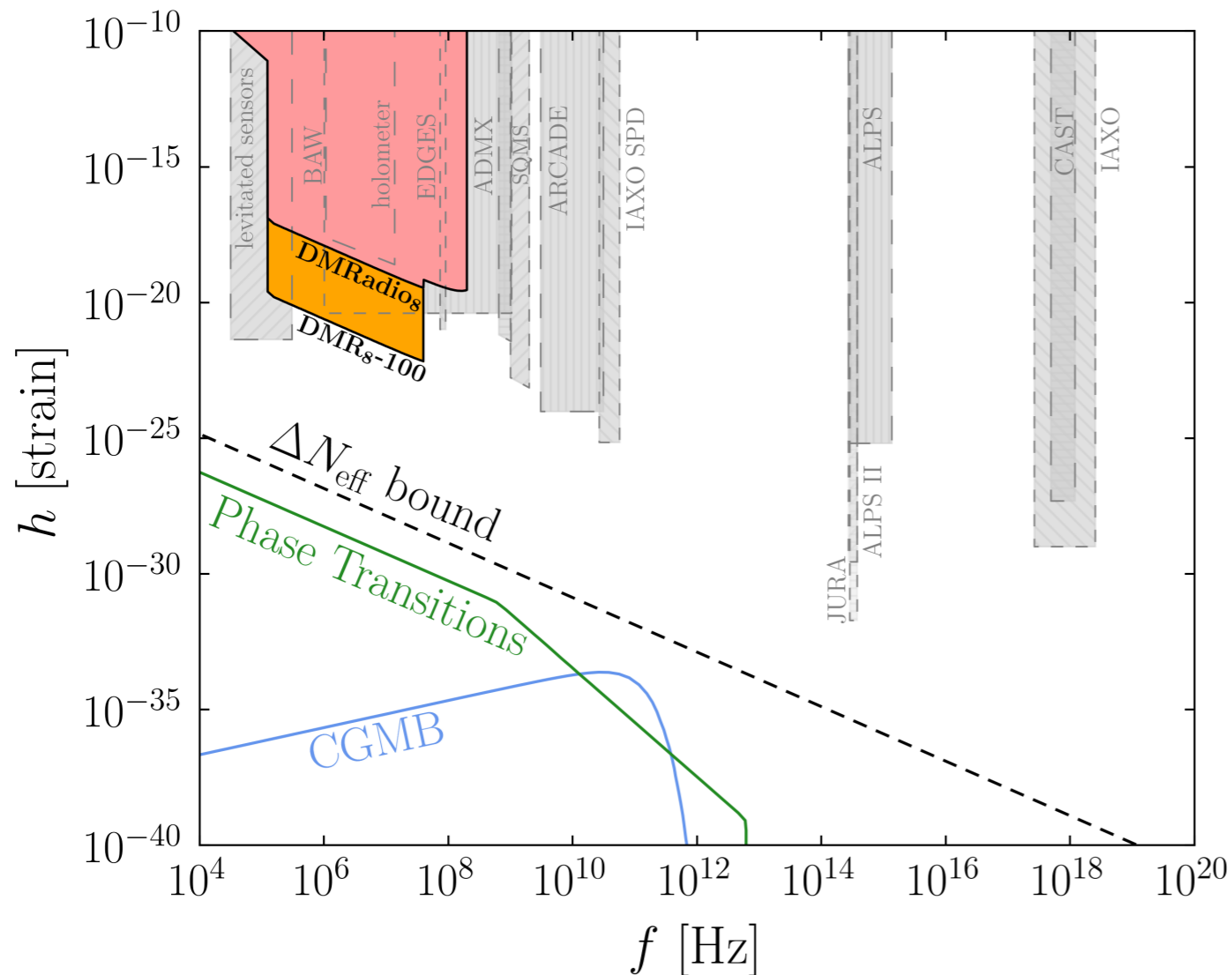


Early Universe



[Aggarwal+ 2020]

Sources: Phase Transitions



Temperature of PT

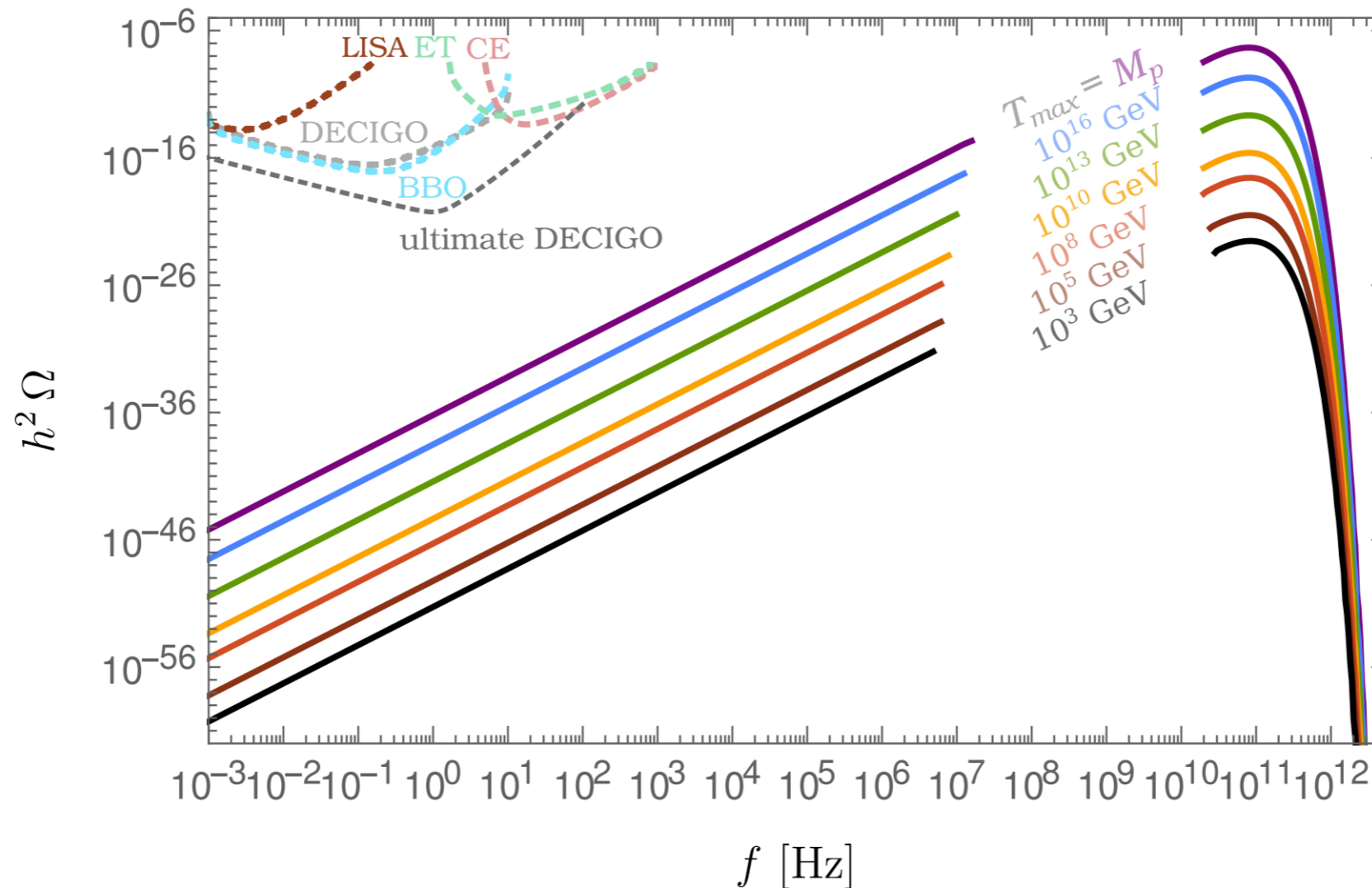
$$f \simeq 3 \text{ GHz} \left(\frac{1}{\epsilon_\star} \right) \left(\frac{T_\star}{10^{16} \text{ GeV}} \right)$$

$$\epsilon_\star = \lambda_\star H_\star$$

$$\lambda_\star = \text{GW wavelength at PT}$$

Sources: CGMB

Blackbody only if $T_{RH} > M_{Pl}$ - arises from non-thermal emission



[Ringwald, Schütte-Engel, Tamarit 2020]

Proper Detector Frame

TT gauge: GW is a plane wave $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Proper Detector Frame: more involved

$$h_{00} = \omega^2 F(\mathbf{k} \cdot \mathbf{r}) \mathbf{b} \cdot \mathbf{r}, \quad b_j \equiv r_i h_{ij}^{\text{TT}} \Big|_{\mathbf{r}=0},$$

$$h_{0i} = \frac{1}{2} \omega^2 [F(\mathbf{k} \cdot \mathbf{r}) - iF'(\mathbf{k} \cdot \mathbf{r})] \left(\hat{\mathbf{k}} \cdot \mathbf{r} b_i - \mathbf{b} \cdot \mathbf{r} \hat{k}_i \right),$$

$$h_{ij} = -i\omega^2 F'(\mathbf{k} \cdot \mathbf{r}) \left(|\mathbf{r}|^2 h_{ij}^{\text{TT}} \Big|_{\mathbf{r}=0} + \mathbf{b} \cdot \mathbf{r} \delta_{ij} - b_i r_j - b_j r_i \right),$$

$$F(\xi) = (e^{i\xi} - 1 - i\xi) / \xi^2 = -1/2 + \mathcal{O}(\xi)$$

See also [Domcke, Garcia-Cely,
Lee, NLR 2306.03125]

Proper Detector Frame

TT gauge: GW is a plane wave $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Proper Detector Frame: more involved

$$\begin{aligned}
 h_{00} &= \omega^2 F(\mathbf{k}\cdot\mathbf{r}) \\
 h_{0i} &= \frac{1}{2}\omega^2 [F(\mathbf{k}\cdot\mathbf{r}) - F(\mathbf{k}\cdot\mathbf{r}_0)] \\
 h_{ij} &= -i\omega^2 F'(\mathbf{k}\cdot\mathbf{r}) \left[\frac{1}{2}(\delta_{ij} - \hat{k}_i\hat{k}_j) \right]
 \end{aligned}$$

$$\begin{aligned}
 g_{\mu\nu}(x) &= \underbrace{g_{\mu\nu}(x_0)}_{=\eta_{\mu\nu}} + \underbrace{(x-x_0)^\alpha \partial_\alpha g_{\mu\nu}(x_0)}_{=0 \text{ } (\because \Gamma_{\nu\rho}^\mu(x_0)=0)} \\
 &\quad + \underbrace{(x-x_0)^\alpha (x-x_0)^\beta \partial_\alpha \partial_\beta g_{\mu\nu}(x_0)}_{\mathcal{O}(\omega^2 R^2)} + \dots
 \end{aligned}$$

$$F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2 = -1/2 + \mathcal{O}(\xi)$$

See also [Domcke, Garcia-Cely, Lee, NLR 2306.03125]

