MAX-PLANCK-INSTITUT FÜR PHYSIK → Ap. Ag≥

A look inside the proton

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Divide et impera

How can we describe an LHC event?

- QCD loves to split particles, how do we treat them? •
- How and where can we absorb the collinear divergence? •
- What is the role of PDFs in a phenomenological calculation? •
- "Shut up and calculate" a cross-section •



An LHC event looks like...



CMS Experiment at the LHC, CERN Data recorded: 2012-Sep-03 20:11:56.343965 GMT Run / Event / LS: 202178 / 412076062 / 328





[www.i2u2.org/elab/cms/ispy-webgl]











Short-scale interaction







Short-scale interaction







Thanks to the asymptotic freedom, we can use pQCD for the hard process:

$$\hat{\sigma} = \sigma_{LO} + \alpha_s \sigma_{NLO} + \alpha_s^2 \sigma_{NNLO} + \dots$$







IR divergences

How we can describe an LHC event?

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Final-state splitting

In the collinear ($\theta \ll 1$) and soft ($E \ll p$) limit





 $z = energy fraction \in [0,1]$ $k_t = \text{transverse mom} \in [0, +\infty)$

$$\sim \sigma_h \alpha_s \frac{\mathsf{d}E}{E} \frac{\mathsf{d}\theta^2}{\theta^2} = \sigma_h \alpha_s \frac{\mathsf{d}z}{1-z} \frac{\mathsf{d}k_t^2}{k_t^2}$$

$$\sim -\sigma_h \alpha_s \frac{\mathrm{d}z}{1-z} \frac{\mathrm{d}k_t^2}{k_t^2}$$

If we are inclusive, the divergent real and virtual corrections cancel out!









Initial-state splitting



Cross-sections with incoming partons are not collinear safe!



 σ_{is}

 $d\sigma_h$



The hard contributions have a different longitudinal momentum!

$$d\sigma_{h+g} \simeq \sigma_h(zp) \ \alpha_s \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$
$$d\sigma_{h+V} \simeq -\sigma_h(p) \ \alpha_s \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$\sim \alpha_{s} \int_{0}^{Q^{2}} \frac{\mathrm{d}k_{t}^{2}}{k_{t}^{2}} \int_{0}^{1} \left[\sigma_{h}(zp) - \sigma(p) \right] \frac{\mathrm{d}z}{1 - z}$$

$$\rightarrow \infty \qquad \text{finite}$$

$$\kappa_{T} \rightarrow 0$$

Regularise PDFs

- How we can describe an LHC event?
- QCD loves to split particles, how do we treat them? •
- What is the role of PDFs in a phenomenological calculation?
- "Shut up and calculate" a cross-section \bullet



How and where can we absorb the collinear divergence?

Introduce a factorisation cut-off

The divergence arises from a soft transverse scale treated in pQCD:

- Introduce a cut-off μ_F for the splitting
- Use a PDF for treating the non-perturbative region below $k_T < \mu_F$

$$\sigma_{is} = \int_{0}^{1} dx \ [\sigma_{is}]_{cutoff} \ f(x) \sim \alpha_{s} \int_{\mu_{F}^{2}}^{Q^{2}} \frac{dk_{T}^{2}}{k_{T}^{2}} \int_{0}^{1} dx \int_{0}^{1} \frac{dz}{1-z} \left[\sigma_{h}(zxP) - \sigma_{h}(xP)\right] f(x)$$

 μ_F is an arbitrary scale that factorises proton non-perturbative dynamics from the perturbative hard process



now finite





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 μ_F is an arbitrary scale that factorises proton non-perturbative dynamics from the perturbative hard process



now finite







DGLAP flow

Similarly to the running of $\alpha_s(\mu_R)$, we can change μ_F and get a Renormalisation Group Flow for PDFs:

> Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations

Also gluons represent a proton component! DGLAP is a matrix in flavour space









From clata to clata

- How we can describe an LHC event?
- QCD loves to split particles, how do we treat them? \bullet
- How and where can we absorb the collinear divergence? •

What is the role of PDFs in a pheno calculation?

