Glauber Gluons in SCET

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MIT

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based on work with Ira Rothstein,
(with earlier collaboration with Junegone Chay)

Outline

- Glauber Gluons: What? and Why?
- Review of "Glauber Region" in classic factorization

  literature: Collins et al, Bodwin

- Glauber Gluons in EFT

  power counting, gauges, ...

  literature: Liu & Ma, Idilbi & Majumder

here:

- direct computation of Glauber graphs (rules for loop integrals, matching IR, obtaining unambiguous results)
- 0-bin's (avoid double counting of other modes)
- forward scattering, glaubers in inclusive processes, glaubers in exclusive processes

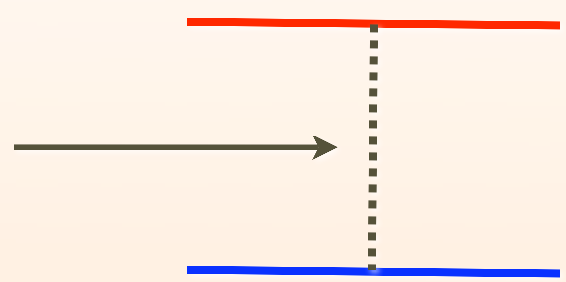
Glauber Introduction

- a "Coulomb" gluon for forward scattering of collinear particles

Glauber:

$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$

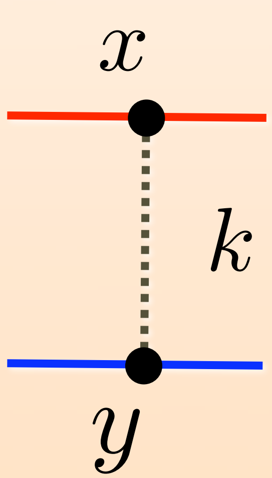
$+ \quad - \quad \perp$



$+ \quad - \quad \perp$

n-collinear: $p^\mu \sim Q(\lambda^2, 1, \lambda)$

$\bar{n}$-collinear: $p^\mu \sim Q(1, \lambda^2, \lambda)$



momentum space: $\dfrac{1}{k_\perp^2}$

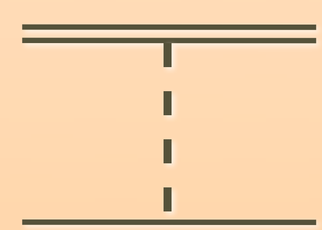
forward scattering that changes $\perp$ - momenta, but leaves collinear directions and large momenta intact

position space: $\delta(x^+ - y^+)\delta(x^- - y^-)\,(x_\perp - y_\perp)^{2\epsilon}\Gamma(-\epsilon)$

instantaneous in x^+ and x^-, ie. t and z

offshell, purely virtual, never appears in final states

- NRQCD analogy: potential gluons

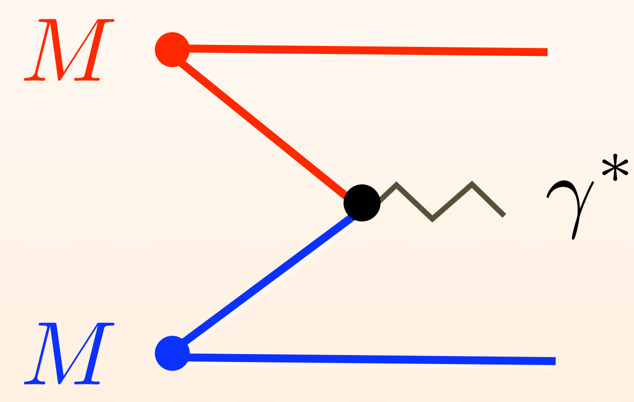


$\dfrac{1}{\vec{k}^2}$

- where can it show up?

<u>forward scattering</u> : ✓

✓ <u>hard scattering</u>:

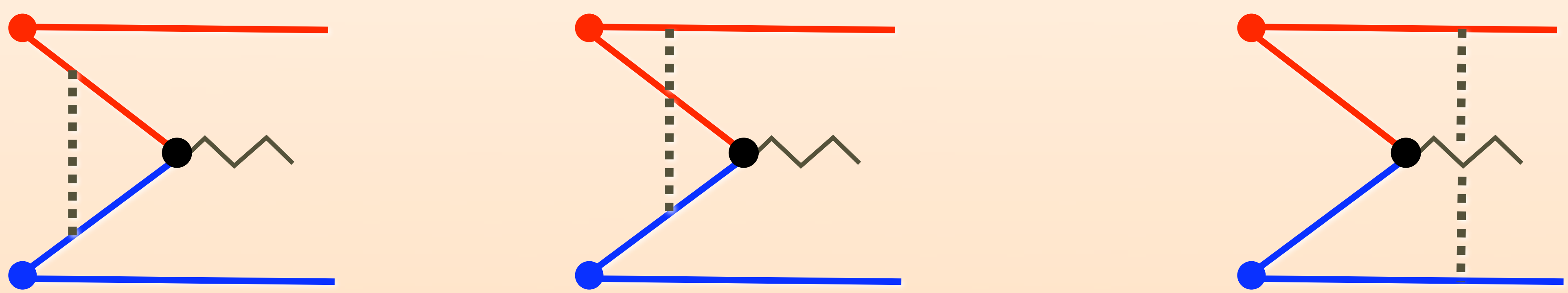


inclusive $MM \to X\gamma^*$

"active" or "spectators"

active-active **active-spectator** **spectator-spectator**

✓

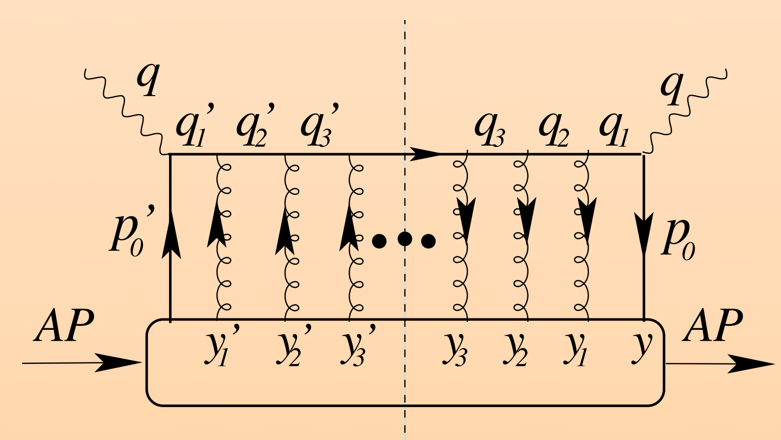


can potentially spoil factorization

<u>as an external field</u>:

eg. $e^- + \text{nucleus} \to e^- + \text{Jet}(k_\perp) + X$

Idilbi & Majumder (cf. SCET 2009)