Proper Detector Frame

In the TT gauge GW satisfies

$$h_{0\mu} = 0, \quad h^\mu{}_\mu = 0, \quad \partial^\mu h_{\mu\nu} = 0$$

Treating the GW as a plane wave, takes the form

$$h_{ij}^{\text{TT}} = \left[(U_i U_j - V_i V_j) h^+ + (U_i V_j + V_i U_j) h^\times \right] \frac{e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}}{\sqrt{2}},$$

$$\hat{\mathbf{k}} = s_{\theta_h} \hat{\mathbf{e}}_\rho^{\phi_h} + c_{\theta_h} \hat{\mathbf{e}}_z, \quad \mathbf{V} = \hat{\mathbf{e}}_\phi^{\phi_h}, \quad \mathbf{U} = \mathbf{V} \times \hat{\mathbf{k}}$$

Incident angle (θ_h, ϕ_h)

Convention

TT frame is not a locally inertial coordinate system:
description of experimental apparatus complex

Proper Detector Frame

Use Fermi normal coordinates

Locally inertial coordinates
along a geodesic [Fermi 1922]

$$h_{ij} = -2 \sum_{n=0}^{\infty} \frac{n+1}{(n+3)!} \hat{R}_{ikjl, m_1 \dots m_n} r_k r_l r_{m_1} \dots r_{m_n},$$

$$h_{0i} = -2 \sum_{n=0}^{\infty} \frac{n+2}{(n+3)!} \hat{R}_{0kil, m_1 \dots m_n} r_k r_l r_{m_1} \dots r_{m_n},$$

$$h_{00} = -2 \sum_{n=0}^{\infty} \frac{n+3}{(n+3)!} \hat{R}_{0k0l, m_1 \dots m_n} r_k r_l r_{m_1} \dots r_{m_n}$$

\hat{R} is evaluated at the
coordinate origin

[Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

Proper Detector Frame

Proper detector frame:
Fermi normal coordinates transformed to the non-inertial reference frame of the detector

[Ni, Zimmermann 1978]

Non-inertial corrections (Earth's gravity, Coriolis effect, etc) are irrelevant at higher frequencies - effectively can just use Fermi normal coordinates

Proper Detector Frame

All orders currents for a toroidal magnetic field

$$j_\phi = \frac{\omega^2 B_{\max} R}{\rho} \left[\frac{e^\kappa}{\kappa} - \frac{2e^\kappa}{\kappa^2} + \frac{2(e^\kappa - 1)}{\kappa^3} \right] (z h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} - \rho h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0}),$$

$$j_\rho = \frac{\omega^2 B_{\max} R}{\rho} \left(\left[-\frac{1}{2} - \frac{1}{\kappa} + \frac{2e^\kappa}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (\rho h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} + z h_{zz}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + \left[\frac{e^\kappa}{\kappa} + \frac{2}{\kappa^2} + \frac{2(1 - e^\kappa)}{\kappa^3} \right] (z h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} + z h_{zz}^{\text{TT}}|_{\mathbf{r}=0}) \right. \\ \left. + i k_z \left[\frac{1}{2\kappa} + \frac{1}{2\kappa^2} - \frac{1 + 2e^\kappa}{\kappa^3} + \frac{3(e^\kappa - 1)}{\kappa^4} \right] r_i r_j h_{ij}^{\text{TT}}|_{\mathbf{r}=0} \right),$$

$$\kappa = i\mathbf{k} \cdot \mathbf{r}$$

$$h_{\rho\rho}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left(-h^+ (\sin^2(\phi - \phi_h) - \cos^2(\phi - \phi_h) \cos^2 \theta_h) + 2h^\times \cos \theta_h \cos(\phi - \phi_h) \sin(\phi - \phi_h) \right),$$

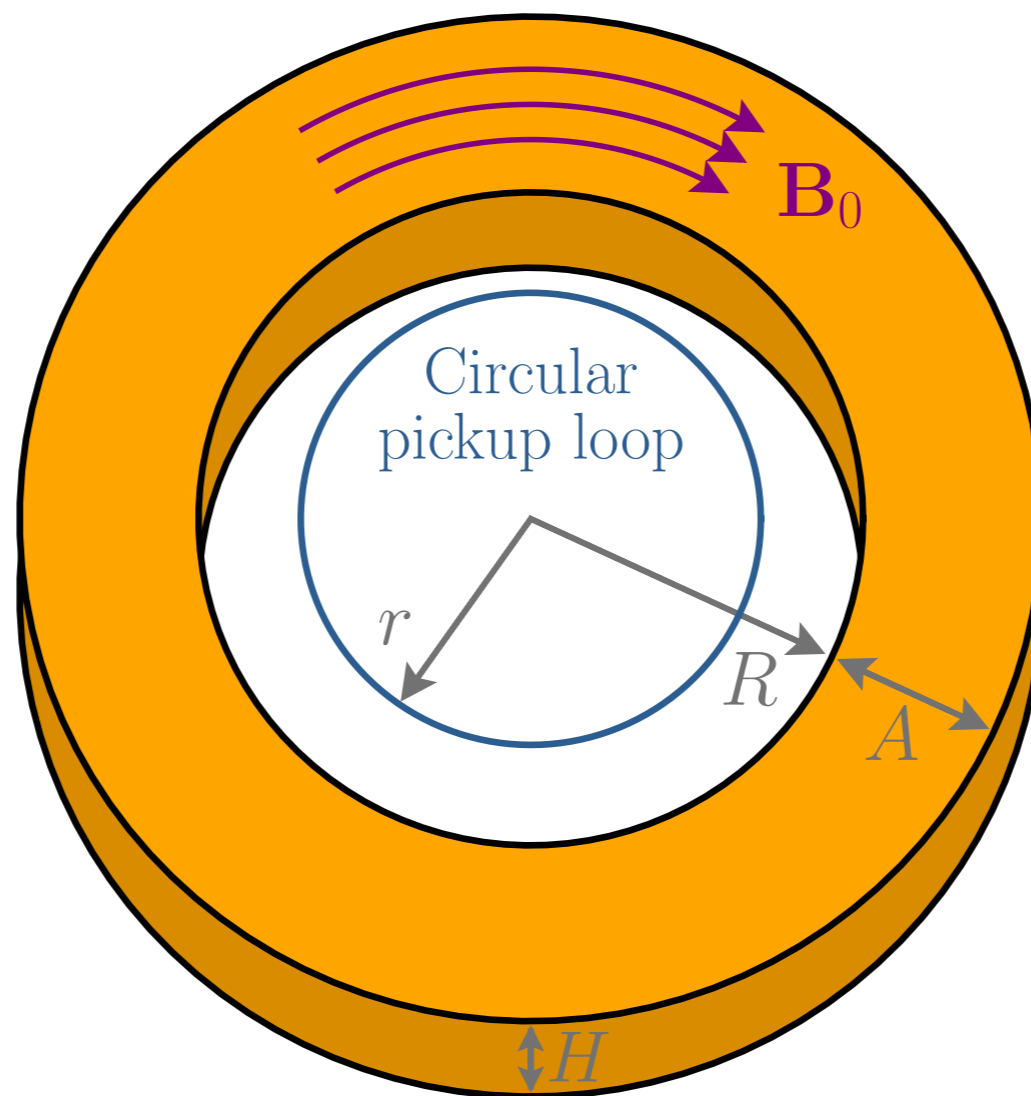
$$h_{\rho\phi}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left(-h^+ (1 + \cos^2 \theta_h) \sin(\phi - \phi_h) \cos(\phi - \phi_h) + h^\times \cos(2(\phi - \phi_h)) \cos \theta_h \right),$$

$$h_{\rho z}^{\text{TT}}|_{\mathbf{r}=0} = -\frac{e^{-i\omega t}}{\sqrt{2}} \left(h^+ \cos \theta_h \sin \theta_h \cos(\phi - \phi_h) + h^\times \sin \theta_h \sin(\phi - \phi_h) \right),$$

$$h_{\phi z}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} \left(h^+ \cos \theta_h \sin \theta_h \sin(\phi - \phi_h) - h^\times \sin \theta_h \cos(\phi - \phi_h) \right),$$

$$h_{zz}^{\text{TT}}|_{\mathbf{r}=0} = \frac{e^{-i\omega t}}{\sqrt{2}} h^+ \sin^2 \theta_h.$$

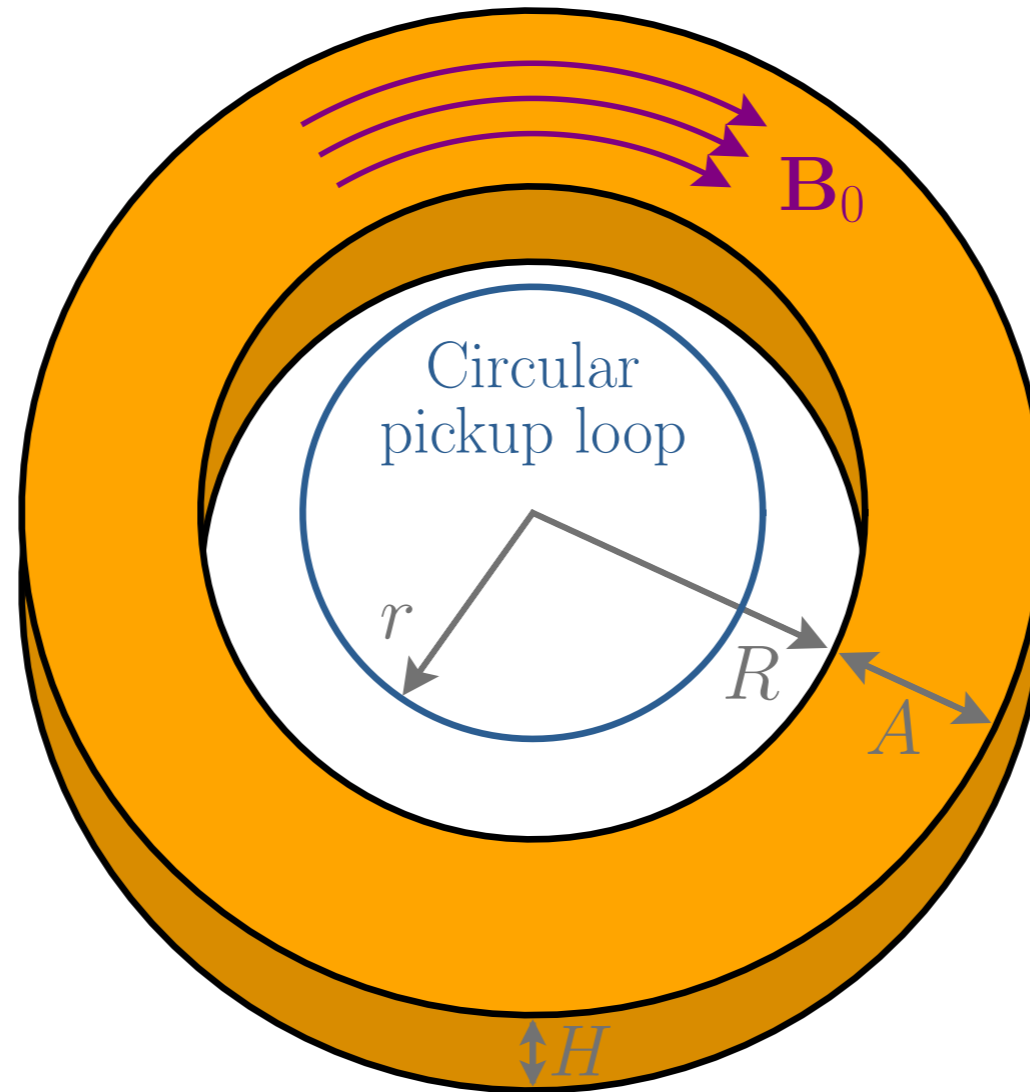
Detection Strategy



cf. axion
 $\Phi_a(t) \sim g_{a\gamma\gamma}(\partial_t a)B_0V$

$$\Phi_h(t) \simeq \frac{ie^{-i\omega t}}{16\sqrt{2}} \omega^3 h^\times B_0 \pi r^2 R A (A + 2R) \sin^2 \theta_h \sim \omega^3 h B_0 V^{5/3}$$