"Shut up and calculate" a cross-section





10/11







The LHC master formula

- How we can describe an LHC event? \bullet
- QCD loves to split particles, how do we treat them? \bullet
- How and where can we absorb the collinear divergence? •
- What is the role of PDFs in a pheno calculation? •

"Shut up and calculate" a cross-section

The LHC master formula $\sigma(P_1 P_2 \to b\bar{b}H + X) = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \sum_n \alpha_s^n(\mu_R) \,\hat{\sigma}^{(n)}(x_1 x_2 E_{CM}^2, \mu_R^2, \mu_F^2)$

 $x_1 =$ momentum fraction of the parton

 f_i = probability to find the parton *i* in the proton







The LHC master formula

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11/11



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The LHC master formula $\sigma(P_1 P_2 \to b\bar{b}H + X) = \sum \left[dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \sum \alpha_s^n(\mu_R) \,\hat{\sigma}^{(n)}(x_1 x_2 E_{CM}^2, \mu_R^2, \mu_F^2) \right]$

 $x_1 =$ momentum fraction of the parton

 f_i = probability to find the parton *i* in the proton





Additional non perturbative effects

11/11















Thank you for your attention!







Backup slides (1) PWG and MiNNLO



Shower Monte Carlo

The Parton Shower formalism is based on **collinear factorisation** with a probabilistic description of the splitting process.

Similarly to a radioactive decay, the probability of evolving between two scales and emitting no gluons is $\begin{bmatrix} dt' \\ dt'$

$$\Delta_t = \exp\left[-\right]$$

Using this form factor we can deduce the SMC prediction with the first emission

$$\begin{split} \langle \mathcal{O} \rangle &= \int d\Phi_n B(\Phi_n) \left[\mathcal{O}(\Phi_n) \Delta_{t_0} + \int_{t_0} \frac{dt}{t} dz d\varphi \mathcal{O}(\Phi_n, \phi_r) \Delta_t \frac{\alpha}{2\pi} P(z) \right] \quad \mathbf{e} \\ &\simeq \int d\Phi_n B(\Phi_n) \left[\mathcal{O}(\Phi_n) + \int_{t_0} \frac{dt}{t} dz d\varphi \left(\mathcal{O}(\Phi_n, \phi_r) - \mathcal{O}(\Phi_n) \right) \frac{\alpha_s}{2\pi} P(z) \right] \end{split}$$

Marchesini, Webber [NPB238(1984)1] Sjostrand [PLB157(1985)321] Altarelli, Parisi [NPB126(1977)298]



$$\frac{dt'}{t'}dz'd\varphi'\frac{\alpha_s}{2\pi}P(z') = \frac{non-radiation}{probability}$$

 $exp(-\lambda t) \lambda \delta t = probability$ the 1st rad





NLO



 \checkmark NLO accuracy for inclusive observables

Reduced theoretical uncertainty

✓ Correct quantum interference

- Wrong shape for small- p_T region
- Description only at the parton level
- Computationally expensive



SMC (LOPS)

- Total normalisation accurate only at LO
- Poor description at high- p_T
- Partial interference through shower ordering
- \checkmark Sudakov suppression of small- p_T emissions (LL resummation)
- \checkmark Simulate high-multiplicity events at the hadron level
- Computationally cheap

HERWIG, SHERPA, PYTHIA, ...

Approaches are complementary: combine them in a consistent way



Matching problem



tested solutions.

POWHEG Idea

result.



- **Double counting** can be easily solved by applying a cut in phase space:
- **Reject hard jets** produced by PS with $p_T > Q_m$
- But how can we obtain smooth distributions without a critical dependence on the matching
- MC@NLO [Frixione, Webber, 2002] and POWHEG [Nason, 2004] are two fully
 - Write a simplified Monte Carlo that generates just one emission (the hardest one) which alone gives the correct NLO
 - $\Delta^{pwg} = \exp \left| \right| \text{ exact real-radiation probability above } p_T \right|$