Traditional Factorization Approach to Glaubers

Collins, Soper, Sterman 1985
Bodwin 1985
Collins, Soper, Sterman 1988

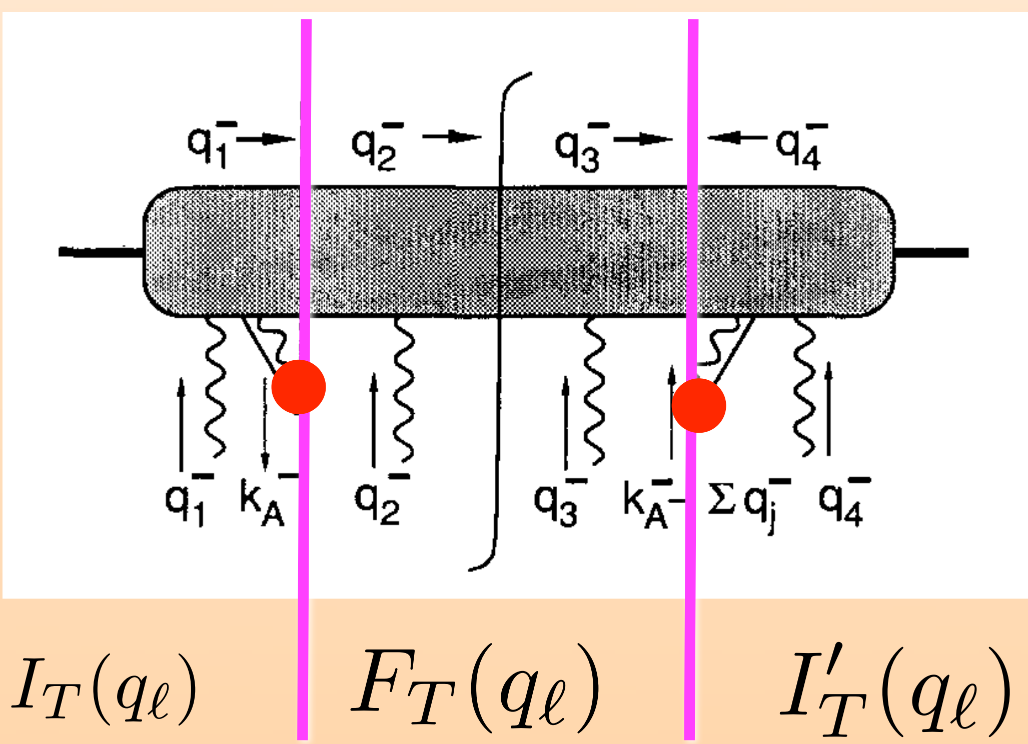
inclusive $p\bar{p} \to X\ell^+\ell^-$

Glaubers arise as an obstacle when canceling soft gluons to derive

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2 dY} = \sum_{i,j}\int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} H_{ij}^{\rm incl}\Big(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, q^2, \mu\Big)\ f_i(\xi_a,\mu)\, f_j(\xi_b,\mu)$$

➡ prove that in sum of graphs that we can deform contours out of "Glauber Region" of momenta at leading power

$\bar{n}$-collinear jet with soft attachments:



$I_T(q_\ell)$ | $F_T(q_\ell)$ | $I'_T(q_\ell)$

$$I_T(q_\ell) = \prod \frac{1}{\sum_\ell (q_\ell^- + i0) - \sum_j \frac{k_{j\perp}^2}{2k_j^+}}$$

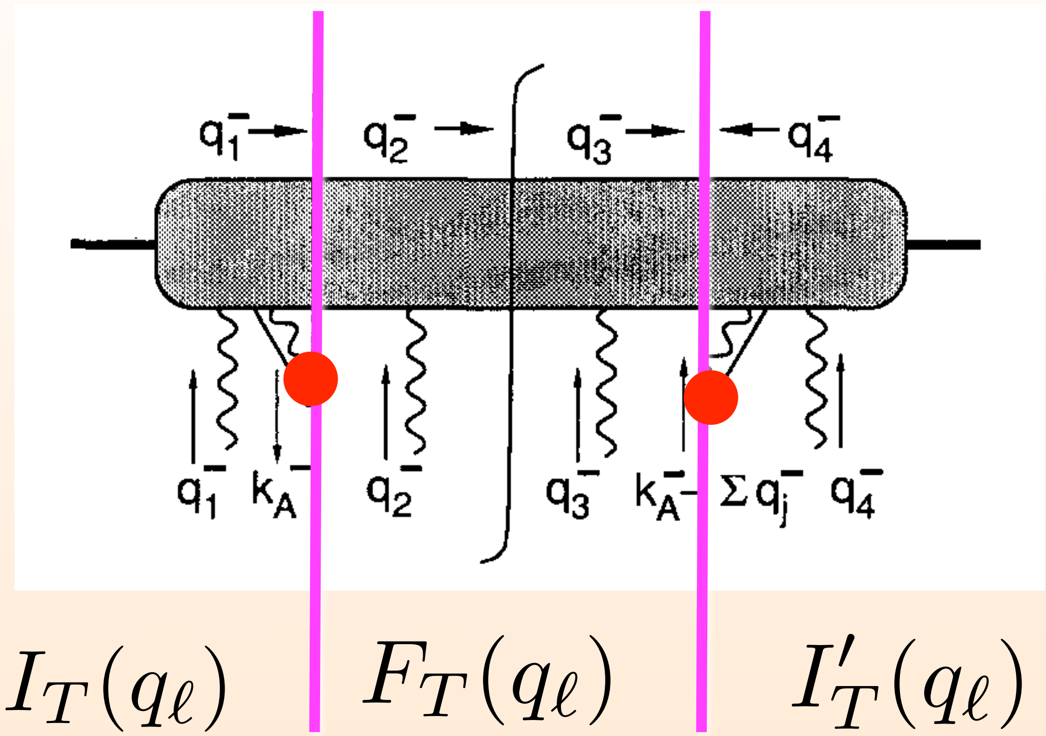
$$F_T(q_\ell) = \prod \frac{1}{\sum_\ell (q_\ell^- - i0) - \sum_j \frac{k_{j\perp}^2}{2k_j^+}} \quad \cdots$$

want to deform contours to soft region

$q_\ell^- \sim |q_\ell^\perp|$, drop $q_\ell^\perp$'s

but we are trapped by final state poles

$\bar{n}$-collinear jet with soft attachments:



$I_T(q_\ell)$ | $F_T(q_\ell)$ | $I'_T(q_\ell)$

Must prove that "R" = Rest of the graph (n-collinear, soft, hard) is independent of "V" = which soft vertices are on left or right of the cut.

Then take $\sum_{\text{cuts}}$ for $\bar{n}$-collinear to cancel final state interactions so we are free to deform the contours.

Must integrate over $\perp$- momenta internal to the jet and for external partons.

"R" is independent of "V":

Use x- ordered pert. theory. For any two V's can match up orderings where I_T and I'_T agree. So only $F_T(q_\ell, V)$. Summing over all final states this V dependence cancels.

Comments: Soft and Glauber are mixed, no separate identity.

Proof relies heavily on sum over final states. Hence its hard to extend to cases that are not fully inclusive.