Detection Strategy

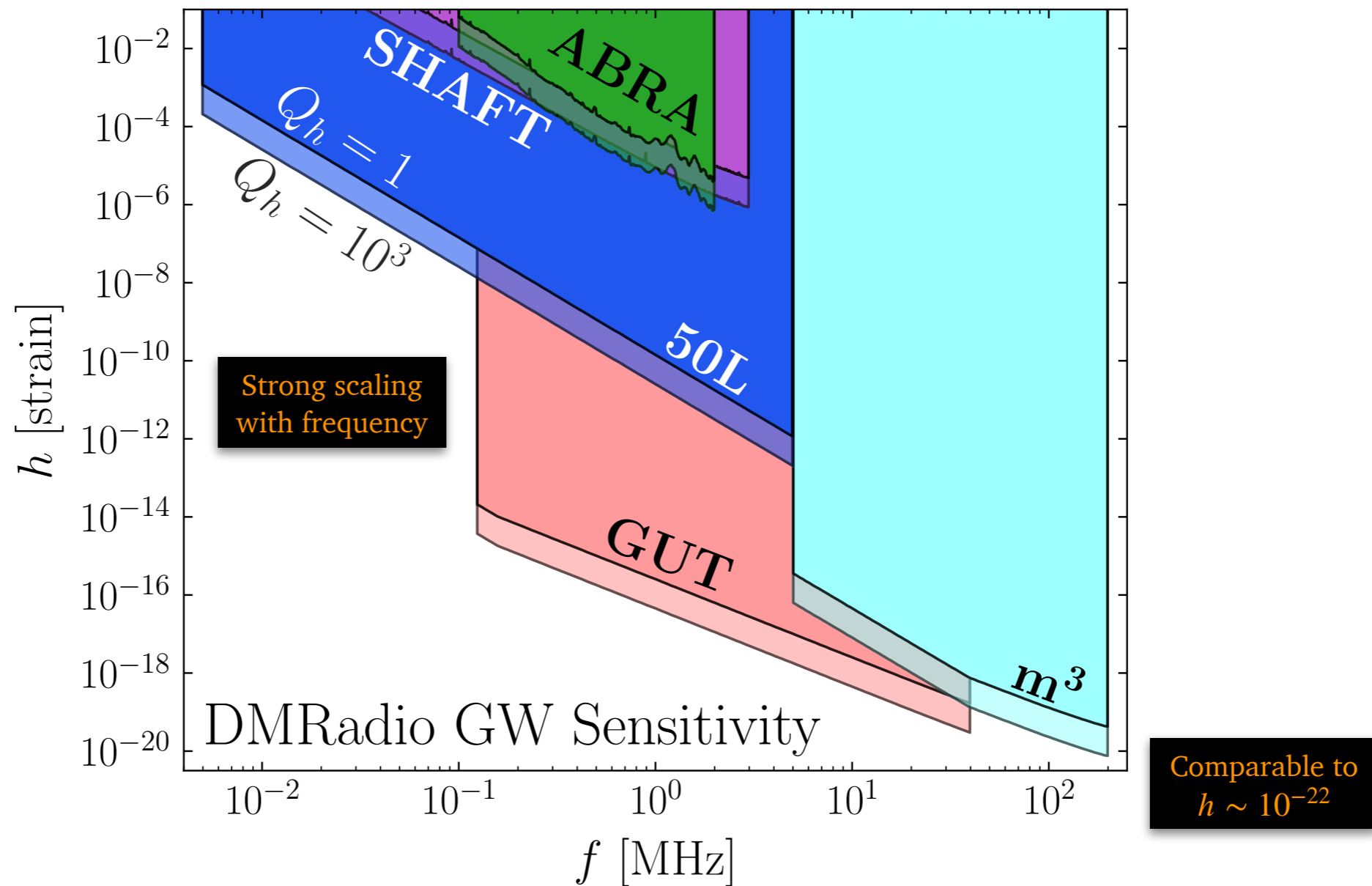


$\mathcal{O}(\omega^2)$ contribution
has vanished!

$$\Phi_h(t) \simeq \frac{ie^{-i\omega t}}{16\sqrt{2}} \omega^3 h^\times B_0 \pi r^2 R A (A + 2R) \sin^2 \theta_h \sim \omega^3 h B_0 V^{5/3}$$

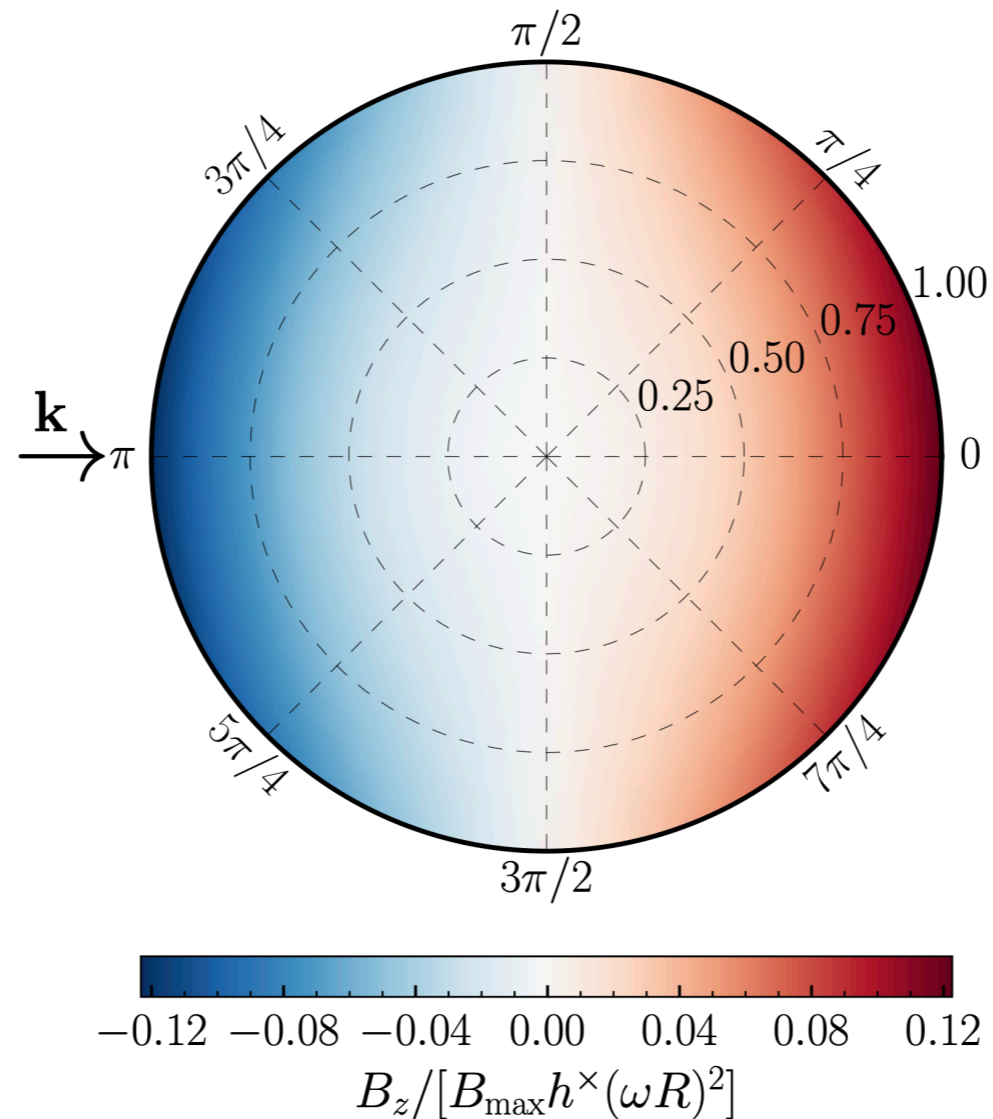
Circular pickup loop reach

Sensitivity set by $\Phi_h = \Phi_a (Q_a/Q_h)^{1/4}$



Optimized Reach

What happened to the leading contribution?