POWHEG in a nutshell

The exact NLO prediction is

$$\langle \mathcal{O} \rangle = \int d\Phi_n \mathcal{O}(\Phi_n) \overline{B}(\Phi_n) + \int d\Phi_n d\phi_{rad}$$

Comparing with the SMC

$$\langle \mathcal{O} \rangle_{SMC} \simeq \int d\Phi_n \left[\mathcal{O}(\Phi_n) B(\Phi_n) + \frac{B(\Phi_n)}{t} \int_{t_0} \frac{dt}{t} dz d\varphi \left(\mathcal{O}(\Phi_n, \phi_r) - \mathcal{O}(\Phi_n) \right) \frac{\alpha_s}{2\pi} P(z) \right],$$

we deduce the Sudakov form factor and the shower formula in POWHEG



 $\bar{B} = B + V + \int d\phi_{rad} R$

 $\left(\mathcal{O}(\Phi_n,\phi_{rad})-\mathcal{O}(\Phi_n)\right)R(\Phi_n,\phi_{rad})$





IR divergences

The radiation of a massless particle produces divergences: a manifestation of the degeneration of these states















virtual contribution.

order in perturbation theory!



C. Biello, Multiscale improved calculations for LHC



Resummation from factorisation

Consider a physical quantity $\mathcal{O}(M^2, m^2)$ in which m^2 measures the distance from the IR region.

If
$$m^2 \ll M^2$$
, $\mathcal{O}(M^2, m^2) =$

 $\mathcal{O} \text{ is } \mu - \text{independent } \Rightarrow \frac{1}{H} \frac{d \ln H}{d \ln \mu^2} = -\frac{1}{S} \frac{d \ln S}{d \ln \mu^2} =: \gamma(\mu^2)$

Solving the differential equation,

$$\sqrt{10^2 \text{ for } m^2 \rightarrow 0}$$

C. Biello, Multiscale improved calculations for LHC





$$\left[-\int_{m^2}^{M^2} \frac{dq^2}{q^2} \gamma(q^2)\right]$$

Sudakov form factor: it captures at all order the log-enhanced terms











Transverse momentum resummation

What is the probability that a boson is produced with transverse momentum $< p_T$?

$$\mathscr{P} \simeq -\frac{\#\alpha_s \ln^2 \frac{Q}{p_T}}{p_T} + \mathscr{O}(\alpha_s^2) \to \exp\left[-\frac{\#\alpha_s \ln^2 \alpha_s}{p_T}\right]$$

for small p_T we need to sum up the logs

In general we have a tower of logs

m = n

m = n - 1

$$\exp\left[-\sum_{n,m}\alpha_s^n\ln^m\frac{Q}{p_T}\right]$$

$$m = n + 1 \longrightarrow$$
 Leading Logs (LL)

$$\rightarrow$$
 Next-To-LL (NLL)

$$\rightarrow$$
 Next-To-NLL (NNL







We can introduce the Sudakov factor, but we don't have to spoil the accuracy:

$$\mathcal{F}\sigma_{Xj} = \sigma_1 + \alpha_s(\sigma_2 + S_1\sigma_1) \neq \sigma_{Xj}^{FO} \left(1 + \mathcal{O}(\alpha_s^2)\right)$$

We define the MiNLO cross section in order to recover the merging

$$\sigma_{MiNLO} = \mathscr{F}\left(\sigma_1(1 - \alpha_s S_1) + \alpha_s \sigma_2\right) = \sigma_{Xj}^{FO}\left(1 + \mathcal{O}(\alpha_s^2)\right)$$

We can keep track of logs fixing the scales and defining the MiNLO Sudakov

$$\mathscr{F}(p_T, Q) = \exp\left\{-\int_{p_T}^{Q} \frac{dq^2}{q^2} \left[A\left(\alpha_s(q)\right)\ln\frac{Q^2}{q^2} + B\left(\alpha_s(q)\right)\right]\right\}$$

MiNLO



Hamilton, Nason, Zanderighi [1206.3572] Hamilton, Nason, Oleari, Zanderighi [1212.4504]