Modes for this talk:

$$q_\perp \sim Q\lambda$$

perturbative

	$+ \; - \; \perp$	
Glauber	$(\lambda^2, \lambda^2, \lambda)$	
n-collinear	$(\lambda^2, 1, \lambda)$	jets
$\bar{\text{n}}$-collinear	$(1, \lambda^2, \lambda)$	
Usoft	$(\lambda^2, \lambda^2, \lambda^2)$	
Soft	$(\lambda, \lambda, \lambda)$	

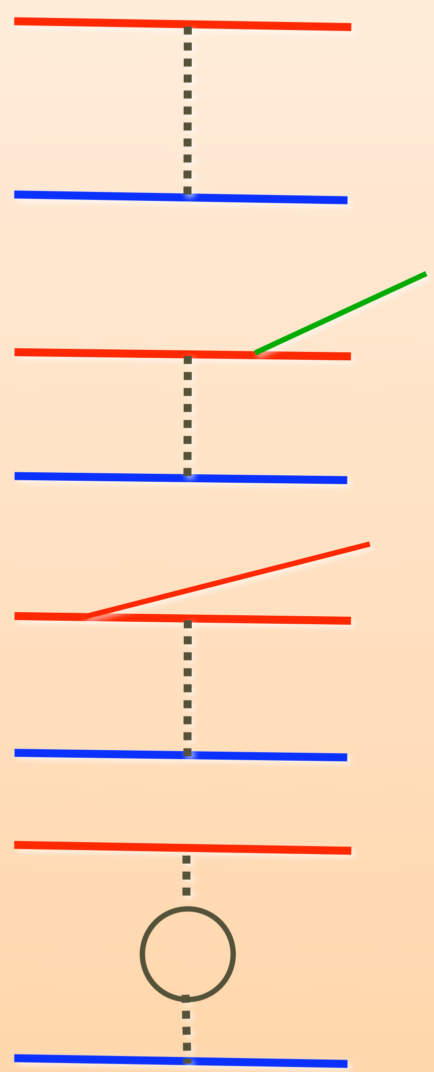
SCET with Glauber

glauber

usoft

collinear

soft

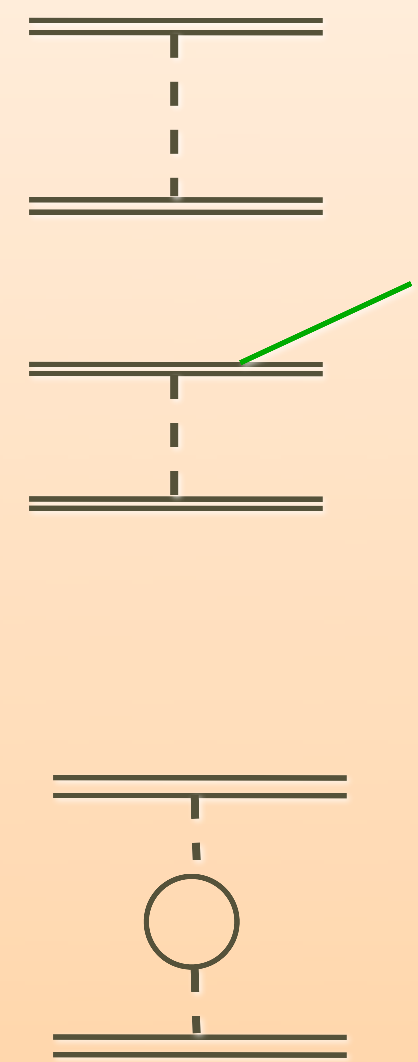


NRQCD analogy

potential

usoft

soft



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Usoft	$(\lambda^2, \lambda^2, \lambda^2)$	
Soft	$(\lambda, \lambda, \lambda)$	

SCET zero-bins

glauber $G - G_U$

usoft U no subt.

collinear $C - C_U - C_G + C_{G_U}$

soft $S - S_U - S_G + S_{G_U}$

NRQCD zero-bins

Manohar & I.S.

potential P subtractions are power suppressed

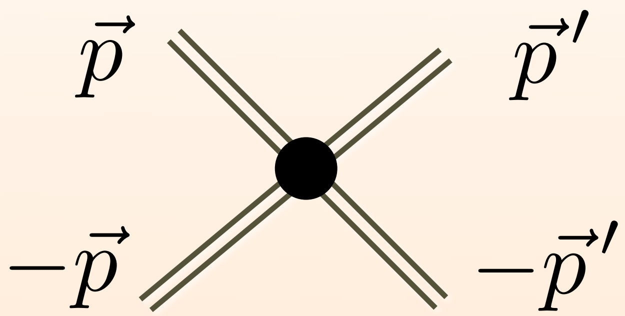
usoft U no subt.

soft $S - S_U - S_P + S_{P_U}$

Glauber Lagrangians

In NRQCD the potential gluon is an offshell mode, and does not need to be added to the Lagrangian.

Luke, Manohar, Rothstein
Pineda & Soto



$$\mathcal{L}_P = -\sum_{\vec{p},\vec{p}\,'} V(\vec{p},\vec{p}\,')\ \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}}$$

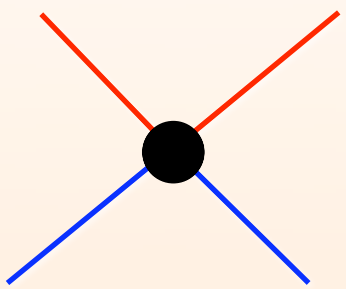
In NRQCD there is no sense in talking about Gauge Transformations for potential gluons. Matching to V is gauge independent (using full theory gauge symmetry & e.o.m.).

We can talk about power counting for V, without introducing a potential gluon field.

Iterations of V yield Green's function for Schroedinger Equation

Glauber Lagrangians

Apply this to Glaubers:



$$\mathcal{L}_G = -\sum_{p_\perp, p'_\perp} V(p_\perp, p'_\perp)\bar{\xi}_{n,p'_\perp}\,\xi_{n,p_\perp}\,\bar{\xi}_{\bar{n},-p'_\perp}\,\xi_{\bar{n},-p_\perp}$$

$$V(p_\perp, p'_\perp) = \frac{8\pi\alpha_s}{(p'_\perp - p_\perp)^2} + \ldots$$

any covariant gauge

No "glauber gauge transformations".

But an auxiliary Glauber Lagrangian is useful for calculations:

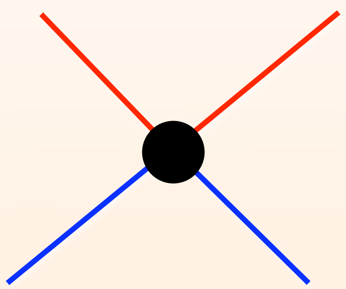
$$\begin{aligned}\mathcal{L}_G^{\rm aux} &= \bar{\xi}_n \frac{\bar{n}\!\!\!/}{2}\, n\cdot A_G\, \xi_n + \bar{\xi}_{\bar{n}} \frac{n\!\!\!/}{2}\, \bar{n}\cdot A_G\, \xi_{\bar{n}} \\ &+ A_G^\mu\, \mathcal{P}_\perp^2\, A_{G\mu} + \ldots\end{aligned}$$

Can pick: $A_G^\mu \sim \lambda^2$ (consistent with Liu & Ma, Idilbi & Majumder)

If A_G^μ is treated like an external source, as it was in Idilbi & Majumder, then we can consider $\mathcal{L}_G^{\rm aux}$ terms to be source couplings.