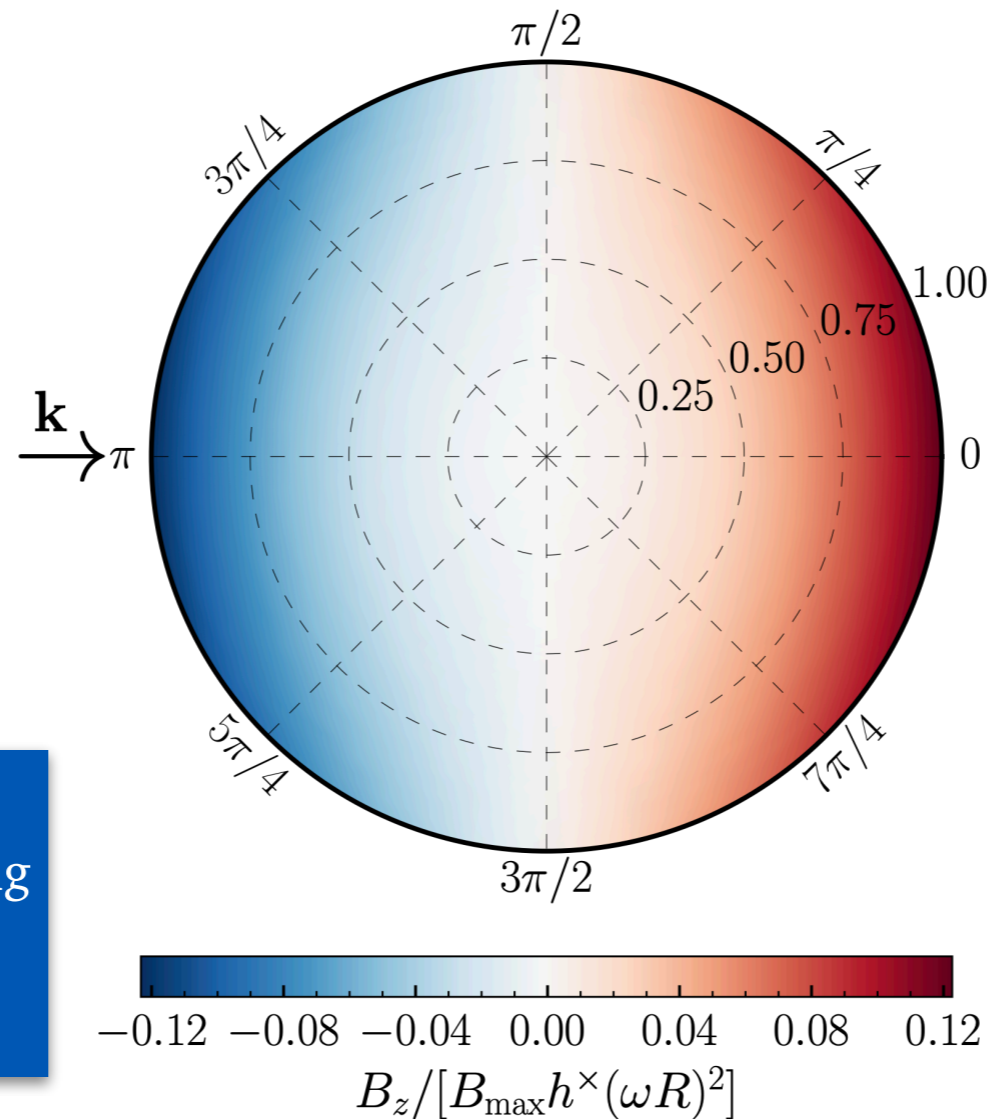


Leading contribution vanishes when integrated over a circular pickup loop

But why? Symmetry imposes selection rules GW must satisfy

Optimized Reach

What happened to the leading contribution?



Leading contribution vanishes when integrated over a circular pickup loop

Selection rules: determine leading terms without calculating
Apply also to scalar (ϕF^2) vs axion ($a F \tilde{F}$)

But why? Symmetry imposes selection rules GW must satisfy

What could CAST see?

Detecting gravitons requires an extremely large flux

$$p(g \rightarrow \gamma) \simeq 4.6 \times 10^{-35} \left(\frac{B}{9 \text{ T}} \right)^2 \left(\frac{L}{9.26 \text{ m}} \right)^2$$

Could be produced by a nearby PBH inspiral, naively require

$$m \simeq 2 \times 10^{-15} M_{\odot}, \quad d \simeq 1.4 \times 10^6 \text{ m} \simeq 0.2 R_{\oplus}$$

At this mass, PBHs can be 100% of DM [Carr, Kühnel 2021]
Final merger generate $f \sim \text{keV}$ gravitons

Distance incorporates the extremely short duration of such a merger

But in principle possible, unlike examples Dyson gave

What could CAST see?

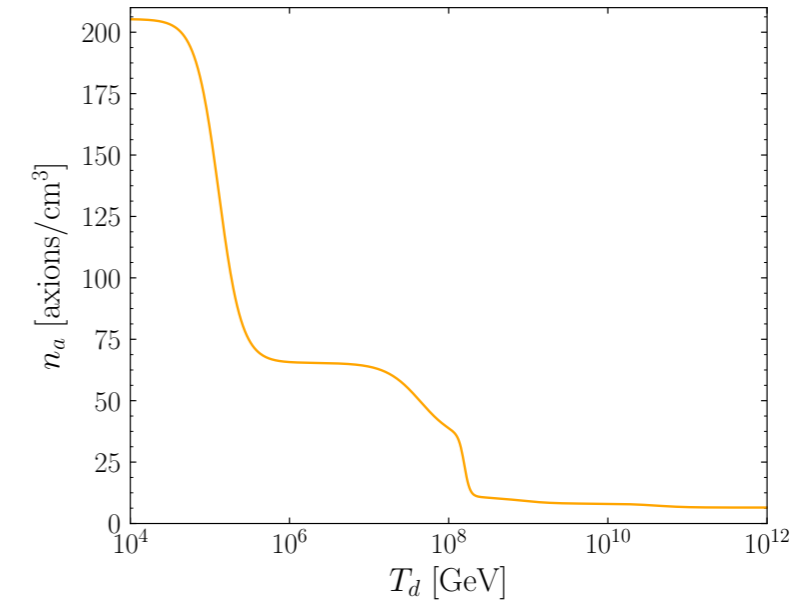
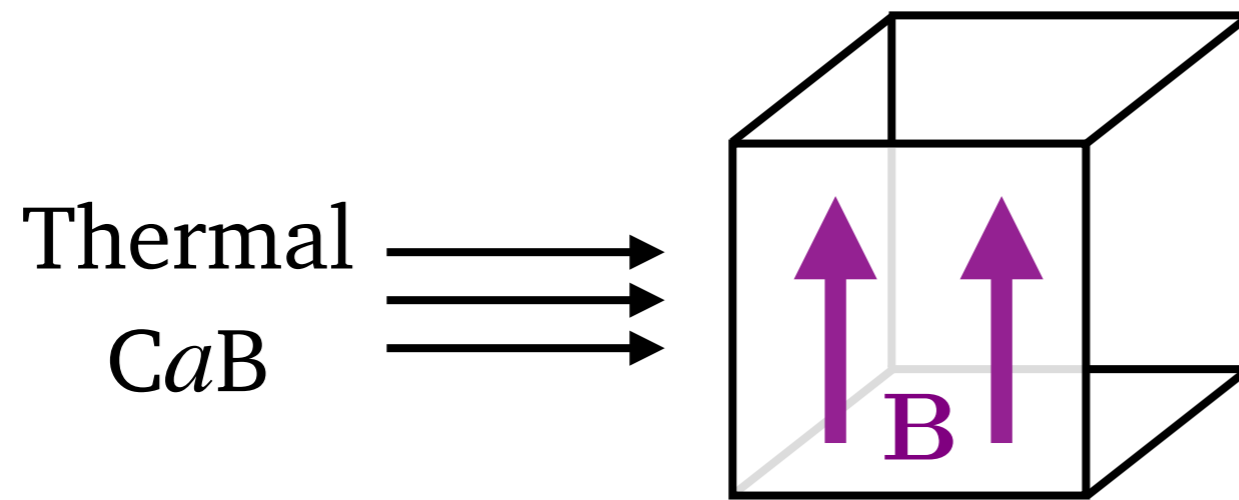
Would such a detection prove a GW is made of quanta? **No**

Even if the GW was perfectly antibunched, 1σ evidence for a quantised signal would require the following number of events

$$N \gtrsim p(g \rightarrow \gamma)^{-2} \simeq 10^{70}$$

Cf. the proposal on the last slide: $N = 1$

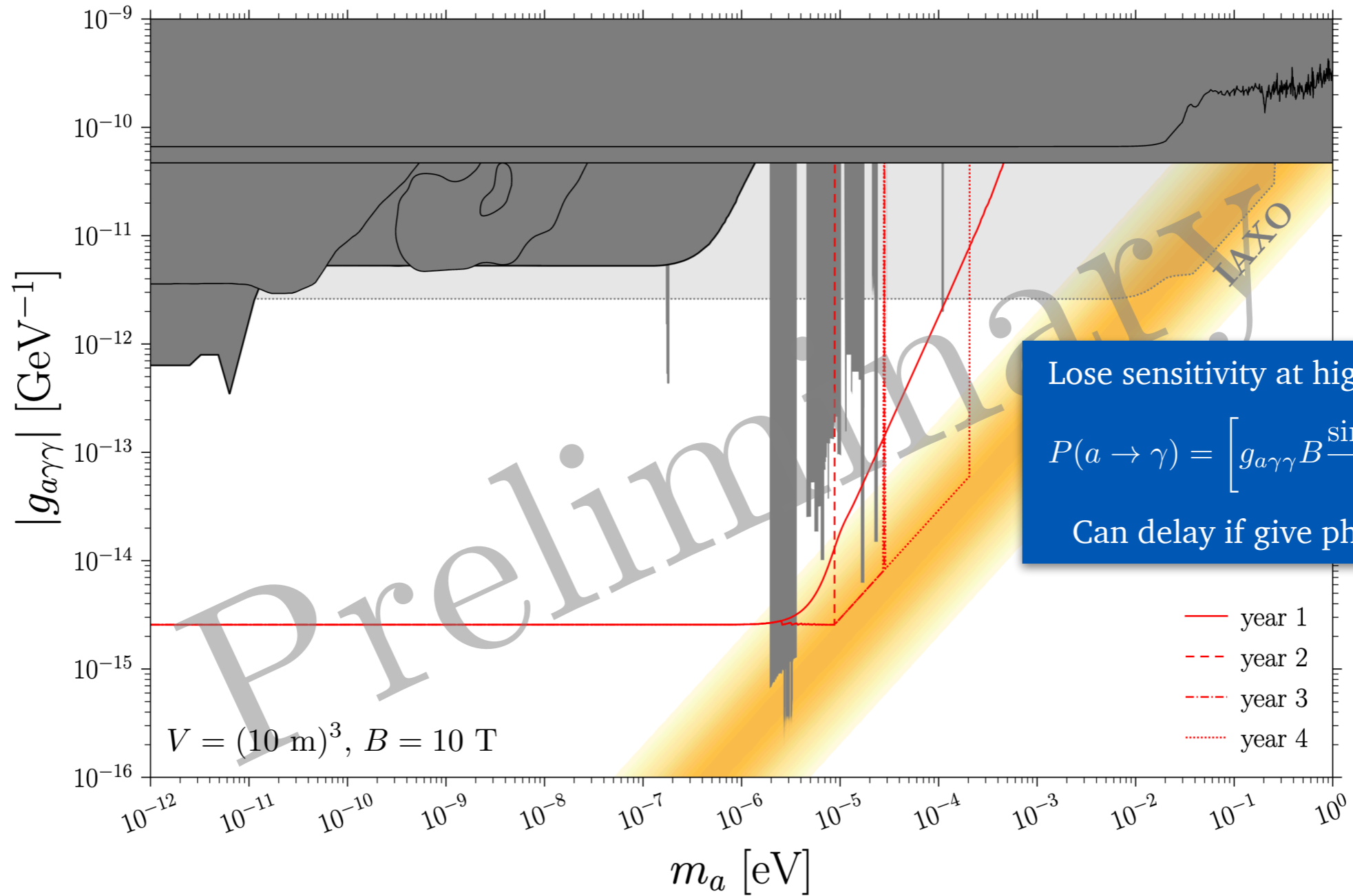
Detecting the Thermal CaB



$$N(a \rightarrow \gamma) \simeq 2.5 \times 10^9 \left(\frac{g_{a\gamma\gamma}}{10^{-10} \text{ GeV}^{-1}} \right)^2 \left(\frac{B}{10 \text{ T}} \right)^2 \left(\frac{L}{10 \text{ m}} \right)^4 \left(\frac{t_{\text{exp}}}{1 \text{ year}} \right)$$

Potentially detectable!