MINNLO

NLO Xi -> NNLO X

 $\frac{d\sigma^{sing}}{dp_T^2 d\Phi_X} = \frac{d}{dp_T^2} \left\{ \mathscr{F}(p_T, Q) \mathscr{L}(p_T) \right\} =: \mathscr{F}(p_T, Q) D(p_T)$ Sudakov form factor Luminosity: it also contains



Monni, Nason, Re, Wiesemann, Zanderighi [1206.3572]







FORHEG
machinery
Deee EVENTS (a) NLO
with the har
radiation including

$$\bar{B}(\Phi_{Xj}) = B(\Phi_{Xj}) + \left[V(\Phi_{Xj}) + \int d\phi_{rad}R(\Phi_{Xj})\right]$$

 $\overline{B(\Phi_{Xj})} = \overline{\mathcal{F}(p_T, Q)} \left\{ B(\Phi_{Xj}) \left(1 - \alpha_s S_1\right) + \left[V(\Phi_{Xj}) + \int d\phi_{rad} R(\Phi_{Xjj})\right] + \left[D_3 \text{-term}\right] F(\Phi_{Xj}) \right\}$



Monni, Nason, Re, Wiesemann, Zanderighi [1908.06987]

NNLO+PS timeline

Using the MiNNLO method, there was an increasing number of processes with predictions accurate at NNLO+PS.

NNLOPS MiNNLOPS

MiNNLO was firstly implemented for a color singlet production. The method can be extended to more complex processes.





 \mathbf{O}

We are working on QQF extension.





 $-\overline{Q}$ e.g. $b\overline{b}H$ production

Backup slides (2) bbH production



Higgs production via bottom fusion



- allows a direct evaluation of the bottom Yukawa coupling
- is enhanced in SUSY theories with large tan β and it can become the dominant channel

two ways of performing computations:





Although it is not the main production channel, the Higgs creation via bottom fusion



For all processes that feature bottom quarks at the hard-process level there are







decoupling/massive scheme

- It does not resum possibly large collinear logs
- Computing higher orders is more difficult due to higher multiplicity
- ✓ Mass effects $O(m_h/m_H)$ are there at any order
- \checkmark Straightforward implementation in MC event generators at LO and NLO

massless scheme

- ✓ DGLAP evolution resum initial state logs into f_b
- ✓ Computing higher orders is easier
 - Neglecting $O(m_b/m_H)$, it yields less accurate description of bottom kinematic distribution
 - Implementation in MC depends on the gluon splitting model in the PS







Historical LO comparisons

Large differences in the predictions were first observed at the leading order: the effect of collinear resummation is extremely large.



For $\mu_F = m_H/4$, FO computations in the different schemes become compatible, indeed the collinear logs have a small effect. This also improved the convergence of the perturbation series.

The improvement of the compatibility opens the possibility to match together the predictions at least at the inclusive level (Santander matching, FONLL...)





Differences between schemes

results.







- Lot of progress in understanding the origin of the differences. The predictions can be merged into a consistent picture by taking into account two main
 - 1. At NLO, the resummation effects of collinear logs are important only at high Bjorken-*x*
 - 2. The possibly large ratios m_H^2/m_h^2 are always accompanied by universal phase space factors f

$$\ln^{2} \frac{m_{H}^{2} f}{m_{b}^{2}} = \ln^{2} \frac{\tilde{\mu}^{2}}{m_{b}^{2}}, \quad \tilde{\mu} < m_{H}$$

FONLL

FONLL matches the flavour schemes $\sigma^{FONNL} = \sigma^{4FS} + \sigma^{5FS}$ – double couting.

For a consistent subtraction, we have to express the two cross-sections in terms of the same α_{s} and PDFs.

 Currently, the flavour matching for bbH is performed at

 $\text{FONNL}_C := \text{N}^3 \text{LO}_{5FS} \bigoplus \text{NLO}_{4FS}$.