Glauber Lagrangians

Apply this to Glaubers:



$$\mathcal{L}_G = -\sum_{p_\perp, p'_\perp} V(p_\perp, p'_\perp)\bar{\xi}_{n,p'_\perp}\,\xi_{n,p_\perp}\,\bar{\xi}_{\bar{n},-p'_\perp}\,\xi_{\bar{n},-p_\perp}$$

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higher order: $\partial A_G A_G A_G$

$\bar{\xi}_n A_{G\perp}^2 \xi_n$

$\partial A_G A_G A_{usoft}$

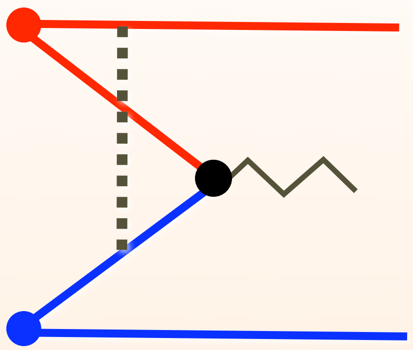
(consistent with Liu & Ma, Idilbi & Majumder)

leading order:

$\partial A_G A_{soft} A_{soft}$

$\bar{q}_{soft}\, A_G\, q_{soft}$

Glaubers do not generically give Wilson lines:



loop momentum: $k^\mu \sim (\lambda^2, \lambda^2, \lambda)$

red propagators see k^+

blue propagators see k^-

everyone can see $k_\perp$

$$\int d^d k \frac{1}{k_\perp^2 \left[-k^+ + p_{1\bar{q}}^+ - \frac{(k_\perp^2 - p_{1\bar{q}}^\perp)^2}{p_{1\bar{q}}^-} + i0 \right] \left[k^+ + p_{1q}^+ - \frac{(k_\perp^2 + p_{1q}^\perp)^2}{p_{1q}^-} + i0 \right] \left[-k^- + p_{2\bar{q}}^- - \frac{(k_\perp - p_{2\bar{q}}^\perp)^2}{p_{1q}^-} + i0 \right]}$$

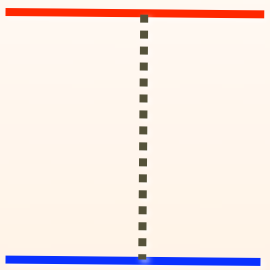
no shift in k^+ can makes this look like a Wilson line

shifting k^- this looks like a Wilson line

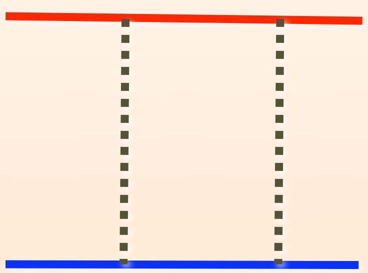
Hence the “proof” of Liu and Ma of decoupling of Glaubers in SCET via Wilson lines fails.

Lets Calculate !

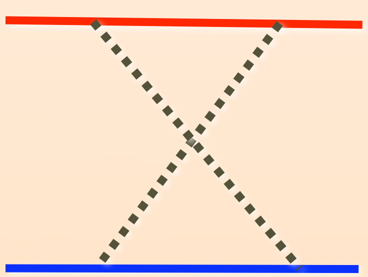
Forward Scattering Graphs (abelian)



$$= -i\frac{8\pi\alpha_s}{q_\perp^2}$$



$$\int đ^d k \frac{1}{k_\perp^2 (k_\perp - q_\perp)^2 \big[k^+ + p^+ - \frac{(p_\perp + k_\perp)^2}{p^-} + i0\big]\big[-k^- + p'^- - \frac{(p'_\perp - k_\perp)^2}{p'^-} + i0\big]}$$



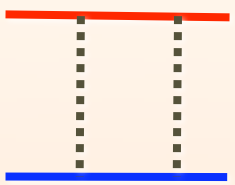
$$\int đ^d k \frac{1}{k_\perp^2 (k_\perp - q_\perp)^2 \big[-k^+ + p^+ - \frac{(q_\perp + p_\perp - k_\perp)^2}{p^-} + i0\big]\big[-k^- + p'^- - \frac{(p'_\perp - k_\perp)^2}{p'^-} + i0\big]}$$

$$\int \frac{dk^+}{2\pi}\ \frac{1}{k^+ + A + i0} = \ ?$$

dim.reg. is irrelevant here

not removed by 0-bin subtraction

$$\int \frac{dk^+}{2\pi}\,\frac{1}{k^+ + A + i0} = \frac{-i}{2}$$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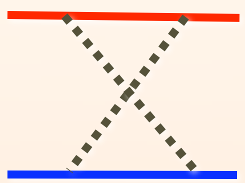


$$= \left(\frac{-i}{2}\right)^2 + \left(\frac{-i}{2}\right)^2 = -\frac{1}{2}$$

Principal Value:

$$\frac{1}{k^+ + i0} = PV\frac{1}{k^+} - i\pi\delta(k^+)$$

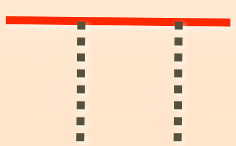
Parity Average:

$$\frac{1}{2}\left[\frac{1}{k^+ + i0} + \frac{1}{-k^+ + i0}\right] = -i\pi\delta(k^+)$$

Cutoff at Large Momentum:

$$\int \frac{dk^+}{2\pi}\frac{\theta(\Lambda^2 - k^{+2})}{k^+ + i0} = \frac{1}{2\pi}\ln\left(\frac{\Lambda + i0}{-\Lambda + i0}\right) = \frac{-i}{2}$$

Add the Two Graphs first: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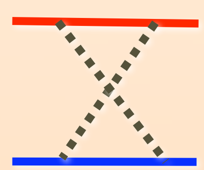
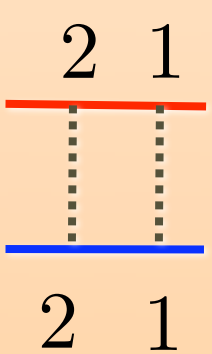


$$\int \frac{dk^+}{2\pi}\left[\frac{1}{k^+ + A + i0} + \frac{1}{-k^+ + B + i0}\right] = \int \frac{dk^+}{2\pi}\frac{(B+A)}{(k^+ + A + i0)(-k^+ + B + i0)} = -i$$

$\frac{1}{2}\Big[$


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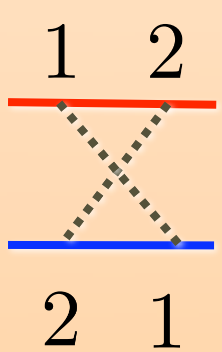

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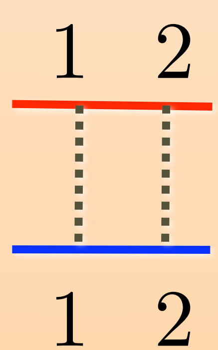