Detecting the Thermal CaB



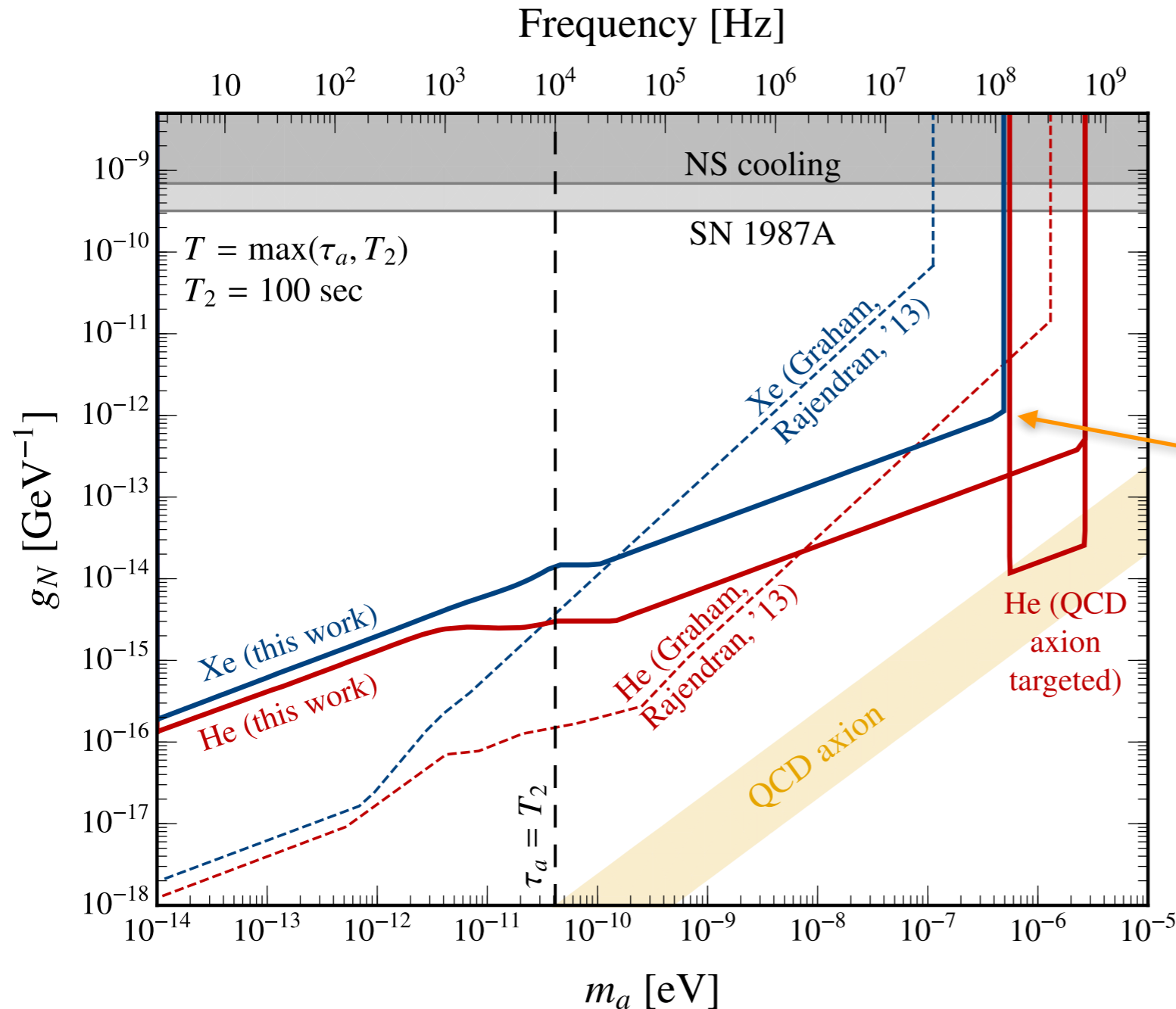
Lose sensitivity at higher masses as

$$P(a \rightarrow \gamma) = \left[g_{a\gamma\gamma} B \frac{\sin(m_a^2 L / 2\omega_a)}{m_a^2 / \omega_a} \right]^2$$

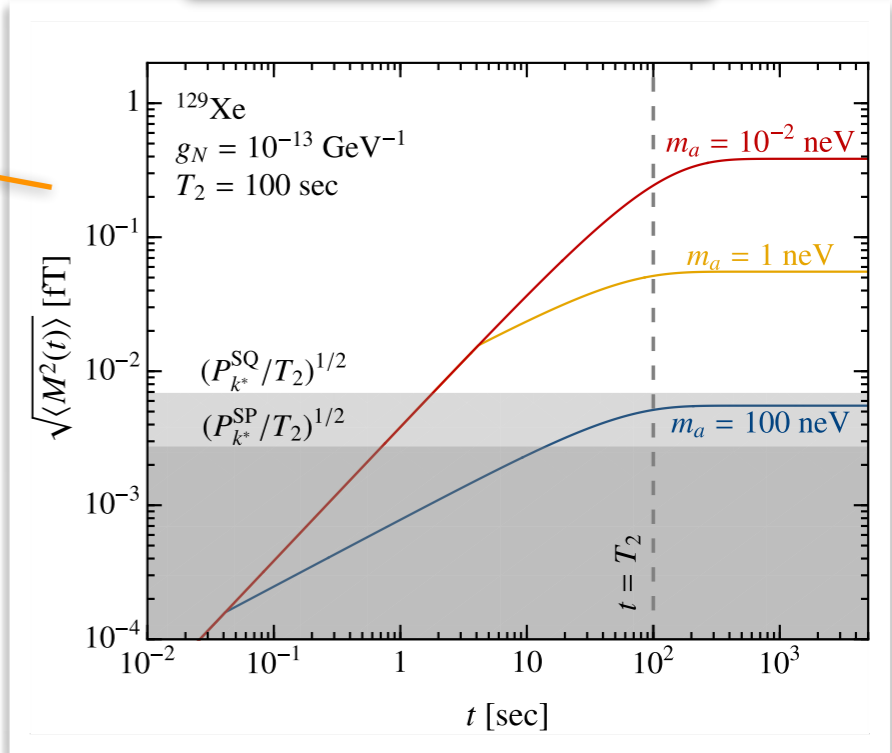
Can delay if give photon a mass

Axion NMR

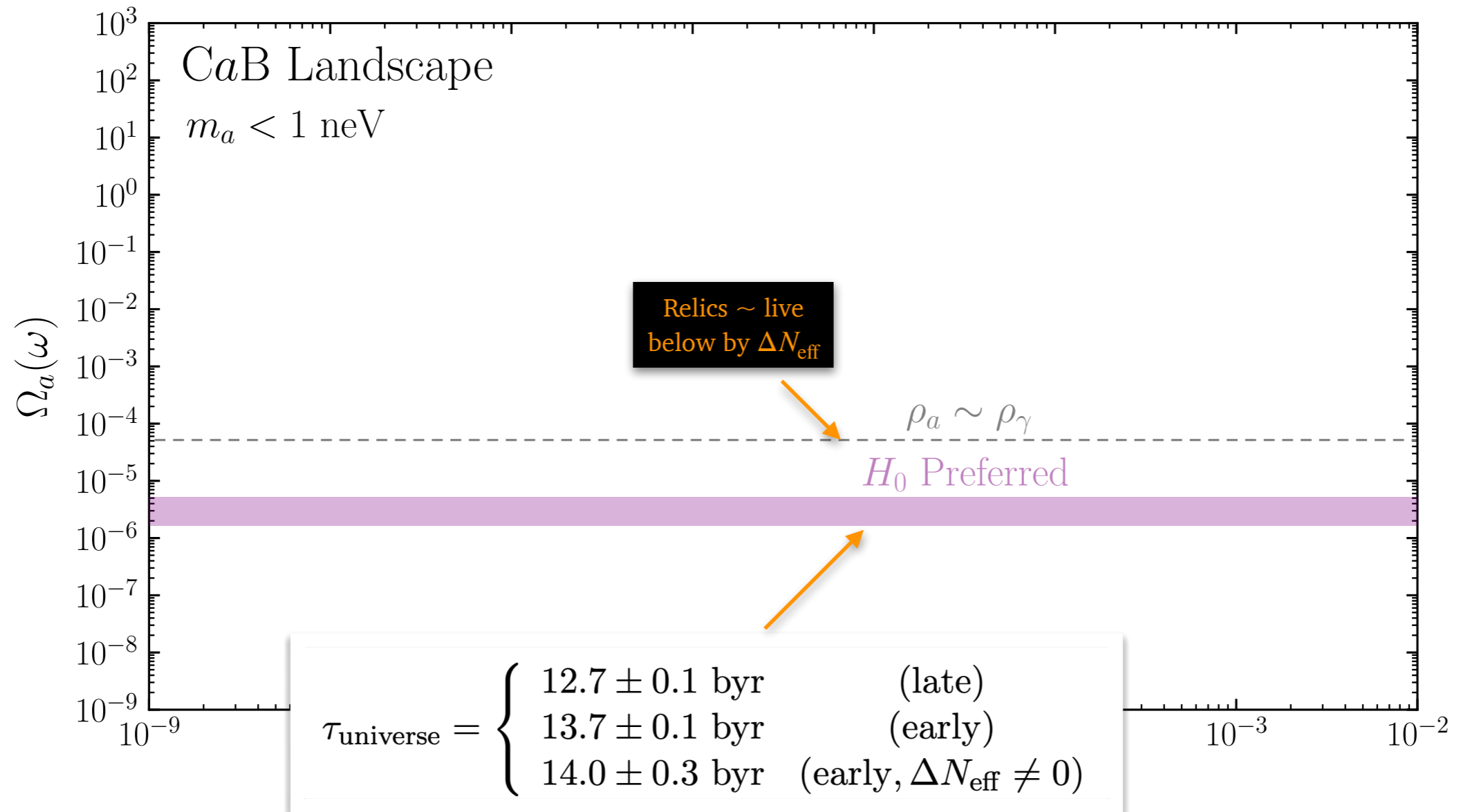
$$\mathcal{L} \supset g_N (\partial_\mu a) \bar{N} \gamma^\mu \gamma_5 N$$



Axion signal continues to grow for $\tau_a < T < T_2$



Hubble Tension



See e.g. [Valentino+ “In the Realm of the Hubble tension – a Review of Solutions” 2021]
 Not a finalist in the H_0 Olympics
 [Schoneberg+ 2021]