Differential FONLL applied for Z+b-jet $d\sigma^{FONLL} = d\sigma^{5FS} + \left(d\sigma_{m_b}^{4FS} - d\sigma_{m_b \to 0}^{4FS} \right)$ Forte, Napoletano, Ubiali [1508.01529] Forte, Napoletano, Ubiali [1607.00389]



[Gauld, Gehrmann-De Ridder, Glover, Huss, Majer, 2005.03016]





Exclusive observables

Recent developments in fully differential calculations, for example:

- 1. Introduce an unphysical scale μ_b in order to switch from 4FS to 5FS in a region where mass effects and collinear logs are not crucial [Bertone, Glazov, Mitov, Papanastasiou, Ubiali, 1711.03355]
- 2. Massive 5FS at NLO [Krauss, Napoletano, 1712.06832]
- 3. Differential FONLL applied for Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, Huss, Majer, 2005.03016] $d\sigma^{FONLL} = d\sigma^{5FS}$



$$S + \left(d\sigma_{m_b}^{4FS} - d\sigma_{m_b \to 0}^{4FS} \right)$$

H(bb) production in 5FS

pp->H(bb)@LHC 13 TeV do/dptH [fb] 10² Using the Sudakov factor $\mathcal{F}(p_T, Q)$, we can MiNNLO FOatQ (LHE) 10 MiNNLO pt (LHE) resum the logs at low transverse momenta. NNLO+NNLL [1403.7196] 10^{0} Including the 2-loop correction, we obtained inclusive predictions for Higgs accurate at NNLO. 10^{-1} The non-singular part of the cross-section was 10⁻² evaluated with two different scale choices as 10⁻³ $d\sigma/d\sigma_{MINNLO}$ FOatQ (LHE) done for the diphoton production. 1.4 Gavardi, Oleari, Re [2204.12602] 1.2 0.8 100 50 150 200 Use p_T as reference for scales or ptH

$$\bar{B}(\Phi_{Xj}) = \mathscr{F}(p_T, Q) \left\{ B(\Phi_{Xj}) \left(1 - \alpha_s S_1 \right) + \left[V(\Phi_{Xj}) + \left[d\phi_{rad} R(\Phi_{Xjj}) \right] + \left[D_3 \text{-term} \right] F(\Phi_X) \right\} \right\}$$

evaluate them at the hard scale Q

C. Biello (MPP), Multiscale improved calculations for LHC





CB, Sankar, Wiesemann, Zanderighi in progress

ICTP Summer School on Particle Physics, Student Talks







H(bb) production in 5FS

We implemented the MiNNLO method in 5 flavour-scheme for the Higgs production via bottom fusion in SM.

- Direct access to bottom Yukawa coupling
- Although it is a subdominant channel, it has effects on the shape of the transverse momentum spectrum
- In SUSY models, this channel can be enhanced

We started performing NLOps predictions in POWHEG framework for the Hj production.





ICTP Summer School on Particle Physics, Student Talks

Heavy flavours inside the proton?

flavours in the proton.





Thanks to the gluon splittings, we can have a DGLAP component of heavy





Higgs production via bottom fusion

We implemented a MC code for the Higgs production via bottom fusion in SM.

WHY?

- Direct access to bottom Yukawa coupling
- In SUSY models, this channel \bullet can be enhanced

HOW?

- We used NNPDF4.0 as PDFs sets in the **5FS** with bottom component included
- We used the MiNNLO method in POWHEG framework

















Data for PDF fits











PDFs renormalisation flow: DGLAP





Similarly to the running of $\alpha_s(\mu_R)$, we can change μ_F and get a Renormalisation Group Equation for PDFs:

> Dokshitzer-Gribov-Lipatov-Altarelli–Parisi (DGLAP) eqs



DGLAP flavour structure

Also gluons represent a proton component! DGLAP is a matrix in flavour space



 $P_{\overline{9}g} = P_{\overline{9}g}$ $P_{qq}^{LO}(z) = C_F\left(\frac{1+z^2}{1-z}\right)$ $\to C_F\left(\frac{1}{1-z}\right)_+$



$$P_{qq'} = 0 (a LO)$$

 $P_{ij}@LO$ DGLAP, 1972-77 P_{ii} @NLO Curci et al, 1980 P_{ij} @ N²LO Moch at al, 2004 P_{ii} @ N³LO Moch et al, 2020-