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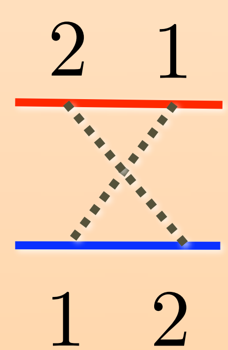
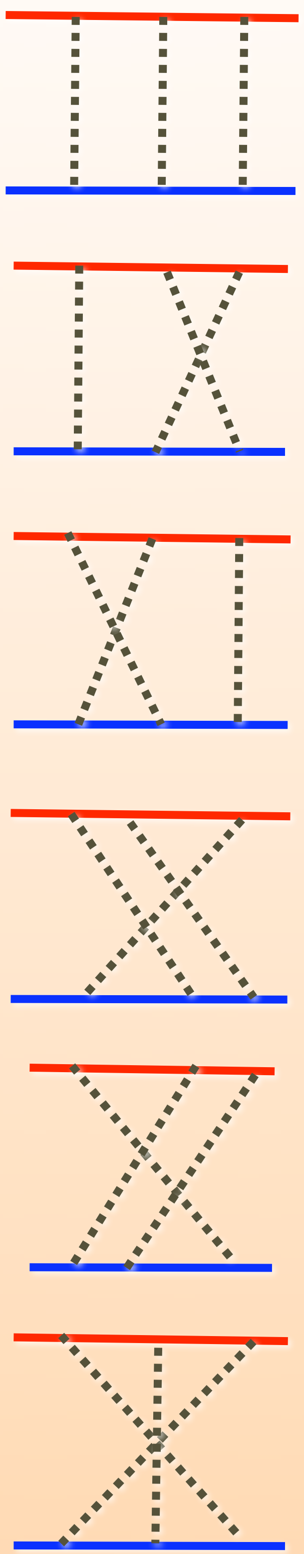


$\Big]$

$$= \frac{1}{2}(-i)^2 = -\frac{1}{2}\int \frac{d^n k_\perp}{(2\pi)^n}\frac{1}{k_\perp^2 (q_\perp - k_\perp)^2}$$

Three Glaubers



new wrinkle: order of integration

$$\int \frac{dk^+ d\ell^+}{(2\pi)^2} \frac{1}{(k^+ + A + i0)(k^+ - \ell^+ + B + i0)} \quad = 0 \text{ ?} \quad = \left(\frac{-i}{2}\right)^2 \text{ ?}$$

neither of these are correct.

Add Graphs first:

plus integrals = $(-i)^2$

avg. over bottom perms, then plus and minus integrals $= \frac{1}{3!}(-i)^4$

sum

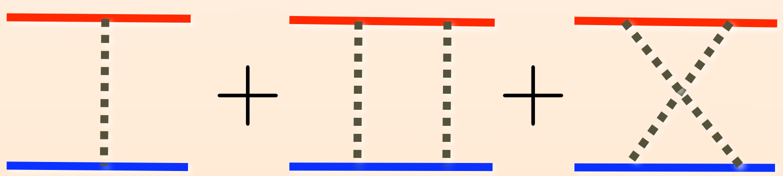
$$= \frac{1}{3!}(-i)^4 \int \frac{d^n k_\perp d^n \ell_\perp}{(2\pi)^{(}2n)} \frac{1}{k_\perp^2 (\ell_\perp - k_\perp)^2 (q_\perp - \ell_\perp)^2}$$

$\perp$ Propagators $\dfrac{1}{q_\perp^2}\,,\ \displaystyle\int \frac{d^n k_\perp}{(2\pi)^n}\frac{1}{k_\perp^2(q_\perp-k_\perp)^2}\,,\ \int \frac{d^n k_\perp d^n \ell_\perp}{(2\pi)^{(}2n)}\frac{1}{k_\perp^2(\ell_\perp-k_\perp)^2(q_\perp-\ell_\perp)^2}$

Fourier transform to de-convolute them

$$i\tilde{\phi}_G = i\,\alpha_s\, e^{\epsilon\gamma_E} 2^{-2\epsilon}\Gamma(-\epsilon)\,\mu^{2\epsilon}|x_\perp|^{2\epsilon}$$

All Glaubers



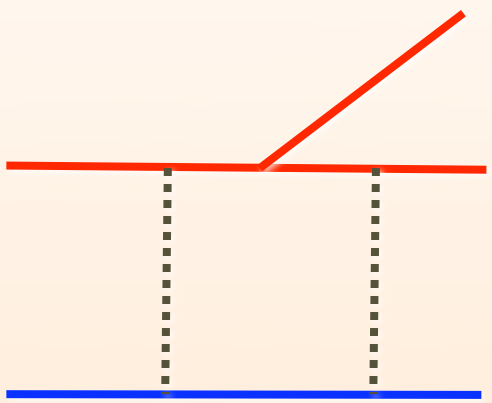
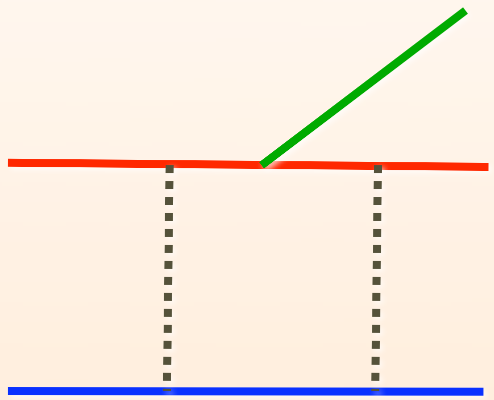
$$+\ldots \;= \sum_{m=0}^{\infty}\frac{1}{(m+1)!}(i\tilde{\phi}_G)^{m+1} = e^{i\tilde{\phi}_G} - 1$$

Forward Scattering Amplitude: $\displaystyle\int d^2x_\perp\; e^{iq_\perp\cdot x_\perp}\Big(e^{i\tilde{\phi}_G(x_\perp)} - 1\Big)$

(well known eikonal result)

Why are these the correct graphs to sum?

What about collinear or usoft radiation?

 $= 0$  $= 0$

same for crossed graphs

same for virtual graphs

same for higher orders

$$\int \frac{dk^+}{2\pi} \frac{1}{(k^+ + A + i0)(k^+ + B + i0)} = 0$$

Radiation does not interfere with the Glauber exponentiation!

Fwd scattering: $e^{i\tilde{\phi}_G} - 1 = i\tilde{\phi}_G - \frac{1}{2}\tilde{\phi}_G^2 + \ldots$

Hard scattering: $H\ e^{i\tilde{\phi}_G}$

Hard Scattering sets a reference time (t=0), distinguishes initial and final state

Provides an "End" for the exponentiation (ie. a graph with no Glauber exchange)

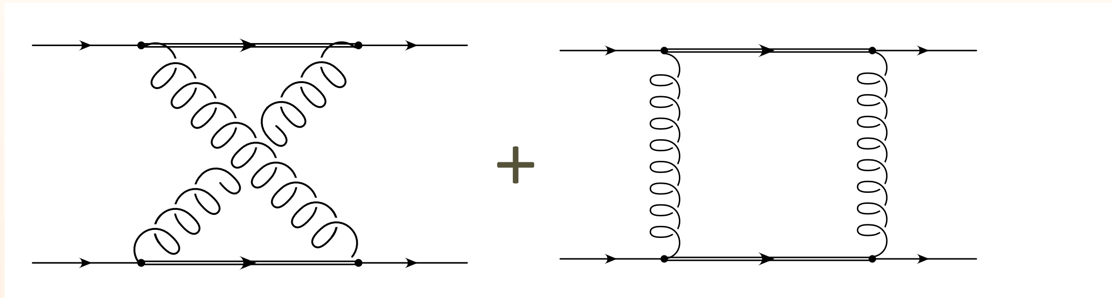
$x_\perp$ space is important. FT of a phase is not a phase.