Bose Enhancement

Relevant when $f_a \gg 1$

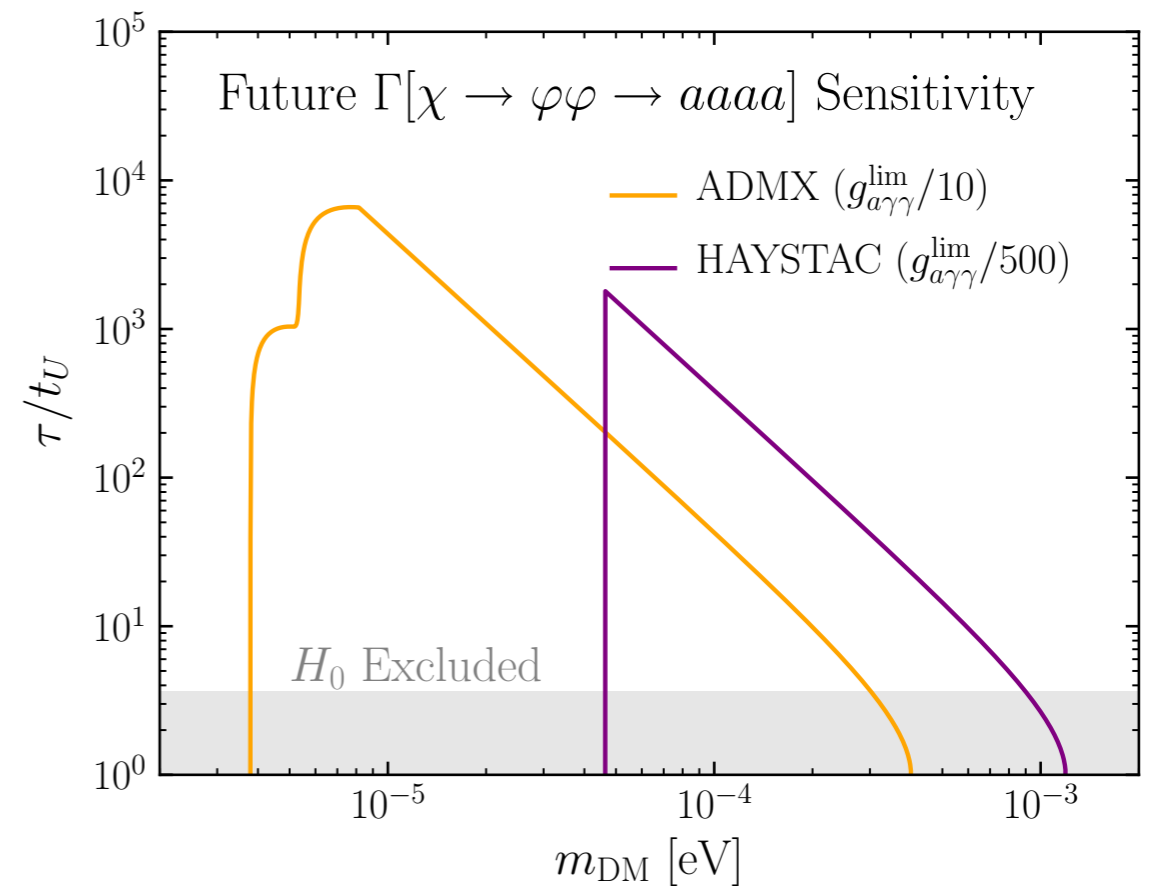
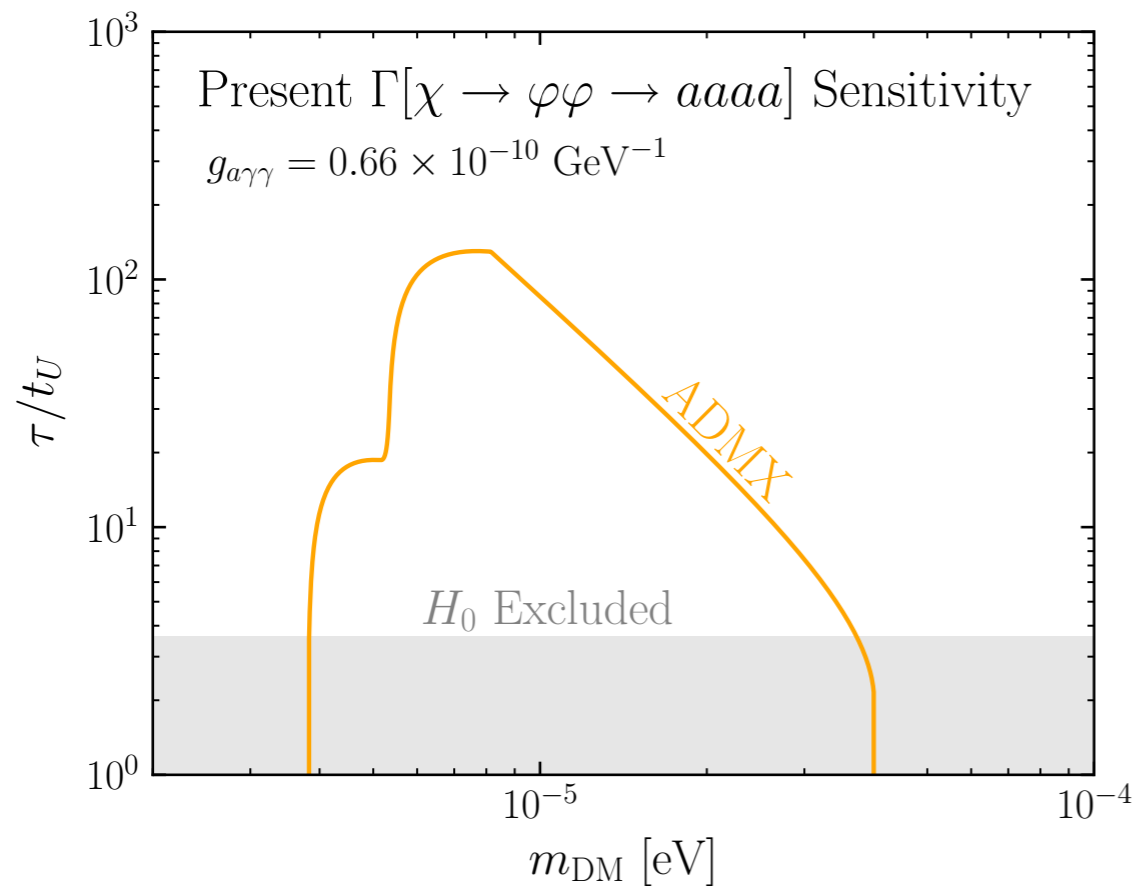
$$f_a = \frac{2\pi^2}{\omega^3} \frac{d\rho_a}{d\omega} \simeq 4 \times 10^{10} \left(\frac{Q_a}{1} \right) \left(\frac{\rho_a}{\rho_\gamma} \right) \left(\frac{\bar{\omega}}{1 \mu\text{eV}} \right)^{-4}$$