Nonabelian? Due to the non-abelian exponentiation theorem (Gatheral, Frankel & Taylor) we will get exponentiation.
There are nonabelian corrections to the phase.
It will not be one-loop exact.

How do 0-bins change the abelian calculation?

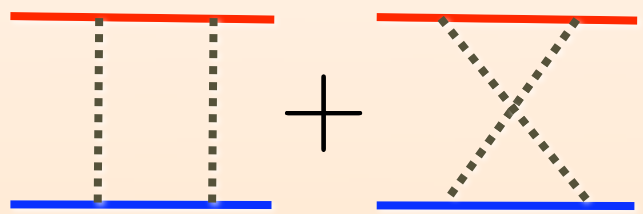
Consider Full Theory matching with 0-bin terms in SCET:

full:



$$= \frac{i\pi}{t}\left[\frac{1}{\epsilon_{\rm IR}} + \ln\frac{\mu^2}{-t}\right]$$

Glauber:



$$\qquad G \qquad\qquad\qquad\qquad -G_U$$

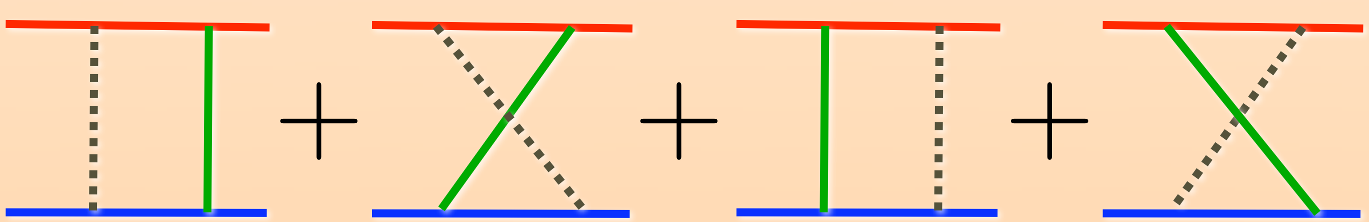
$$= \frac{i\pi}{t}\left[\frac{1}{\epsilon_{\rm IR}} + \ln\frac{\mu^2}{-t}\right] + \frac{i\pi}{t}\left[\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right]$$

$$= \frac{i\pi}{t}\left[\frac{1}{\epsilon_{\rm UV}} + \ln\frac{\mu^2}{-t}\right]$$

counterterms also exponentiate

$$\tilde{\phi}_G \to \phi_G$$

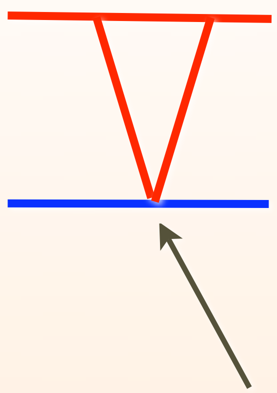
Usoft:



$$= \frac{i\pi}{t}\left[\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right]$$

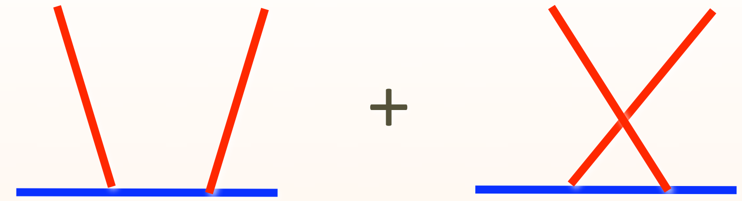
has the IR divergence

Collinear?



no such vertex at LO

(such a vertex would spoil factorization)



$$= \frac{1}{p^+} + \frac{1}{-p^+} = 0$$

zerobin $\quad p^+ \neq 0$

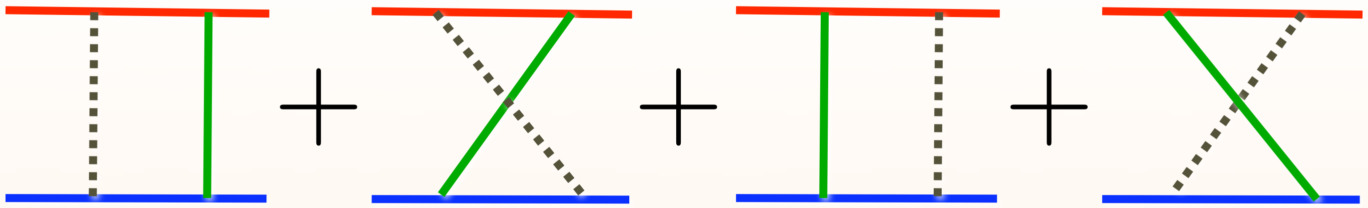
cf. Neubert & Hill

A bit strange from the point of view of the Threshold Expansion.

In that case the full forward scattering box result comes from collinear. (Smirnov)

But in SCET one uses the equations of motion (& momentum conservation) to show that the above vertex is absent.

So there need not be a simple correspondence.

Is  $= \frac{i\pi}{t}\left[\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right]$

consistent with the SCET usoft field redefinition?

Yes: but must be careful with path dependence here.

2
Chay, Kim, Lee
Arnesen, Kundu, IS

$$\mathcal{L}_G = -\sum_{p_\perp, p'_\perp} V(p_\perp, p'_\perp)\bar{\xi}_{n,p'_\perp}\xi_{n,p_\perp}\bar{\xi}_{\bar{n},-p'_\perp}\xi_{\bar{n},-p_\perp}$$

$$\rightarrow \quad \mathcal{L}_G = -\sum_{p_\perp, p'_\perp} V(p_\perp, p'_\perp)\bar{\xi}_{n,p'_\perp} Y_n^{\infty} \xi_{n,p_\perp}\bar{\xi}_{\bar{n},-p'_\perp} Y_{\bar{n}}^{\infty} \xi_{\bar{n},-p_\perp}$$

$$Y_n^{\infty} = P\exp\Big(ig\int_{-\infty}^{+\infty} ds\ n\cdot A_{us}(ns)\Big)$$

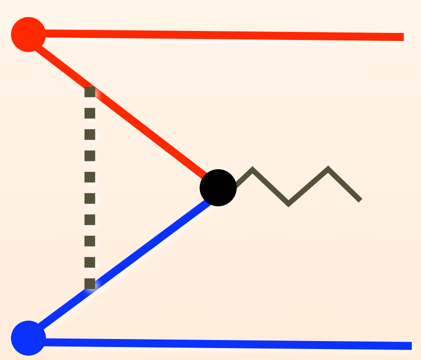
$$Y_{\bar{n}}^{\infty} = P\exp\Big(ig\int_{-\infty}^{+\infty} ds\ \bar{n}\cdot A_{us}(\bar{n}s)\Big)$$

Our Usoft contains a Glauber region and that is fine! When we consider hard scattering, this is a phase that does not cancel out until we square the amplitude.

Hard Scattering, “Ends”, and 0-bins

Hard Scattering, "Ends", and 0-bins

Active - Active



$$G \qquad\qquad G_U$$

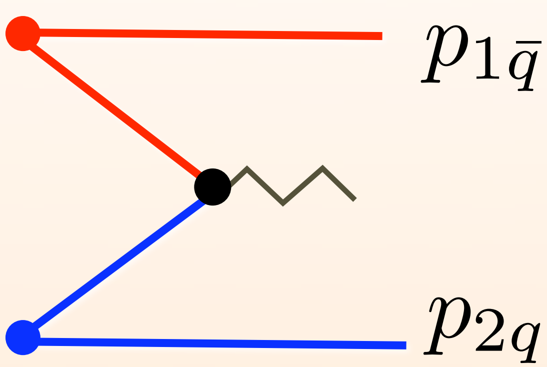
$$\int \frac{dk_\perp^n}{(2\pi)^n}\frac{dk^+dk^-}{(2\pi)^2}\frac{1}{k_\perp^2}\left[\frac{1}{(k^+ + A + i0)(-k^- + B + i0)} - \frac{1}{(k^+ + A' + i0)(-k^- + B' + i0)}\right]$$

has $k_\perp$ — no $k_\perp$

$$= 0$$

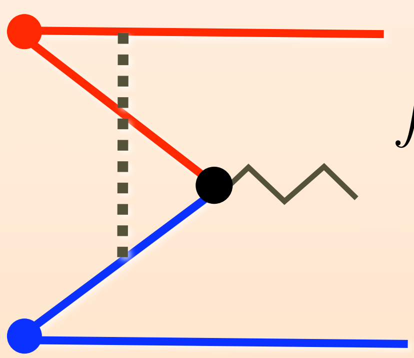
consistent with dropping Glaubers in standard matching computation

Active - Spectator



$$= \left(\frac{p_{1q}^- p_{1\bar{q}}^-}{p_1^-}\right)\left(\frac{p_{2q}^+ p_{2\bar{q}}^+}{p_2^+}\right) \frac{1}{p_{2q}^{\perp 2}} \frac{1}{p_{1\bar{q}}^{\perp 2}}$$

FT $\rightarrow$ $E(x_{1\perp})E(x_{2\perp})$



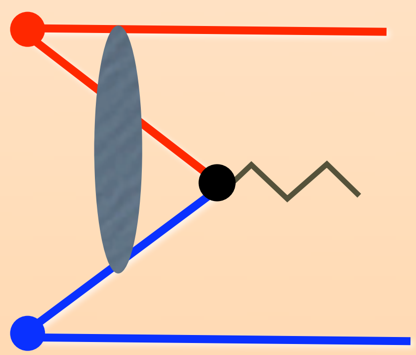
$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k_\perp^2 \left[-k^+ + p_{1\bar{q}}^+ - \frac{(k_\perp^2 - p_{1\bar{q}}^\perp)^2}{p_{1\bar{q}}^-} + i0\right]\left[k^+ + p_{1q}^+ - \frac{(k_\perp^2 + p_{1q}^\perp)^2}{p_{1q}^-} + i0\right]\left[-k^- + p_{2\bar{q}}^- - \frac{(k_\perp - p_{2\bar{q}}^\perp)^2}{p_{1q}^-} + i0\right]}$$

$= (\text{-i}/2)$

$$= \left(\frac{p_{1q}^- p_{1\bar{q}}^-}{p_1^-}\right)\left(\frac{p_{2q}^+ p_{2\bar{q}}^+}{p_2^+}\right) \frac{1}{p_{2q}^{\perp 2}} \int \frac{d^n k_\perp}{(2\pi)^n} \frac{1}{k_\perp^2 (k_\perp - p_{1\bar{q}}^\perp)^2}$$

G_U converts this IR pole to UV just like in fwd. scatt. graphs

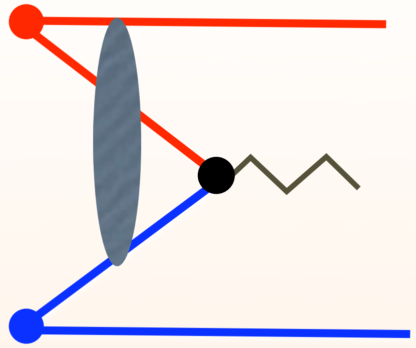
Sum up Glaubers



FT $\rightarrow$ $E(x_{1\perp})E(x_{2\perp})\ \hat{G}(x_{1\perp})$

$$\hat{G}(x_\perp) = e^{i\phi_G(x_\perp)}$$

$$\hat{G}(x_\perp) = e^{i\phi_G(x_\perp)}$$

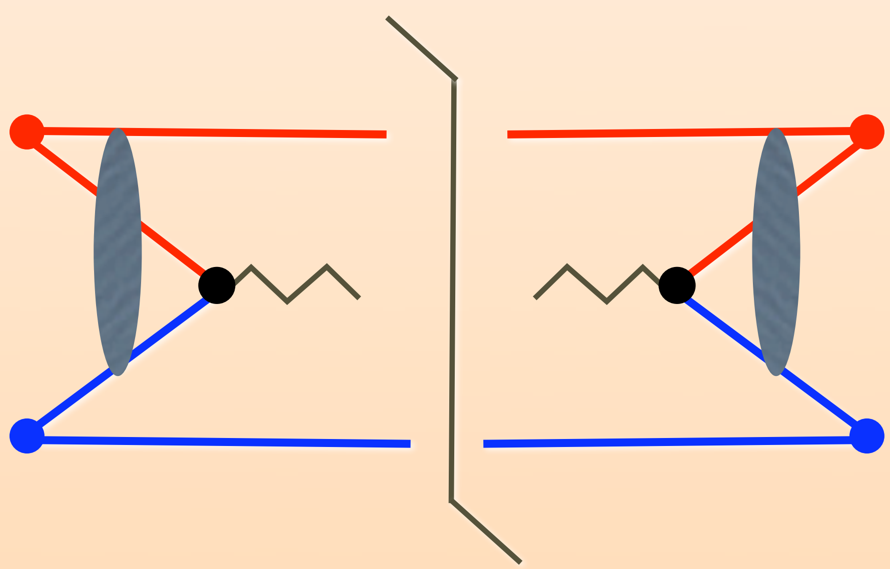


$$= E(x_{1\perp})E(x_{2\perp})\,\hat{G}(x_{1\perp})$$

overall phase

Phase Space: $\int dp_{1\bar{q}}^{\perp 2}\ \left|A(p_{1\bar{q}}^{\perp})\right|^2 = \int dx_{1\perp}^2\ \left|A(x_{1\perp})\right|^2$

(so do not measure $p_{1\bar{q}}^{\perp}$)



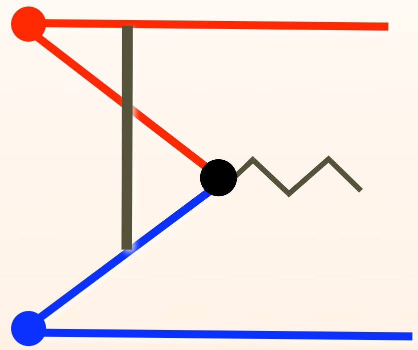
$$\propto \int d^2x_{1\perp}\ \left|\hat{G}(x_{1\perp})\right|^2$$

Glauber phase cancels

Active - Spectator cross check

(basically consistent with Liu & Ma (2010))