Large over the entire range we consider

Bose Enhancement

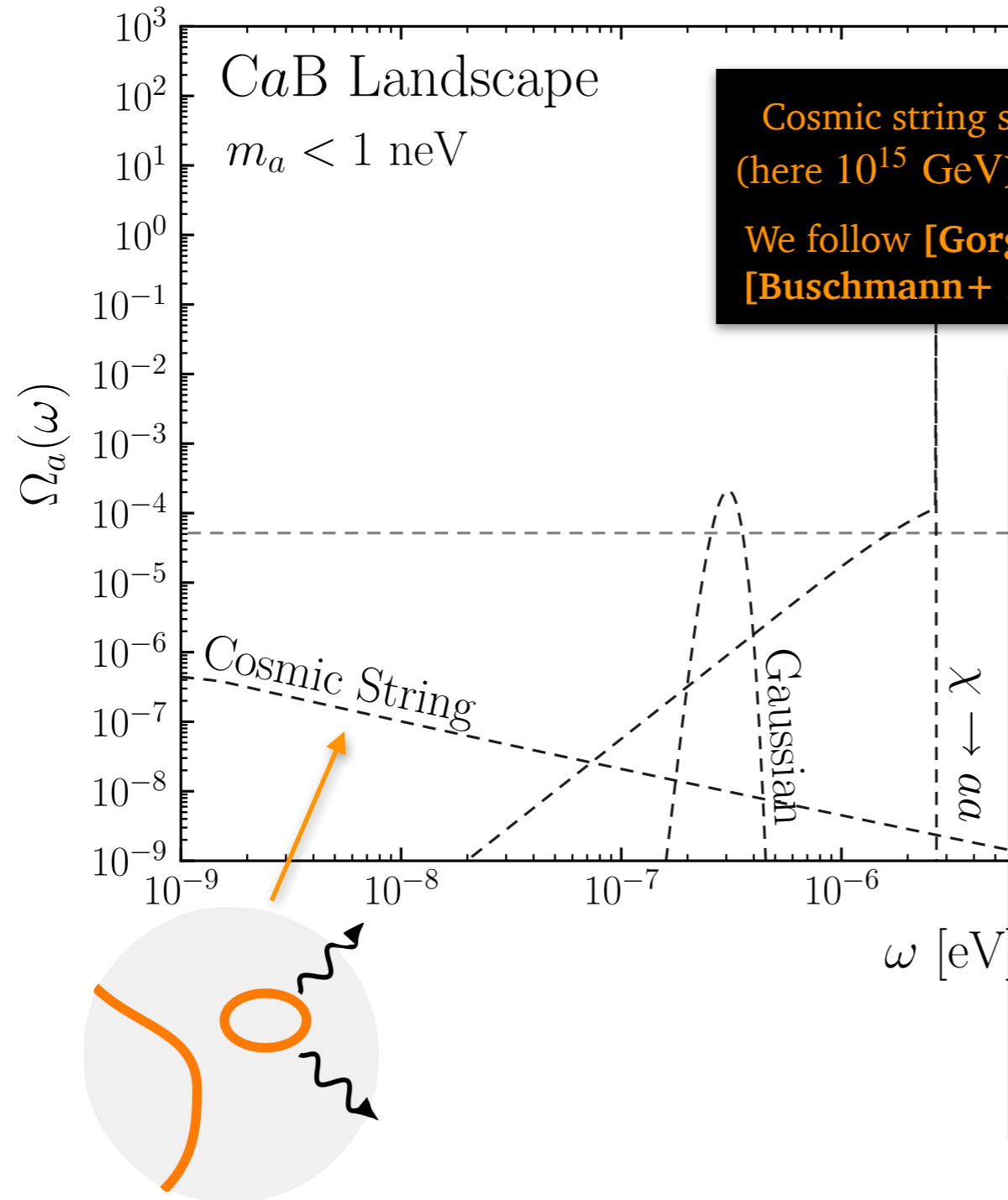
Can lift with cascade decays

Exploit daily modulation of the signal

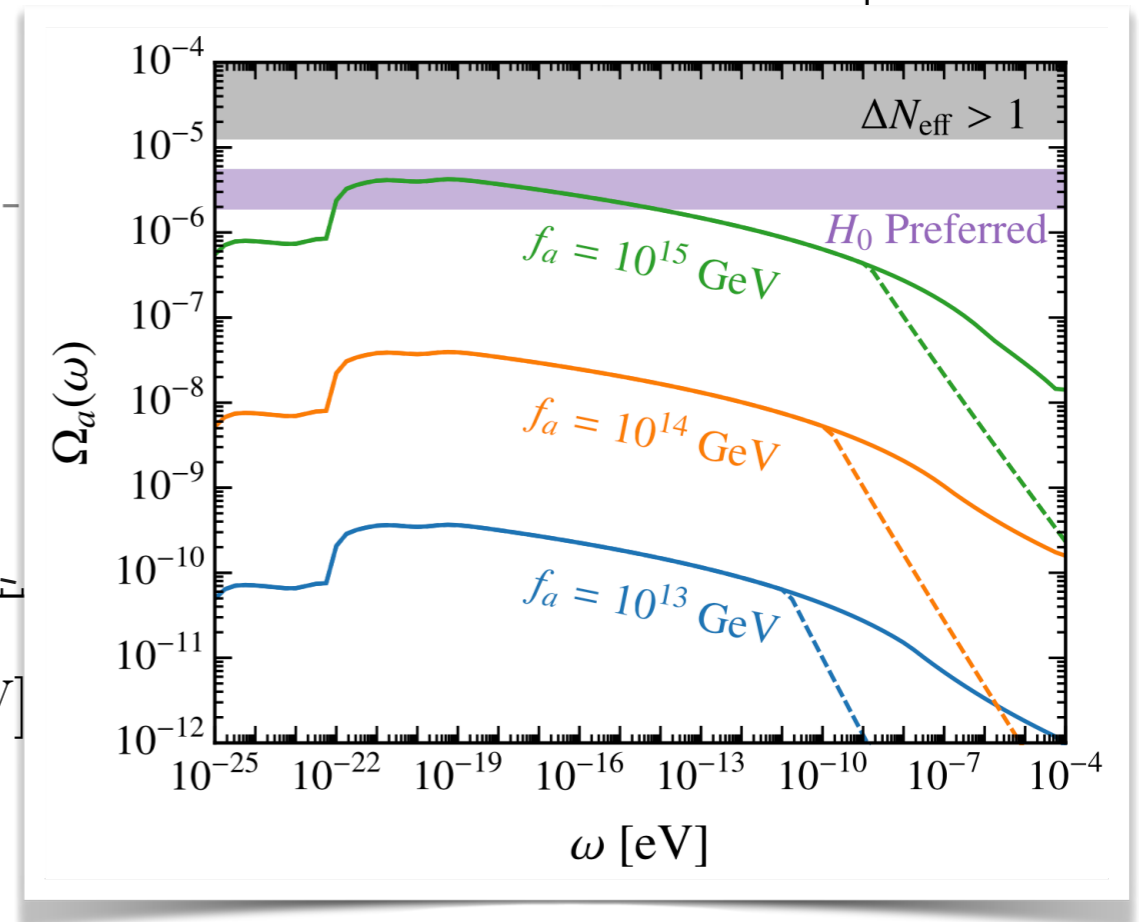


Finalizing this analysis with ADMX

Cosmic Strings



Cosmic string spectrum depends on the symmetry breaking scale f_a (here 10^{15} GeV) and the exact distribution is an area of active debate
 We follow [Gorghetto, Hardy, Villadoro 2018, 2020], but see also [Buschmann+ 2021] & [Dine, Fernandez, Ghalsasi, Patel 2020]



Parametric Resonance

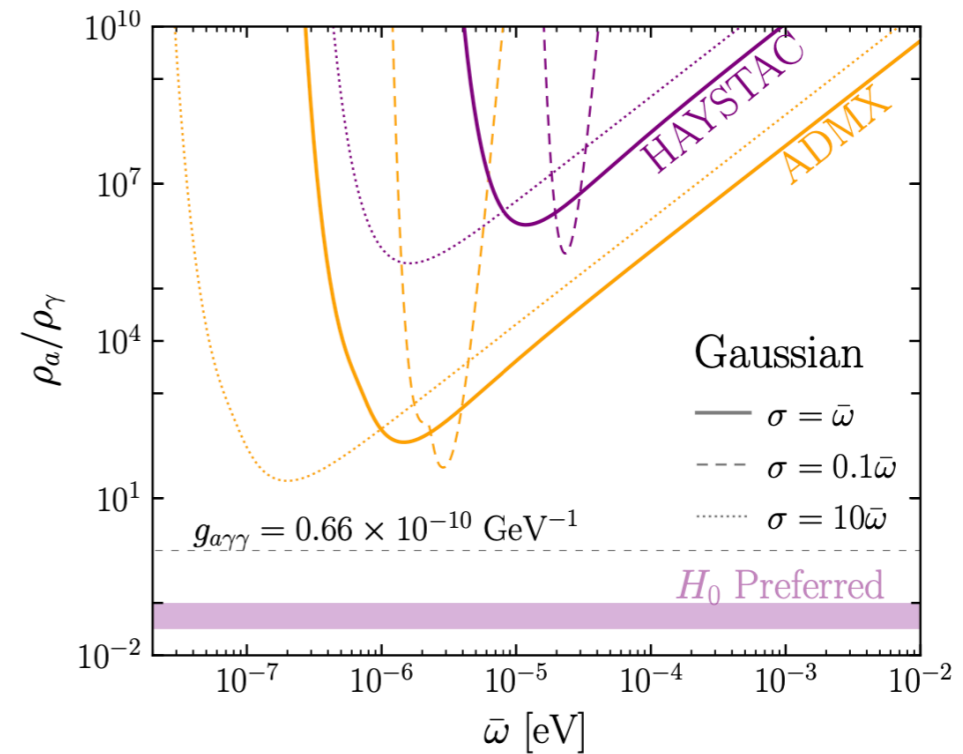
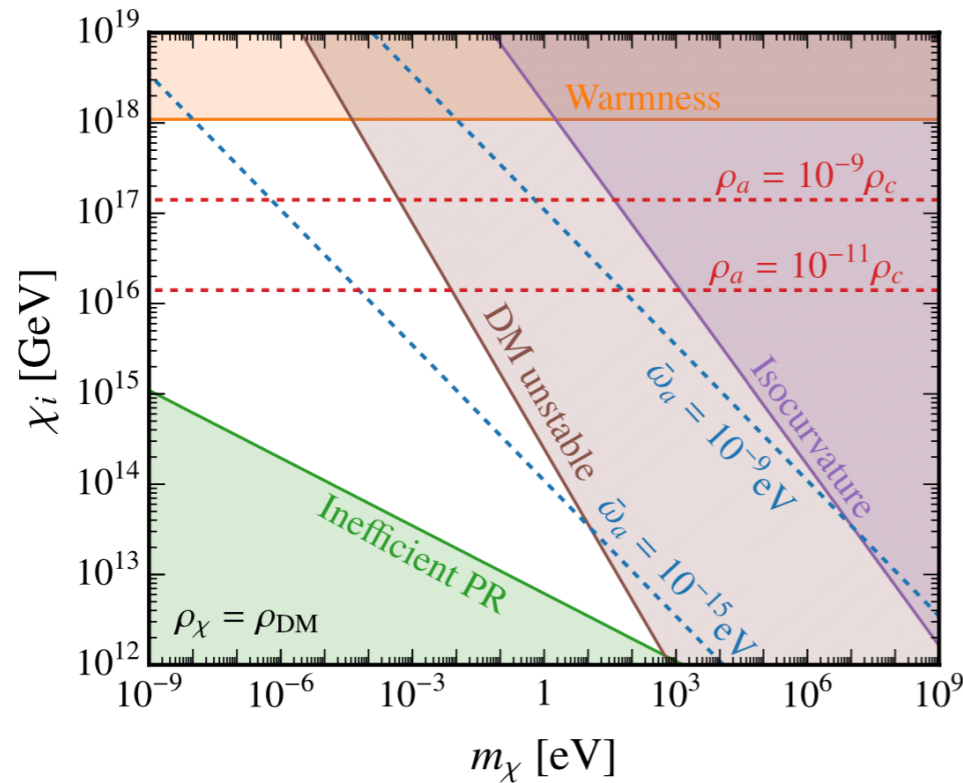
$$V(\Phi) = \lambda^2 \left(|\Phi|^2 - f_a^2/2 \right)^2$$

Oscillations when $m_\chi^{\text{eff}}(\chi_i) \simeq \lambda \chi_i \sim H$

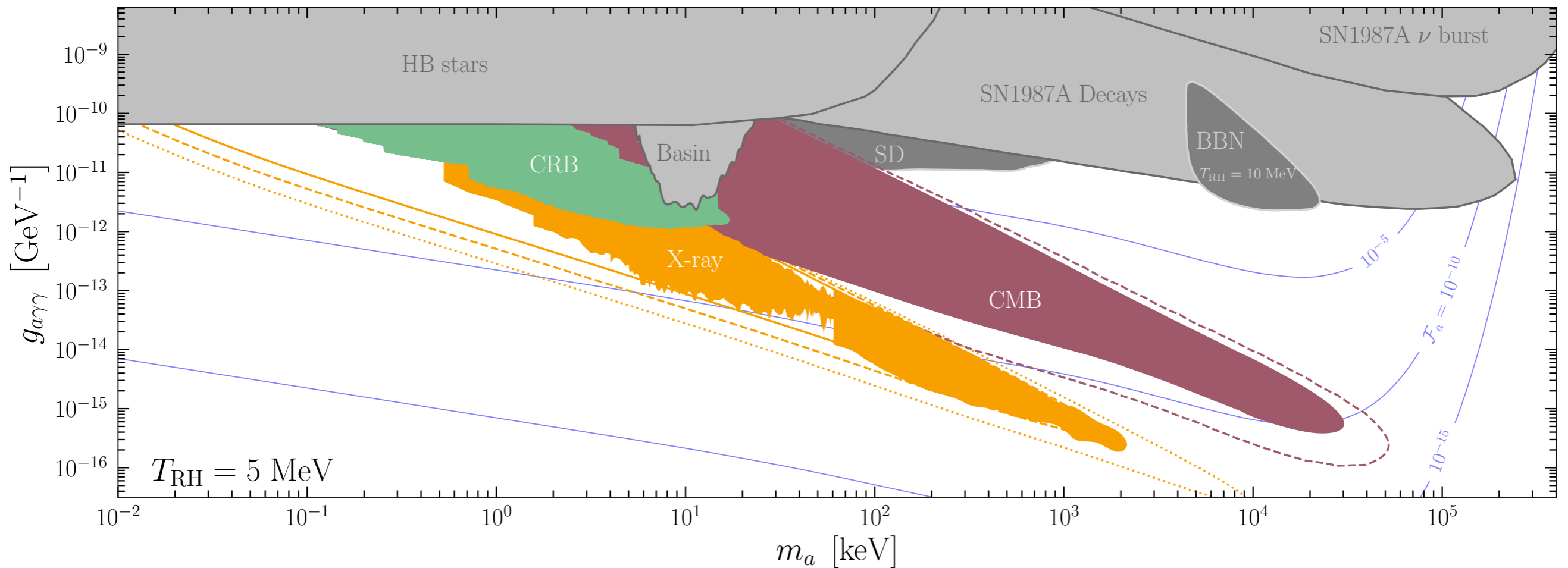
Typical energy: $\bar{\omega}_a \sim m_\chi^{\text{eff}}(\chi_i) \left(\frac{s(T_0)}{s(T_{\text{osc}})} \right)^{1/3} \sim 10^{-15} \text{ eV} \left(\frac{m_\chi^{\text{eff}}(\chi_i)}{\text{MeV}} \right)^{1/2}$

Energy density: $\Omega_a \sim 3 \times 10^{-7} \left(\frac{\chi_i}{M_{\text{Pl}}} \right)^2$ ← detectable?