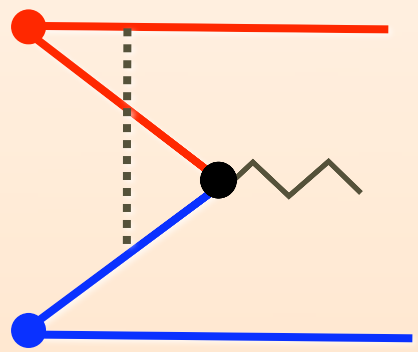
full theory scalar graph



$$\mathrm{Im} \quad = \frac{1}{8\pi}\frac{1}{s't}\left[\frac{1}{\epsilon_{\mathrm{IR}}} + \ln\left(\frac{\mu^2}{p_{1\bar{q}}^{\perp 2}}\right)\right]$$

$$t = p_{1\bar{q}}^{\perp 2}(1 + p_{1q}^-/p_{1\bar{q}}^-)$$

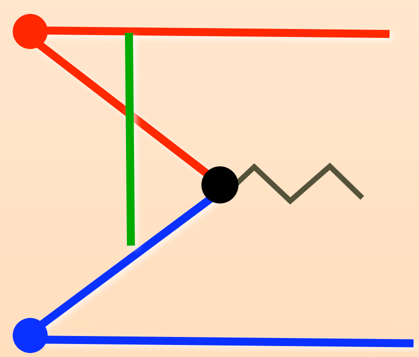
Glauber scalar graph



$$\mathrm{Im} \quad = \frac{1}{8\pi}\frac{1}{s't}\left[\frac{1}{\epsilon_{\mathrm{UV}}} + \ln\left(\frac{\mu^2}{p_{1\bar{q}}^{\perp 2}}\right)\right]$$

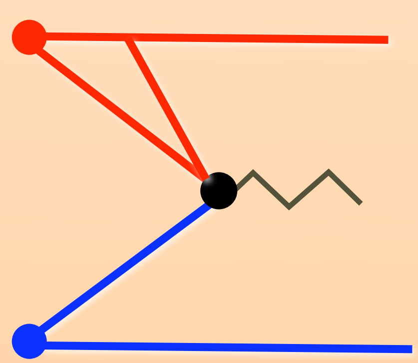
with G_U

Usoft scalar graph



$$\mathrm{Im} \quad = \frac{1}{8\pi}\frac{1}{s't}\left[\frac{1}{\epsilon_{\mathrm{IR}}} - \frac{1}{\epsilon_{\mathrm{UV}}}\right]$$

collinear scalar graph



$$C - C_U - C_G + C_{G_U}$$

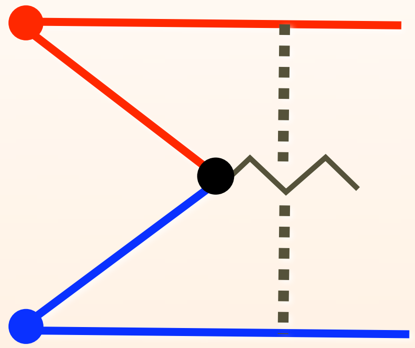
$$\mathrm{Im} = 0$$

Note: a phase in collinear would be bad since it would be hard to see it cancel

Spectator - Spectator

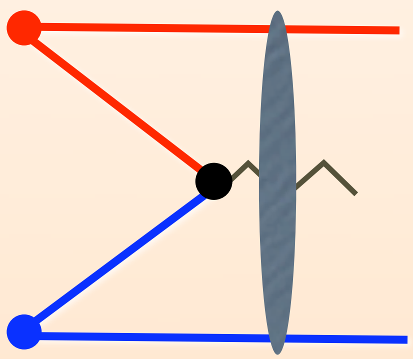
two poles for k^+

two poles for k^-



$$= \left(\frac{p_{1q}^- p_{1\bar{q}}^-}{p_1^-}\right)\left(\frac{p_{2q}^+ p_{2\bar{q}}^+}{p_2^+}\right) \int \frac{d^n k_\perp}{(2\pi)^n} \frac{1}{(k_\perp + p_{2q}^\perp)^2 (k_\perp - p_{1\bar{q}}^\perp)^2}$$

sum glaubers

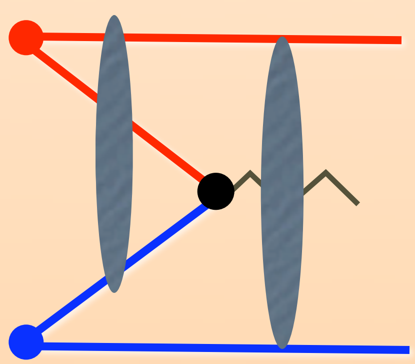


FT

phase again

$$E(x_{1\perp})E(x_{2\perp}) \ \hat{G}(x_{1\perp} - x_{2\perp})$$

Spectator - Spectator & Active - Spectator



FT

$$E(x_{1\perp})E(x_{2\perp}) \ \hat{G}(x_{1\perp} - x_{2\perp}) \ \hat{G}(x_{1\perp})$$

a phase again

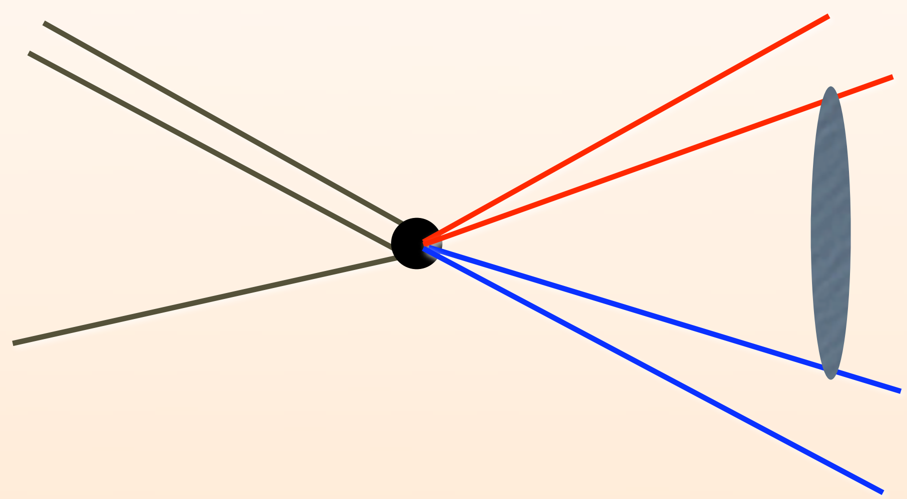
etc.

Continuing in this manner we will get an alternate proof of factorization for inclusive Drell Yan.
(Though recall that my calculations here were abelian.)

Advantage: This method is more readily adapted to determine which measurements on the hadronic final state still allow Glaubers to cancel.

Glaubers in Exclusives?

$B \to \pi\pi$



We factorize the amplitude.

All partons are ACTIVE.

Usoft subtractions are not appropriate here (a mode below confinement scale).

Glaubers sum to phase for active lines $\hat{G}(x_\perp \to 0)$ that is independent of the details of the hard vertex, and still cancel when we square amplitudes.

This is why in SCET that "Regge" effects (Donoghue et.al.) do not spoil factorization at leading power.

Caveats: No where in this talk did I account for rapidity divergences or $\mathrm{SCET_{II}}$ type 0-bin subtractions, which sometimes show up in the exclusive case.

The End