Assume χ dark matter



The Irreducible Axion



The Irreducible Axion

Take $m_a = 10$ keV

Early Universe: photon conversion ($\gamma e \rightarrow ae$)
freezes-in axions

$$\mathcal{F}_a \simeq 10^{-4} \left(\frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \left(\frac{T_{\text{RH}}}{5 \text{ MeV}} \right) \quad (1)$$

$= \rho_a / \rho_{\text{DM}}$

UV dominated

The Irreducible Axion

X-ray constraints at ~ 10 keV require

$$\tau_{\text{DM}} \gtrsim 10^{29} \text{ s} \Rightarrow g_{a\gamma\gamma}^{\text{DM}} \lesssim 7 \times 10^{-19} \text{ GeV}^{-1} \simeq 10^{-8} g_{a\gamma\gamma}^{\text{HB}}$$

Must satisfy $\rho_a/\tau_a \lesssim \rho_{\text{DM}}/\tau_{\text{DM}}$ and $\tau^{-1} \propto g_{a\gamma\gamma}^2$, so

$$\mathcal{F}_a \lesssim \frac{\tau_a}{\tau_{\text{DM}}} = \left(\frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}} \right)^2 \quad (2)$$

The Irreducible Axion

$$\mathcal{F}_a \simeq 10^{-4} \left(\frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \left(\frac{T_{\text{RH}}}{5 \text{ MeV}} \right) \quad (1)$$

$$\mathcal{F}_a \lesssim \frac{\tau_a}{\tau_{\text{DM}}} = \left(\frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}} \right)^2 \quad (2)$$

Combine (1) and (2)

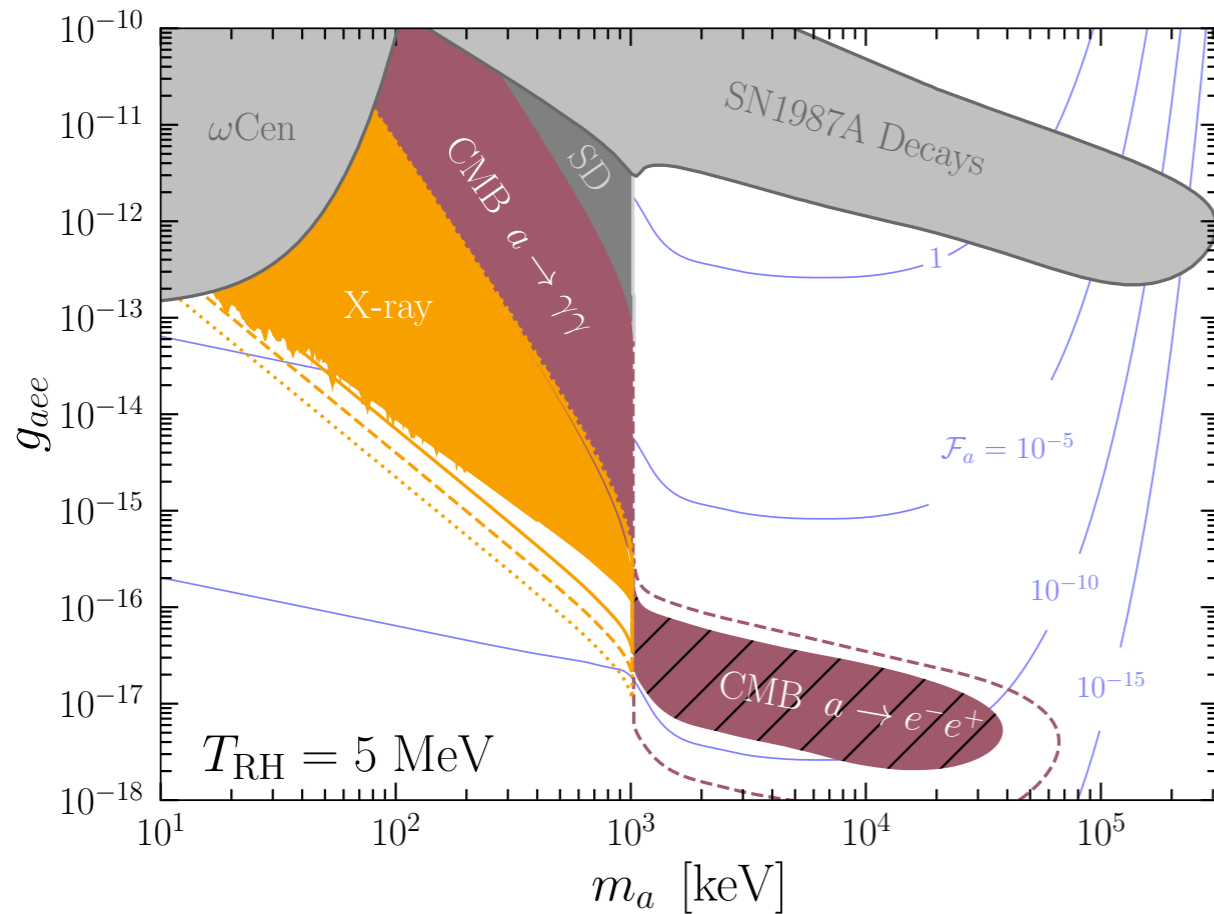
$$10^{-4} \left(\frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \lesssim \left(\frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}} \right)^2$$

$$\Rightarrow \frac{g_{a\gamma\gamma}}{g_{a\gamma\gamma}^{\text{HB}}} \lesssim \left[10^4 \left(\frac{g_{a\gamma\gamma}^{\text{DM}}}{g_{a\gamma\gamma}^{\text{HB}}} \right)^2 \right]^{1/4} \simeq (10^{-12})^{1/4} \simeq 10^{-3}$$

$$\Rightarrow g_{a\gamma\gamma} \lesssim 7 \times 10^{-14} \text{ GeV}^{-1} \ll g_{a\gamma\gamma}^{\text{HB}}$$

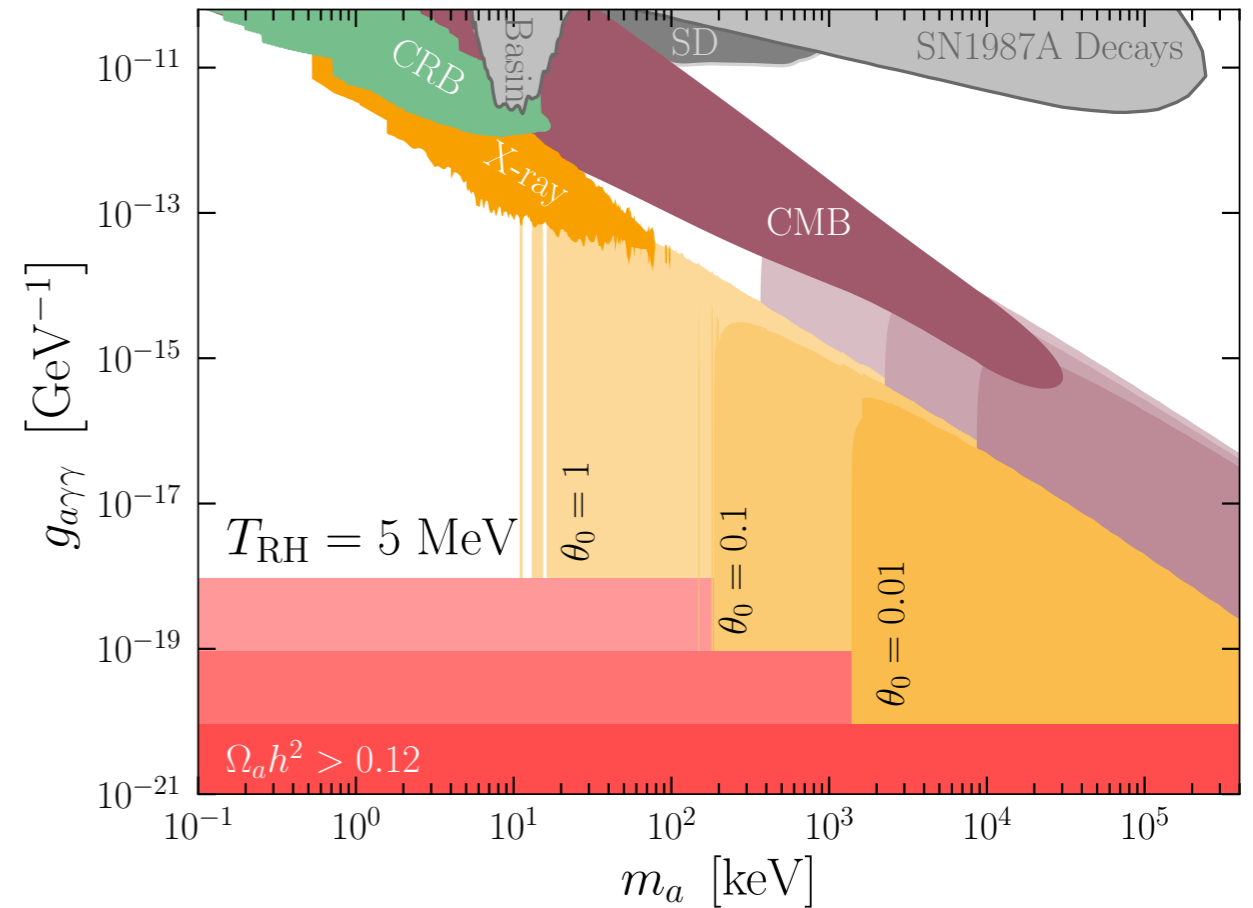
The Irreducible Axion

Electron Couplings



$$\frac{g_{aee}}{2m_e} (\partial_\mu a) \bar{e} \gamma^\mu \gamma_5 e$$

With misalignment



QCD Axion Mass

At $T = 0$ one can compute the axion effective potential

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left(\frac{a}{2f_a} \right)}$$

Expanding out, we find

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 0.5 \frac{m_\pi f_\pi}{f_a}$$

[Cortona, Hardy, Vega, Villadoro 2015]

Axion Dark Matter

In an FRW universe, the axion evolves according to

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

For early times, $H \gg m_a$, $a(t) = a_0$

At late times, $H \ll m_a$

$$a = \left[\frac{R(H = m_a)}{R(t)} \right]^{3/2} a_0 \cos(m_a t)$$

The energy density, $\rho \propto a^2$, behaves like CDM

$$\rho(t) = \rho(H = m_a) \left[\frac{R(H = m_a)}{R(t)} \right]^3$$

See, e.g. [Hook 2018]

Axion Dark Matter

Signal Power of the QCD Axion

$$P_a \simeq 5 \text{ yW} \quad \text{at } 5 \text{ GHz}$$

10^{-24}

For a detailed discussion see [Brubaker 2018]